Que. No.	Sub. Que.	Model Answer	Marks	Total Marks
		Important Instructions to the Examiners: 1) The answers should be examined by key words and not as word-to-word as given in themodel answer scheme. 2) The model answer and the answer written by candidate may vary but the examiner may tryto assess the understanding level of the candidate. 3) The language errors such as grammatical, spelling errors should not be given more importance. (Not applicable for subject English and Communication Skills.) 4) While assessing figures, examiner may give credit for principal components indicated in thefigure. The figures drawn by the candidate and those in the model answer may vary. The examiner may give credit for any equivalent figure drawn. 5) Credits may be given step wise for numerical problems. In some cases, the assumed constantvalues may vary and there may be some difference in the candidate's answers and the model answer. 6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidate's understanding. 7) For programming language papers, credit may be given to any other program based on equivalent concept.		

Que. No.	Sub. Que.	Model Answer	Marks	Total Marks
1.		**Attempt any <u>TEN</u> of the following:**		**20**
	a)	Solve $\begin{vmatrix} 1 & x & x^2 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{vmatrix} = \begin{vmatrix} 5 & 4 \\ 5 & 4 \end{vmatrix}$		
	Ans	$1(4-2)-x(4-1)+x^2(2-1)=20-20$	1	
		$2-3x+x^2=0$	½	
		$(x-1)(x-2)=0$		
		$x=1$ or $x=2$	½	**2**
		--		
	b)	If $A=\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix}$, find $2A+3B-4I$,		
		Where I is the unit matrix of order two		
	Ans	$2A+3B-4I=2\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}+3\begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix}-4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$		
		$=\begin{bmatrix} 4 & 6 \\ 8 & 14 \end{bmatrix}+\begin{bmatrix} 3 & 9 \\ 12 & 18 \end{bmatrix}-\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$	1	
		$=\begin{bmatrix} 3 & 15 \\ 20 & 28 \end{bmatrix}$	1	**2**
		--		
	c)	If $A=\begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$ Prove that $A^2-3A=2I$,		
		Where I is the unit matrix of order two		
	Ans	$A^2=AA=\begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}=\begin{bmatrix} 8 & 12 \\ 3 & 5 \end{bmatrix}$	1	
		$3A=\begin{bmatrix} 6 & 12 \\ 3 & 3 \end{bmatrix}$	½	
		$A^2-3A=\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}=2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=2I$	½	**2**
		--		
	d)	If $A=\begin{bmatrix} 1 & 2 \\ 5 & 3 \end{bmatrix}$, $B=\begin{bmatrix} 2 & 6 \\ -3 & 4 \end{bmatrix}$ then verify that $(AB)'=B'A'$		
	Ans	$AB=\begin{bmatrix} 1 & 2 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 2 & 6 \\ -3 & 4 \end{bmatrix}=\begin{bmatrix} -4 & 14 \\ 1 & 42 \end{bmatrix}$	½	

Que. No.	Sub. Que.	Model Answer	Marks	Total Marks
1.		$(AB)' = \begin{bmatrix} -4 & 1 \\ 14 & 42 \end{bmatrix}$	½	
		$B'A' = \begin{bmatrix} 2 & -3 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix}$	½	
		$= \begin{bmatrix} -4 & 1 \\ 14 & 42 \end{bmatrix}$	½	**2**
		--		
	e)	Resolve into partial fraction $\frac{1}{x^2+3x+2}$		
	Ans	$\frac{1}{x^2+3x+2} = \frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$	½	
		$\therefore 1 = A(x+2) + B(x+1)$		
		Put $x=-1$	½	
		$A = 1$		
		Put $x=-2$	½	
		$B = -1$		
		$\therefore \frac{1}{(x+1)(x+2)} = \frac{1}{x+1} + \frac{-1}{x+2}$	½	**2**
		--		
	f)	Prove that $\cos 2\theta = \cos^2\theta - \sin^2\theta$		
	Ans	$\cos 2\theta = \cos(\theta + \theta)$	½	
		$= \cos\theta \cos\theta - \sin\theta \sin\theta$	1	
		$= \cos^2\theta - \sin^2\theta$	½	**2**
		--		
	g)	Without using calculator , find the value of $\sin 15^0$		
	Ans	$\sin 15^0 = \sin(60^0 - 45^0)$		
		$= \sin 60^0 \cos 45^0 - \cos 60^0 \sin 45^0$	1	
		$= \frac{\sqrt{3}}{2} \frac{1}{\sqrt{2}} - \frac{1}{2} \frac{1}{\sqrt{2}}$		
		$= \frac{\sqrt{3}-1}{2\sqrt{2}}$	1	**2**
		OR		
		$\sin 15^0 = \sin(45^0 - 30^0)$		

Que. No.	Sub. Que.	Model Answer	Marks	Total Marks
1.		$=\sin 45^0 \cos 30^0 - \cos 45^0 \sin 30^0$	1	
		$=\frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}\frac{1}{2}$		
		$=\frac{\sqrt{3}-1}{2\sqrt{2}}$	1	**2**
	h)	Prove that $\frac{\tan 420^0+\tan 300^0}{1-\tan 420^0 \tan 660^0}=0$		
	Ans	$\frac{\tan 420^0+\tan 300^0}{1-\tan 420^0 \tan 660^0}$		
		$=\frac{\tan 420^0+\tan 300^0}{1-\tan 420^0 \tan\left(360^0+300^0\right)}$	½	
		$=\frac{\tan 420^0+\tan 300^0}{1-\tan 420^0 \tan 300^0}$	½	
		$=\tan\left(420^0+300\right)$	½	
		$=\tan\left(720^0\right)$	½	**2**
		$=0$		
		OR		
		$\frac{\tan 420^0+\tan 300^0}{1-\tan 420^0 \tan 660^0}$		
		$=\frac{\tan\left(4\times 90^0+60^0\right)+\tan\left(3\times 90^0-30^0\right)}{1-\tan 420^0 \tan 660^0}$	½	
		$=\frac{\tan\left(60^0\right)-\cot\left(30^0\right)}{1-\tan 420^0 \tan 660^0}$	½	
		$=\frac{\sqrt{3}-\sqrt{3}}{1-\tan 420^0 \tan 660^0}$	½	
		$=0$	½	**2**
	i)	If $\sin 80+\sin 50=2\sin A\cos A$ then find A and B		
	Ans	Consider $\sin 80+\sin 50=2\sin A\cos A$	--	**--**
		$\therefore 2\sin 65\cos 15=2\sin A\cos A$		
		In this case we can not find B		
	j)	Prove that: $\sin(n+1)A\sin(n+2)A+\cos(n+1)A\cos(n+2)A=\cos A$		
	Ans	$\sin(n+1)A\sin(n+2)A+\cos(n+1)A\cos(n+2)A$		

