WINTER – 14 EXAMINATIONS

Subject Code: 17422 **Model Answer** **Total Pages: 46**

Important Instruction to Examiners:-

1) The answers should be examined by key words & not as word to word as given in the model answers scheme.

2) The model answers & answers written by the candidate may vary but the examiner may try to access the understanding level of the candidate.

3) The language errors such as grammatical, spelling errors should not be given more importance.

4) While assessing figures, examiners, may give credit for principle components indicated in the figure.

The figures drawn by candidate & model answer may vary. The examiner may give credit for any equivalent figure drawn.

5) Credit may be given step wise for numerical problems. In some cases, the assumed contact values may vary and there may be some difference in the candidate's answers and model answer.

6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidates understanding.

7) For programming language papers, credit may be given to any other programme based on equivalent concept.

Important notes to examiner

Q. 1 B) c) Calculate the forces in the members AB, BD & DC for the truss given in the question paper, member BD is not possible as per the figure given in question paper therefore forces in BC & BF are calculated. Some student may calculate single force in other member as member BD given in question paper is not correct, examiner shall give proportionate marks accordingly.

Two solutions for members BC & BF are given Examiner shall consider any one force either BC or BF.

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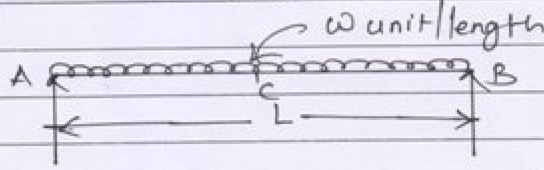
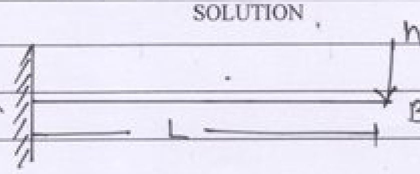
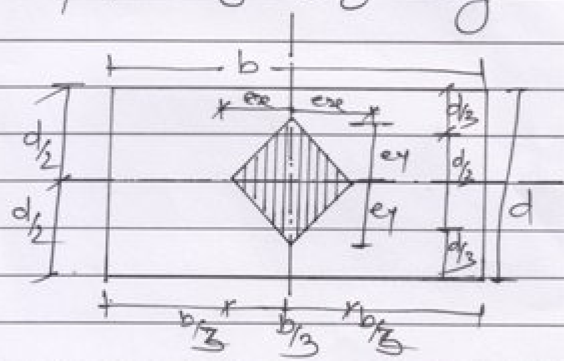
Q.NO	SOLUTION	MARKS
Q.1(A)	Attempt any SIX of the following.	12
a)	Define Direct stress with expression.	
	Direct stress :– It is defined as the ratio of direct load to the cross-sectional area and which gives the compressive stresses only.	01 M
	$\therefore$ Direct Stress $= \sigma_0 = \dfrac{\text{Direct load}}{\text{cross-sectional Area}}$	01 M
	$\sigma_0 = P/A$.	
b)		
	$\text{Max}^m \text{ slope} = \dfrac{dy}{dx} = \theta_A = \theta_B = \dfrac{\omega L^3}{24EI}$	1 M
	$\text{Max}^m \text{ Deflection} = y_{max} = y_C = \dfrac{5}{384} \cdot \dfrac{\omega L^4}{EI}$.	1 M
c)	The relation between slope, Deflection and radius of curvature.	
	$\dfrac{1}{R} = \dfrac{(d^2y/dx^2)}{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}$	2 M

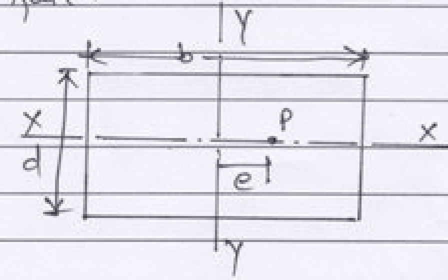
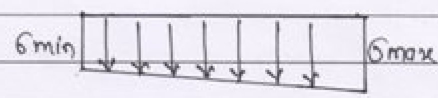
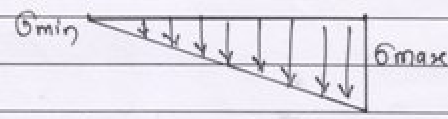
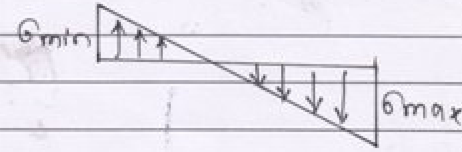
Q .NO	SOLUTION	MARKS
Q1 d)		
	Deflection at free End $= Y_B = \frac{WL^3}{3EI}$	2 M
e)	principle of Superposition :- If the number of forces/moments are acting simultaneously on a body, then their combined effect on body is equal to the algebraic sum of the effect of the individual forces/moments considered seperately	2 M
f)	Carry over factor :- The ratio of moment produced at a joint to the moment applied at the other joint, without displacing it. is called as Carry over factor	2 M
g)	Stiffness factor :- It is the moment required at end of beam to produce unit rotation at that End without translation of either End	2 M.