Que. No.	Sub. Que.	Model Answer	Marks	Total Marks
1.		$=\cos\left(\left(n+1\right)A-\left(n+2\right)A\right)$	1	
		$=\cos\left(nA+A-nA-2A\right)$		
		$=\cos\left(-A\right)$	½	
		$=\cos A$	½	**2**
		---		
	k)	Find distance between parallel lines $3x+2y-6=0$ and $3x+2y-12=0$		
	Ans	$p=\left\|\dfrac{c_2-c_1}{\sqrt{a^2+b^2}}\right\|$		
		$=\left\|\dfrac{-12-\left(-6\right)}{\sqrt{3^2+2^2}}\right\|$	1	
		$=\left\|\dfrac{-6}{\sqrt{13}}\right\|$		
		$=\dfrac{6}{\sqrt{13}}$	1	**2**
		---		
	l)	Evaluate $\begin{vmatrix}3 & 4 & 2\\ 12 & 16 & 8\\ -5 & -6 & 0\end{vmatrix}$		
	Ans	$\begin{vmatrix}3 & 4 & 2\\ 12 & 16 & 8\\ -5 & -6 & 0\end{vmatrix}$		
		$=3\left(0-\left(-48\right)\right)-4\left(0-\left(-40\right)\right)+2\left(-72-\left(-80\right)\right)$	1	
		$=0$	1	**2**

2.		**Attempt any <u>FOUR</u> of the following:**		**16**
	a)	Solve the following equations by using Cramer's rule		
		$x+y-z=0, 2x+y+3z=9, x-y+z=2$		
	Ans	$D=\begin{vmatrix}1 & 1 & -1\\ 2 & 1 & 3\\ 1 & -1 & 1\end{vmatrix}=1\left(1+3\right)-1\left(2-3\right)-1\left(-2-1\right)=8$	1	
		$D_x=\begin{vmatrix}0 & 1 & -1\\ 9 & 1 & 3\\ 2 & -1 & 1\end{vmatrix}=0\left(1+3\right)-1\left(9-6\right)-1\left(-9-2\right)=8$	½	

Que. No.	Sub. Que.	Model Answer	Marks	Total Marks
2.		$D_y=\begin{vmatrix}1&0&-1\\2&9&3\\1&2&1\end{vmatrix}=1(9-6)-0(2-3)-1(4-9)=8$	½	
		$D_z=\begin{vmatrix}1&1&0\\2&1&9\\1&-1&2\end{vmatrix}=1(2+9)-1(4-9)+0(-2-1)=16$	½	
		$\therefore x=\frac{D_x}{D}=\frac{8}{8}=1$	½	
		$\therefore y=\frac{D_y}{D}=\frac{8}{8}=1$	½	
		$\therefore z=\frac{D_z}{D}=\frac{16}{8}=2$	½	**4**
		--		
	b)	Find x,y,z if $\left\{\begin{bmatrix}1&3&2\\2&0&1\\3&1&2\end{bmatrix}+2\begin{bmatrix}3&0&2\\1&4&5\\2&1&0\end{bmatrix}\right\}\begin{bmatrix}1\\2\\3\end{bmatrix}=\begin{bmatrix}x\\y\\z\end{bmatrix}$		
	Ans	$\left\{\begin{bmatrix}1&3&2\\2&0&1\\3&1&2\end{bmatrix}+2\begin{bmatrix}3&0&2\\1&4&5\\2&1&0\end{bmatrix}\right\}\begin{bmatrix}1\\2\\3\end{bmatrix}=\begin{bmatrix}x\\y\\z\end{bmatrix}$		
		$\left\{\begin{bmatrix}1&3&2\\2&0&1\\3&1&2\end{bmatrix}+\begin{bmatrix}6&0&4\\2&8&10\\4&2&0\end{bmatrix}\right\}\begin{bmatrix}1\\2\\3\end{bmatrix}=\begin{bmatrix}x\\y\\z\end{bmatrix}$	½	
		$\begin{bmatrix}7&3&6\\4&8&11\\7&3&2\end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix}=\begin{bmatrix}x\\y\\z\end{bmatrix}$	1	
		$\begin{bmatrix}7+6+18\\4+16+33\\7+6+6\end{bmatrix}=\begin{bmatrix}x\\y\\z\end{bmatrix}$	1	
		$\begin{bmatrix}31\\53\\19\end{bmatrix}=\begin{bmatrix}x\\y\\z\end{bmatrix}$	1	
		$\therefore x=31,\ y=53,\ z=19$	½	**4**
		--		
	c)	If A = $\begin{bmatrix}2&4&4\\4&2&4\\4&4&2\end{bmatrix}$ show that A^2-8A is scalar matrix		

Que. No.	Sub. Que.	Model Answer	Marks	Total Marks
2.	Ans	$A=\begin{bmatrix}2&4&4\\4&2&4\\4&4&2\end{bmatrix}$		
		$A^2=AA=\begin{bmatrix}2&4&4\\4&2&4\\4&4&2\end{bmatrix}\begin{bmatrix}2&4&4\\4&2&4\\4&4&2\end{bmatrix}$	½	
		$=\begin{bmatrix}4+16+16&8+8+16&8+16+8\\8+8+16&16+4+16&16+8+8\\8+16+8&16+8+8&16+16+4\end{bmatrix}$		
		$=\begin{bmatrix}36&32&32\\32&36&32\\32&32&36\end{bmatrix}$	1	
		$8A=8\begin{bmatrix}2&4&4\\4&2&4\\4&4&2\end{bmatrix}=\begin{bmatrix}16&32&32\\32&16&32\\32&32&16\end{bmatrix}$	½	
		$A^2-8A=\begin{bmatrix}36&32&32\\32&36&32\\32&32&36\end{bmatrix}-\begin{bmatrix}16&32&32\\32&16&32\\32&32&16\end{bmatrix}$	1	
		$=\begin{bmatrix}20&0&0\\0&20&0\\0&0&20\end{bmatrix}$	1	
		$\therefore A^2-8A$ is scalar matrix		**4**
		--		
	d)	If A $=\begin{bmatrix}-2&0&1\\1&2&3\end{bmatrix}, B=\begin{bmatrix}0&1\\2&3\\1&1\end{bmatrix}$ show that the matrix AB is non-singular		
	Ans	AB $=\begin{bmatrix}-2&0&1\\1&2&3\end{bmatrix}\begin{bmatrix}0&1\\2&3\\1&1\end{bmatrix}$		
		$=\begin{bmatrix}0+0+1&-2+0+1\\0+4+3&1+6+3\end{bmatrix}$		
		$=\begin{bmatrix}1&-1\\7&10\end{bmatrix}$	2	
		$AB=\begin{vmatrix}1&-1\\7&10\end{vmatrix}=10-(-7)$	1½	
		$=17$		

Que. No.	Sub. Que.	Model Answer	Marks	Total Marks
2.		$\therefore AB \neq 0$	½	
		$\therefore$ AB is non-singular matrix		**4**
		--		
	e)	Resolve into partial fraction $\dfrac{3x-1}{(x-4)(2x+1)(x-1)}$		
	Ans	$\dfrac{3x-1}{(x-4)(2x+1)(x-1)} = \dfrac{A}{x-4} + \dfrac{B}{2x+1} + \dfrac{C}{x-1}$	½	
		$3x-1 = A(2x+1)(x-1) + B(x-4)(x-1) + C(x-4)(2x+1)$		
		Put $x = 4$		
		$3(4)-1 = A(2(4)+1)(4-1)$		
		$11 = A(9)(3)$		
		$11 = A(27)$		
		$\therefore A = \dfrac{11}{27}$	1	
		Put $x = \dfrac{-1}{2}$		
		$3\left(\dfrac{-1}{2}\right) - 1 = B\left(\dfrac{-1}{2} - 4\right)\left(\dfrac{-1}{2} - 1\right)$		
		$\dfrac{-5}{2} = B\left(\dfrac{-9}{2}\right)\left(\dfrac{-3}{2}\right)$		
		$\dfrac{-5}{2} = B\left(\dfrac{27}{4}\right)$		
		$\therefore B = \dfrac{-10}{27}$	1	
		Put $x = 1$		
		$3(1)-1 = C(1-4)(2(1)+1)$		
		$2 = C(-3)(3)$		
		$\therefore C = \dfrac{-2}{9}$	1	
		$\dfrac{3x-1}{(x-4)(2x+1)(x-1)} = \dfrac{\frac{11}{27}}{x-4} + \dfrac{\frac{-10}{27}}{2x+1} + \dfrac{\frac{-2}{9}}{x-1}$	½	**4**
		--		

Que. No.	Sub. Que.	Model Answer	Marks	Total Marks
2.	f)	Solve by Cramer's rule $x+y+z=6,\ 2x-y+3z=9,\ x+2y+3z=14$		
	Ans	$D=\begin{vmatrix}1&1&1\\2&-1&3\\1&2&3\end{vmatrix}=1(-3-6)-1(6-3)+1(4+1)=-7$	1	
		$D_x=\begin{vmatrix}6&1&1\\9&-1&3\\14&2&3\end{vmatrix}=6(-3-6)-1(27-42)+1(18+14)=-7$	½	
		$D_y=\begin{vmatrix}1&6&1\\2&9&3\\1&14&3\end{vmatrix}=1(27-42)-6(6-3)+1(28-9)=-14$	½	
		$D_z=\begin{vmatrix}1&1&6\\2&-1&9\\1&2&4\end{vmatrix}=1(-14-18)-1(28-9)+6(4+1)=-21$	½	
		$\therefore x=\dfrac{D_x}{D}=\dfrac{-7}{-7}=1$	½	
		$\therefore y=\dfrac{D_y}{D}=\dfrac{-14}{-7}=2$ -	½	
		$\therefore z=\dfrac{D_z}{D}=\dfrac{-21}{-7}=3$	½	**4**
		--		
3.		**Attempt any FOUR of the following:**		**16**
	a)	Using matrix inversion method solve the following equations : $x+3y+2z=6$, $3x-2y+5z=5$, $2x-3y+6z=7$		
	Ans	$Let\ A=\begin{bmatrix}1&3&2\\3&-2&5\\2&-3&6\end{bmatrix}$		
		$\|A\|=1(-12+15)-3(18-10)+2(-9+4)$ $\|A\|=3-24-10$ $\therefore \|A\|=-31\neq 0$ $\therefore A^{-1}$ exists	½	
		Matrix of minors $=\begin{bmatrix}\begin{vmatrix}-2&5\\-3&6\end{vmatrix}&\begin{vmatrix}3&5\\2&6\end{vmatrix}&\begin{vmatrix}3&-2\\2&-3\end{vmatrix}\\\begin{vmatrix}3&2\\-3&6\end{vmatrix}&\begin{vmatrix}1&2\\2&6\end{vmatrix}&\begin{vmatrix}1&3\\2&-3\end{vmatrix}\\\begin{vmatrix}3&2\\-2&5\end{vmatrix}&\begin{vmatrix}1&2\\3&5\end{vmatrix}&\begin{vmatrix}1&3\\3&-2\end{vmatrix}\end{bmatrix}$		

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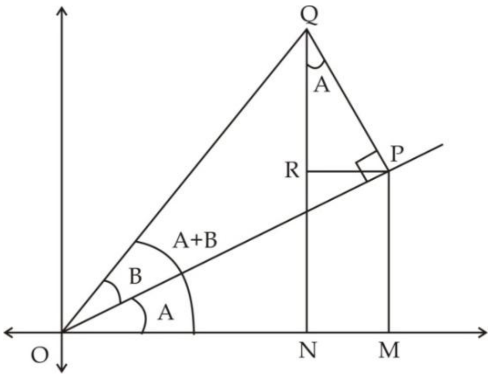
3.		Matrix of minors $=\begin{bmatrix}3&8&-5\\24&2&-9\\19&-1&-11\end{bmatrix}$		
		Matrix of cofactors $=\begin{bmatrix}3&-8&-5\\-24&2&9\\19&1&-11\end{bmatrix}$	1½	
		$\text{Adj}.A=\begin{bmatrix}3&-24&19\\-8&2&1\\-5&9&-11\end{bmatrix}$	½	
		$A^{-1}=\frac{1}{\|A\|}\text{Adj}.A$		
		$A^{-1}=\frac{1}{-31}\begin{bmatrix}3&-24&19\\-8&2&1\\-5&9&-11\end{bmatrix}$	½	
		$\therefore\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{-31}\begin{bmatrix}3&-24&19\\-8&2&1\\-5&9&-11\end{bmatrix}\begin{bmatrix}6\\5\\7\end{bmatrix}$		
		$\therefore\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{-31}\begin{bmatrix}18-120+133\\-48+10+7\\-30+45-77\end{bmatrix}$		
		$\therefore\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{-31}\begin{bmatrix}31\\-31\\-62\end{bmatrix}$	½	
		$\therefore\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-1\\1\\2\end{bmatrix}$		
		$\therefore\ x=-1,\ y=1,\ z=2$	½	**4**
		--		
	b)	Resolve into partial fractions $\frac{2x-3}{(x+1)(x^2+4)}$		
	Ans	$\frac{2x-3}{(x+1)(x^2+4)}=\frac{A}{x+1}+\frac{Bx+C}{x^2+4}$		
		$\therefore 2x-3=(x^2+4)A+(x+1)(Bx+C)$	½	
		$Put\ x=-1$		
		$\therefore 2(-1)-3=\left((-1)^2+4\right)A+0$		

Que. No.	Sub. Que.	Model Answer	Marks	Total Marks
3.		$\therefore -5=5A$		
		$\therefore \boxed{A=-1}$	1	
		Put $x=0$		
		$\therefore 2(0)-3=\left((0)^2+4\right)A+\left((0)+1\right)\left(B(0)+C\right)$		
		$\therefore -3=4A+C$		
		$\therefore -3=-4+C$		
		$\therefore \boxed{C=1}$	1	
		Put $x=1$		
		$\therefore 2(1)-3=\left(1^2+4\right)A+(1+1)\left(B(1)+C\right)$		
		$\therefore -1=5A+2B+2C$		
		$\therefore -1=-5+2B+2$		
		$\therefore \boxed{B=1}$	1	
		$\therefore \boxed{\dfrac{2x-3}{(x+1)\left(x^2+4\right)}=\dfrac{-1}{x+1}+\dfrac{x+1}{x^2+4}}$	½	**4**
		--		
	c)	Resolve into partial fractions $\dfrac{5\cos x-3}{(\cos x+1)(\cos x-3)}$		
	Ans	Put $\cos x=t$	½	
		$\dfrac{5\cos x-3}{(\cos x+1)(\cos x-3)}=\dfrac{5t-3}{(t+1)(t-3)}=\dfrac{A}{(t+1)}+\dfrac{B}{(t-3)}$	½	
		$\therefore 5t-3=(t-3)A+(t+1)B$		
		Put $t=-1$		
		$\therefore 5(-1)-3=(-1-3)A+(-1+1)B$		
		$\therefore -8=-4A+0$		
		$\therefore \boxed{A=2}$	1	
		Put $t=3$		
		$\therefore 5(3)-3=(3-3)A+(3+1)B$		
		$\therefore 12=0+4B$		
		$\therefore \boxed{3=B}$	1	
		$\dfrac{5t-3}{(t+1)(t-3)}=\dfrac{2}{(t+1)}+\dfrac{3}{(t-3)}$	½	**4**
		$\dfrac{5\cos x-3}{(\cos x+1)(\cos x-3)}=\dfrac{2}{(\cos x+1)}+\dfrac{3}{(\cos x-3)}$	½	
		--		

Que. No.	Sub. Que.	Model Answer	Marks	Total Marks
3.	d)	Resolve into partial fractions $\frac{x^4}{x^3+1}$		
	Ans	$x^3+1 \overline{) x^4 }$ quotient x $x^4 + x$ $-\quad -$ $-x$		
		$\frac{x^4}{x^3+1} = x - \frac{x}{x^3+1}$		
		$\frac{x}{x^3+1} = \frac{x}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$	½	
		$\therefore x = (x^2-x+1)A + (x+1)(Bx+C)$ Put $x=-1$ $\therefore -1 = ((-1)^2-(-1)+1)A + (-1+1)(B(-1)+C)$ $\therefore -1 = 3A$		
		$\therefore \boxed{A = -\frac{1}{3}}$	1	
		Put $x=0$ $\therefore 0 = (0^2-0+1)A + (0+1)(B(0)+C)$ $\therefore 0 = A + C$ $\therefore 0 = -\frac{1}{3} + C$		
		$\therefore \boxed{C = \frac{1}{3}}$	1	
		Put $x=1$ $\therefore 1 = (1^2-1+1)A + (1+1)(B+C)$ $\therefore 1 = A + 2B + 2C$ $\therefore 1 = -\frac{1}{3} + 2B + \frac{2}{3}$		
		$\therefore \boxed{B = \frac{1}{3}}$	1	
		$\frac{x}{x^3+1} = \frac{-\frac{1}{3}}{x+1} + \frac{\frac{1}{3}x + \frac{1}{3}}{x^2-x+1}$	½	**4**
		--		

Que. No.	Sub. Que.	Model Answer	Marks	Total Marks
3.	e)	Find inverse of the matrix $A=\begin{bmatrix}1&2&-2\\-1&3&0\\0&-2&1\end{bmatrix}$		
	Ans	$A=\begin{bmatrix}1&2&-2\\-1&3&0\\0&-2&1\end{bmatrix}$		
		$\lvert A\rvert=\begin{vmatrix}1&2&-2\\-1&3&0\\0&-2&1\end{vmatrix}=1(3-0)-2(-1-0)-2(2-0)$		
		$\therefore \lvert A\rvert=1\neq 0$ $\therefore A^{-1}$ exists	1	
		Matrix of minors $=\begin{bmatrix}\begin{vmatrix}3&0\\-2&1\end{vmatrix}&\begin{vmatrix}-1&0\\0&1\end{vmatrix}&\begin{vmatrix}-1&3\\0&-2\end{vmatrix}\\\begin{vmatrix}2&-2\\-2&1\end{vmatrix}&\begin{vmatrix}1&-2\\0&1\end{vmatrix}&\begin{vmatrix}1&2\\0&-2\end{vmatrix}\\\begin{vmatrix}2&-2\\3&0\end{vmatrix}&\begin{vmatrix}1&-2\\-1&0\end{vmatrix}&\begin{vmatrix}1&2\\-1&3\end{vmatrix}\end{bmatrix}$		
		$=\begin{bmatrix}3&-1&2\\-2&1&-2\\6&-2&5\end{bmatrix}$	½	
		Matrix of cofactors $=\begin{bmatrix}3&1&2\\2&1&2\\6&2&5\end{bmatrix}$	1½	
		$\therefore \text{adj}(A)=\begin{bmatrix}3&2&6\\1&1&2\\2&2&5\end{bmatrix}$		
		$\therefore A^{-1}=\frac{1}{\lvert A\rvert}\text{adj}(A)$		
		$=\frac{1}{1}\begin{bmatrix}3&2&6\\1&1&2\\2&2&5\end{bmatrix}$	1	
		$\therefore A^{-1}=\begin{bmatrix}3&2&6\\1&1&2\\2&2&5\end{bmatrix}$		**4**
		--		

Que. No.	Sub. Que.	Model Answer	Marks	Total Marks
3.	f)	If I is an unit matrix of order 3 and $A=\begin{bmatrix}1&2&6\\7&4&10\\1&3&5\end{bmatrix}$		
	Ans	$A^2=AA$		
		$=\begin{bmatrix}1&2&6\\7&4&10\\1&3&5\end{bmatrix}\begin{bmatrix}1&2&6\\7&4&10\\1&3&5\end{bmatrix}$		
		$=\begin{bmatrix}21&28&56\\45&60&132\\27&29&61\end{bmatrix}$	1	
		$3A=3\begin{bmatrix}1&2&6\\7&4&10\\1&3&5\end{bmatrix}=\begin{bmatrix}3&6&18\\21&12&30\\3&9&15\end{bmatrix}$	½	
		$A^2-3A+I=\begin{bmatrix}21&28&56\\45&60&132\\27&29&61\end{bmatrix}-\begin{bmatrix}3&6&18\\21&12&30\\3&9&15\end{bmatrix}+\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$	1	
		$=\begin{bmatrix}19&22&38\\24&49&102\\24&20&47\end{bmatrix}$	1½	**4**
		--		

Que. No.	Sub. Que.	Model Answer	Marks	Total Marks
4.		**Attempt any <u>FOUR</u> of the following:**		**16**
	a)	Prove that $\cos(A+B)=\cos A\cos B-\sin A\sin B.$		
	Ans		1	

Right AngledTriangle	Acute Angle	Trigonometric Ratios
Δ OMP	$\angle$MOP = A	$\sin A=\frac{PM}{OP},\quad \cos A=\frac{OM}{OP}$
Δ OPQ	$\angle$POQ = B	$\sin B=\frac{PQ}{OQ},\quad \cos B=\frac{OP}{OQ}$
Δ PRQ	$\angle$PQR = A	$\sin A=\frac{PR}{PQ},\quad \cos A=\frac{QR}{PQ}$
Δ ONQ	$\angle$NOQ = A+B	$\cos(A+B)=\frac{ON}{OQ}$

$$\cos(A+B)=\frac{ON}{OQ}$$ 1

$$=\frac{OM-MN}{OQ}$$

$$=\frac{OM-PR}{OQ}$$

$$=\frac{OM}{OQ}-\frac{PR}{OQ}$$ 1

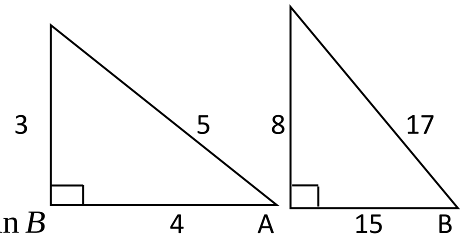
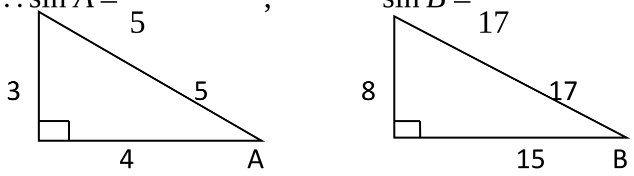
$$=\frac{OM}{OP}\times\frac{OP}{OQ}-\frac{PR}{PQ}\times\frac{PQ}{OQ}$$ 1

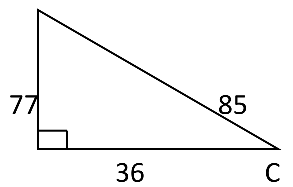
$$=\cos A\cos B-\sin A\sin B$$

Total Marks: 4

Que. No.	Sub. Que.	Model Answer	Marks	Total Marks
4.		**Note:** The above is proved by different ways in several books. Consider all these proof but check whether the method is falling within the scope of curriculum and give appropriate marks in accordance with the scheme of marking. In accordance with the Teacher's Manual published by MSBTE, the result is treated as Fundamental Result which is not proved by the help of any another result. If the above result is proved by students using any another result, suppose using cos (A+B), then this result i.e., cos (A+B) must have been proved first.		
	b)	Prove that $\cos(3A) = 4\cos^3 A - 3\cos A$		
	Ans	$\cos(3A) = \cos(2A + A)$		
		$= \cos 2A\cos A - \sin 2A\sin A$	1	
		$= (2\cos^2 A - 1)\cos A - (2\sin A\cos A)\sin A$	½	
		$= 2\cos^3 A - \cos A - 2\cos A\sin^2 A$	½	
		$= 2\cos^3 A - \cos A - 2\cos A(1 - \cos^2 A)$	½	
		$= 2\cos^3 A - \cos A - 2\cos A + 2\cos^3 A$	½	
		$= 4\cos^3 A - 3\cos A.$	1	**4**
	c)	Without using calculator show that $\cos 20°\cos 40°\cos 60°\cos 80° = \frac{1}{16}$		
	Ans	$\cos 20°\cos 40°\cos 60°\cos 80°$		
		$= \frac{1}{2}(2\cos 20°\cos 40°).\left(\frac{1}{2}\right)\cos 80°$	½	
		$= \frac{1}{4}\left[\cos(20° + 40°) + \cos(20° - 40°)\right]\cos 80°$	1	
		$= \frac{1}{4}\left[\cos(60°) + \cos(-20°)\right]\cos 80°$		
		$= \frac{1}{4}\left[\frac{1}{2}\cos 80° + \cos 20°\cos 80°\right]$	½	
		$= \frac{1}{4}\left[\frac{1}{2}\cos 80° + \frac{1}{2}(2\cos 20°\cos 80°)\right]$		
		$= \frac{1}{8}\left[\cos 80° + \cos(20° + 80°) + \cos(20° - 80°)\right]$	1	
		$= \frac{1}{8}\left[\cos 80° + \cos(180° - 80^0) + \cos(-60°)\right]$		

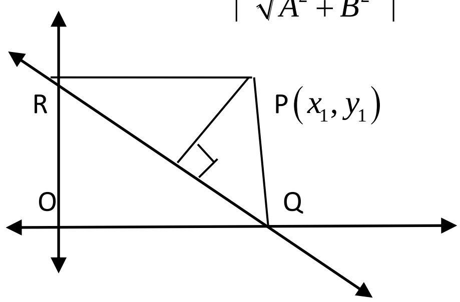
Que. No.	Sub. Que.	Model Answer	Marks	Total Marks
4.		$=\frac{1}{8}\left[\cos 80° - \cos\left(80°\right) + \frac{1}{2}\right]$	½	
		$=\frac{1}{16}$	½	**4**
	d)	Without using calculator show that $\frac{\sin 19° + \cos 11°}{\cos 19° - \sin 11°} = \sqrt{3}$		
	Ans	$\sin 19° = \sin\left(\frac{\pi}{2} - 71°\right) = \cos 71°$	½	
		$\sin 11° = \sin\left(\frac{\pi}{2} - 79°\right) = \cos 79°$	½	
		$\frac{\sin 19° + \cos 11°}{\cos 19° - \sin 11°} = \frac{\cos 71° + \cos 11°}{\cos 19° - \cos 79°}$		
		$= \frac{2\cos\left(\frac{71° + 11°}{2}\right)\cos\left(\frac{71° - 11°}{2}\right)}{2\sin\left(\frac{19° + 79°}{2}\right)\sin\left(\frac{79° - 19°}{2}\right)}$	1	
		$= \frac{2\cos\left(41°\right)\cos\left(30°\right)}{2\sin\left(49°\right)\sin\left(30°\right)}$	1	
		$= \frac{\cos\left(41°\right)\left(\frac{\sqrt{3}}{2}\right)}{\cos\left(41°\right)\left(\frac{1}{2}\right)}$	½	**4**
		$= \sqrt{3}$	½	
	e)	Prove that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$		
	Ans	$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2}\cdot\frac{1}{3}}\right)$	1	
		$= \tan^{-1}\left(\frac{\frac{5}{6}}{1 - \frac{1}{6}}\right)$		

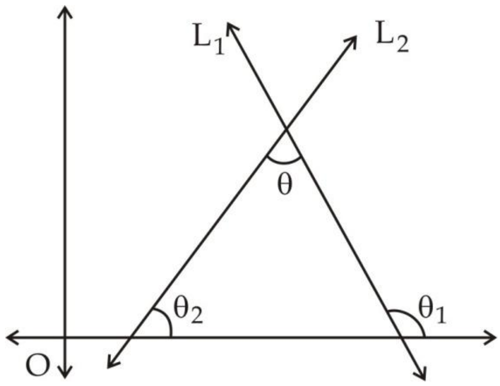
Que. No.	Sub. Que.	Model Answer	Marks	Total Marks
4.		$\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)=\tan^{-1}\left(\frac{\frac{5}{6}}{\frac{5}{6}}\right)$	1	
		$=\tan^{-1}(1)$	1	
		$=\frac{\pi}{4}$	1	**4**
		--		
	f)	Prove that $\sin^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{8}{17}\right)=\sin^{-1}\left(\frac{77}{85}\right)$		
	Ans	Let $A=\sin^{-1}\left(\frac{3}{5}\right)$, $B=\sin^{-1}\left(\frac{8}{17}\right)$		
		$\therefore \sin A=\frac{3}{5}$, $\sin B=\frac{8}{17}$		
		$\cos A=\frac{4}{5},\cos B=\frac{15}{17}$ 	1	
		$\therefore \sin(A+B)=\sin A\cos B+\cos A\sin B$		
		$=\frac{3}{5}\times\frac{15}{17}+\frac{4}{5}\times\frac{8}{17}$	2	
		$=\frac{45}{85}+\frac{32}{85}$		
		$\sin(A+B)=\frac{77}{85}$		
		$\therefore A+B=\sin^{-1}\left(\frac{77}{85}\right)$	½	**4**
		$\therefore \sin^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{8}{17}\right)=\sin^{-1}\left(\frac{77}{85}\right)$	½	
		OR		
		$A=\sin^{-1}\left(\frac{3}{5}\right)$, $B=\sin^{-1}\left(\frac{8}{17}\right)$		
		$\therefore \sin A=\frac{3}{5}$, $\sin B=\frac{8}{17}$	½	
				

Que. No.	Sub. Que.	Model Answer	Marks	Total Marks
4.		$\tan A=\frac{3}{4}$, $\tan B=\frac{8}{15}$		
		$\therefore A=\tan^{-1}\left(\frac{3}{4}\right)$, $B=\tan^{-1}\left(\frac{8}{15}\right)$	½	
		$\therefore \sin^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{3}{4}\right)$, $\sin^{-1}\left(\frac{8}{17}\right)=\tan^{-1}\left(\frac{8}{15}\right)$		
		$\therefore \sin^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{8}{17}\right)$		
		$=\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{8}{15}\right)$		
		$=\tan^{-1}\left(\frac{\frac{3}{4}+\frac{8}{15}}{1-\frac{3}{4}\cdot\frac{8}{15}}\right)$	1	
		$=\tan^{-1}\left(\frac{\frac{45+32}{60}}{\frac{60-24}{60}}\right)$		
		$=\tan^{-1}\left(\frac{77}{36}\right)$	½	
		$Let\ \tan^{-1}\left(\frac{77}{36}\right)=C$		
		$\therefore \tan C=\frac{77}{36}$	½	
		$\therefore \sin C=\frac{77}{85}$		
		$\therefore C=\sin^{-1}\left(\frac{77}{85}\right)$		
		$\therefore \tan^{-1}\left(\frac{77}{36}\right)=\sin^{-1}\left(\frac{77}{85}\right)$		
		$\therefore \sin^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{8}{17}\right)=\sin^{-1}\left(\frac{77}{85}\right)$	1	**4**
				
		--		

Que. No.	Sub. Que.	Model Answer	Marks	Total Marks
5.		**Attempt any FOUR of the following:**		**16**
	a)	Prove that $\cos\left(\frac{\pi}{2}+\theta\right)=-\cos\theta$		
	Ans	$\cos\left(\frac{\pi}{2}+\theta\right)$		
		$=\cos\frac{\pi}{2}\cos\theta-\sin\frac{\pi}{2}\sin\theta$	2	
		$=0\cos\theta-(1)\sin\theta$	1	
		$=-\sin\theta$	1	**4**
		--		
	b)	Prove that $\frac{\sin A+2\sin 2A+\sin 3A}{\cos A+2\cos 2A+\cos 3A}=\tan(2A)$		
	Ans	$\frac{\sin A+2\sin 2A+\sin 3A}{\cos A+2\cos 2A+\cos 3A}$		
		$=\frac{(\sin A+\sin 3A)+2\sin 2A}{(\cos A+\cos 3A)+2\cos 2A}$	½	
		$=\frac{2\sin(2A)\cos(A)+2\sin 2A}{2\cos(2A)\cos(A)+2\cos 2A}$	2	
		$=\frac{2\sin(2A)\left[\cos(A)+1\right]}{2\cos(2A)\left[\cos(A)+1\right]}$	1	
		$=\tan(2A)$	½	**4**
		--		
	c)	Prove that $\frac{\sin 7x+\sin x}{\cos 5x-\cos 3x}=\sin 2x-\cos 2x\cot x$		
	Ans	$\frac{\sin 7x+\sin x}{\cos 5x-\cos 3x}=\frac{\sin 7x+\sin x}{\cos 5x-\cos 3x}$		
		$=\frac{2\sin(4x)\cos(3x)}{2\sin(4x)\sin(-x)}$	1	
		$=\frac{\cos(2x+x)}{-\sin x}$	1	
		$=\frac{\cos(2x)\cos x-\sin(2x)\sin x}{-\sin x}$	1	
		$=\sin 2x-\cos 2x\cot x$	1	**4**
		--		

Que. No.	Sub. Que.	Model Answer	Marks	Total Marks
5.	d)	Prove that $\frac{\sin 9\theta}{\sin 3\theta}-\frac{\cos 9\theta}{\cos 3\theta}=2$		
	Ans	$\frac{\sin 9\theta}{\sin 3\theta}-\frac{\cos 9\theta}{\cos 3\theta}=\frac{\sin(9\theta)\cos(3\theta)-\cos(9\theta)\sin(3\theta)}{\sin(3\theta)\cos(3\theta)}$	1	
		$=\frac{\sin(9\theta-3\theta)}{\sin(3\theta)\cos(3\theta)}$	1	
		$=\frac{\sin(6\theta)}{\sin(3\theta)\cos(3\theta)}$	1	
		$=\frac{2\sin(3\theta)\cos(3\theta)}{\sin(3\theta)\cos(3\theta)}$		
		$=2$	1	**4**
	e)	Prove that $\cos C+\cos D=2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$		
	Ans	We know that,		
		$\cos(A+B)+\cos(A-B)=2\cos A\cos B$	1	
		Put $A+B=C$		
		$A-B=D$		
		$\therefore A=\frac{C+D}{2}$ and	1	
		$B=\frac{C-D}{2}$	1	
		$\therefore \cos C+\cos D=2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$	1	**4**
	f)	If $x>0, y>0$, then prove that $\tan^{-1}x-\tan^{-1}y=\tan^{-1}\left[\frac{x-y}{1+xy}\right]$		
	Ans	Let $\tan^{-1}x=A$ & $\tan^{-1}y=B$		
		$\therefore x=\tan A$ $\therefore y=\tan B$	1	
		Now $\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$	1	
		$\tan(A-B)=\frac{x-y}{1+xy}$	½	
		$(A-B)=\tan^{-1}\left[\frac{x-y}{1+xy}\right]$	½	
		$\therefore \tan^{-1}x-\tan^{-1}y=\tan^{-1}\left[\frac{x-y}{1+xy}\right]$	1	**4**

Que. No.	Sub. Que.	Model Answer	Marks	Total Marks
6.		**Attempt any FOUR of the following:**		16
	a)	If $P(x_1, y_1)$ be any point outside the line $ax+by+c=0,$ then prove that perpendicular distance from the point to the line is $d=\left\|\frac{ax_1+by_1+c}{\sqrt{A^2+B^2}}\right\|$		
	Ans	Let Q$\left(\frac{-c}{a},0\right)$ and R$\left(0,\frac{-c}{b}\right)$ 	½	
		$A(\Delta PQR)=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ \frac{-c}{a} & 0 & 1 \\ 0 & \frac{-c}{b} & 1 \end{vmatrix}=\frac{1}{2}\left[x_1\left(0+\frac{c}{b}\right)-y_1\left(\frac{-c}{a}-0\right)+1\left(\frac{c^2}{ab}\right)\right]$	½	
		$=\frac{1}{2}\left[\frac{x_1c}{b}+\frac{y_1c}{a}+\frac{c^2}{ab}\right]$ $=\frac{1}{2}\frac{c}{ab}(ax_1+by_1+c)$	1	
		$d(QR)=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}=\sqrt{\left(\frac{-c}{a}-0\right)^2+\left(0+\frac{c}{b}\right)^2}$	½	
		$=\sqrt{\frac{c^2}{a^2}+\frac{c^2}{b^2}}$ $=\sqrt{\frac{b^2c^2+a^2c^2}{a^2b^2}}$ $=\frac{c}{ab}\sqrt{a^2+b^2}$	½	
		$A(\Delta PQR)=\frac{1}{2}\times d(QR)\times PM$ $=\frac{1}{2}\times\frac{c}{ab}\sqrt{a^2+b^2}\times PM$		
		$\therefore \frac{1}{2}\frac{c}{ab}(ax_1+by_1+c)=\frac{1}{2}\times\frac{c}{ab}\sqrt{a^2+b^2}\times PM$	½	
		$\therefore PM=\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}$		
		$\therefore PM=\left\|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right\|$ $\because$ distance is positive	½	4
		--		

Que. No.	Sub. Que.	Model Answer	Marks	Total Marks
6.	b)	If m_1 and m_2 are the slope of two lines then prove that angle between two lines is $\theta = \tan^{-1}\left\|\frac{m_1-m_2}{1+m_1m_2}\right\|$		
	Ans			
		Let θ_1 = Inclination of L_1 θ_2 = Inclination of L_2 $\therefore$ Slope of L_1 is $m_1 = \tan\theta_1$ Slope of L_2 is $m_2 = \tan\theta_2$	1	
		from figure, $\theta = \theta_1 - \theta_2$	½	
		$\therefore \tan\theta = \tan(\theta_1-\theta_2)$ $= \frac{\tan(\theta_1)-\tan(\theta_2)}{1+\tan(\theta_1)\tan(\theta_2)}$	1	
		$\tan\theta = \frac{m_1-m_2}{1+m_1\cdot m_2}$	½	
		θ is acute, $\therefore \tan\theta = \left\|\frac{m_1-m_2}{1+m_1\cdot m_2}\right\|$ $\therefore \theta = \tan^{-1}\left\|\frac{m_1-m_2}{1+m_1\cdot m_2}\right\|$	1	4
		--		
	c)	Find the length of perpendicular on the line $3x+4y-6=0$ from the point $(3,4)$		
	Ans	Let $L = 3x+4y-6=0$, point $(x_1,y_1)=(3,4)$ $a=3, b=4, c=-6$	1	
		length of perpendicular from the point to the line is		

Que. No.	Sub. Que.	Model Answer	Marks	Total Marks
6.		$d=\left\|\dfrac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right\|$		
		$=\left\|\dfrac{3(3)+4(4)-6}{\sqrt{3^2+4^2}}\right\|$	2	
		$=\dfrac{19}{5}$ or 3.8	1	**4**
		--		
	d)	Find the equation of straight line passing through the point of intersection of lines $4x+3y=8$ and $x+y=1$ and parallel to the line $5x-7y=3$		
	Ans	$\therefore 4x+3y=8$		
		$x+y=1$		
		$\therefore \quad 4x+3y=8$		
		$-\ 4x+4y=4$		
		$-y=4$	1	
		$y=-4$		
		$\therefore \quad x-4=1$	½	
		$\therefore x=5$		
		$\therefore$ Point of intersection $=(5,-4)$		
		Slope of the line $5x-7y=3$ is,	1	
		$m_0=-\dfrac{a}{b}=-\dfrac{5}{-7}=\dfrac{5}{7}$		
		$\therefore$ Slope of the required line is,		
		$m=m_0=\dfrac{5}{7}$		
		$\therefore$ equation is,		
		$y-y_1=m(x-x_1)$		
		$\therefore\ y+4=\dfrac{5}{7}(x-5)$	1	
		$\therefore 5x-7y-53=0$	½	**4**
		--		
	e)	Find the equation of line passing through the point of intersection of the lines $2x+3y=13, 5x-y=7$ and passing through the point $(1,-1)$		

Que. No.	Sub. Que.	Model Answer	Marks	Total Marks
6.		$2x+3y=13$ $5x-y=7$ $\therefore\ 2x+3y=13$ $15x-3y=21$ $\therefore\ 17x=34$ $\therefore x=2$ $y=3$ $\therefore$ Point of intersection $=(2,\ 3)$ Given point $(1,-1)$ $\therefore$ equation is, $\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$ $\therefore \frac{y-3}{3+1}=\frac{x-2}{2-1}$ $\therefore 4x-y-5=0$	1 1 1 1	**4**
	f) Ans	Find the acute angle between the lines $3x-2y+4=0$ and $2x-3y-7=0$ For $3x-2y+4=0$ slope $m_1=-\frac{a}{b}=\frac{3}{2}$ For $2x-3y-7=0$, slope $m_2=-\frac{a}{b}=-\frac{2}{-3}=\frac{2}{3}$ $\therefore \tan\theta=\left\|\frac{m_1-m_2}{1+m_1\cdot m_2}\right\|$ $=\left\|\frac{\frac{3}{2}-\frac{2}{3}}{1+\left(\frac{3}{2}\right)\cdot\left(\frac{2}{3}\right)}\right\|$ $=\frac{5}{12}$ $\therefore \theta=\tan^{-1}\left(\frac{5}{12}\right)$	1 1 1 1	**4**

Que. No.	Sub. Que.	Model Answer	Marks	Total Marks
		<u>Important Note</u> *In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.* --- ---		