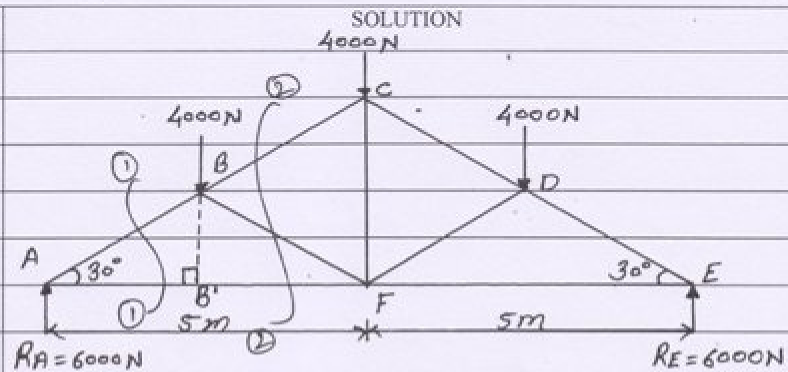
Q.NO	SOLUTION	MARKS
Q1 b)	Redundant frame ; $n > 2j - 3$	1M
	Non-redundant frame ; $n < 2j - 3$ where n = No of members j = No of joints	1M
[B]	Attempt any Two of the following	
a)		2M
	Incase of rectangular section, the external load if acts within middle third part of section, then No tension is produced anywhere in the section is called Middle third rule. OR	2M
OR	For No tension condition	OR
	$e \leq \frac{Z}{A}$	
	$\leq \frac{\frac{1}{6}bd^2}{bd}$	
	$e_y \leq \frac{1}{6}d.$	1M

$\& \; 2e_y \leq \frac{1}{3}d.$

OR

$2e_x = b/3$

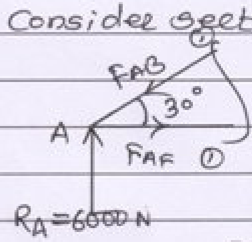
Q.NO	SOLUTION	MARKS
1 (B)	It means that the load can be eccentric, on either side of the geometrical axis, by an amount equal to $d/6$. Thus if the line of action of the load is within the middle third as shown by hatted area in fig, then there will be only compressive stress. is known as middle third Rule.	1M
b)		1M
	 $\sigma_a > \sigma_b$	1M
	 $\sigma_a = \sigma_b$	1M
	 $\sigma_a < \sigma_b$	1M

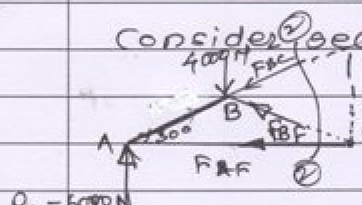
Q.NO.	SOLUTION	MARKS
1 (B) c)		
	i) Support reactions	
	Due to symmetrical loading	
	$R_A = R_E = \frac{\text{Total load}}{2} = \frac{4000 \times 3}{2} = 6000\,N$	$\frac{1}{2}$M
	ii) Geometrical properties	
	Length of member C.F	
	$CF = AF \times \tan 30 = 5 \times \tan 30 = 2.88\,m$	
	Length of member AC	
	$AC = \sqrt{AF^2 + CF^2} = \sqrt{5^2 + 2.88^2} = 5.77\,m$	
	$\therefore AB = BC = \frac{AC}{2} = \frac{5.77}{2} = 2.88\,m$	
	Length of perpendicular BB'	
	$BB' = AB \cdot \sin 30 = 2.88 \cdot \sin 30$	
	$BB' = 1.44\,m$	$\frac{1}{2}$M
Note:- In this question forces in member BD is not correct., therefore forces in		

member AB, BC, BF & DC calculated.

* Examiner shall give proportionate marks to either BC OR BF

Q.NO	SOLUTION	MARKS
1 (B) c)	Length of $AB' = AB \cdot \cos 30$	
	$= 2.88 \cdot \cos 30$	
	$AB' = 2.50\ m$	
	Consider section ①-①	
	$\therefore \Sigma F_y = 0$	
	$\therefore F_{AB} \sin 30 = 6000$	
	$\therefore F_{AB} = 12000\ N$ (comp.)	1M
	$\Sigma F_x = 0$	
	$F_{AB} \cos 30 = F_{AF}$	
	$F_{AF} = 10{,}392.304\ N$ (Tensile)	
	Consider section ②-②.	
	Taking at moment @ F	
	$\therefore \Sigma M_F = 0$	
	$\therefore 6000 \times 5 - 4000 \times 2.5 - F_{BC} \sin 30 \times 2.5 - F_{BC} \cos 30 \times 1.44 - F_{BF} \cos 30 \times 1.44 + F_{BF} \sin 30 \times 2.5 = 0$	$F_{BC} \rightarrow$ 1M; $F_{DC} \rightarrow$ 1M
	$20000 - 1.25 F_{BC} - 1.25 F_{BC} = 0$	
	$F_{BC} = \frac{20000}{2.50} = 8000\ N = F_{DC}$ (comp) (Due to symmetry)	
OR	$\Sigma M_A = 0$	OR
	$4000 \times 2.5 - F_{BC} \cos 30 \times 1.44 + F_{BC} \sin 30 \times 2.5 - F_{BF} \cos 30 \times 1.44 - F_{BF} \sin 30 \times 2.5 = 0$	
	$10000 + 0 \times F_{BC} - 2.5 F_{BF} = 0$	

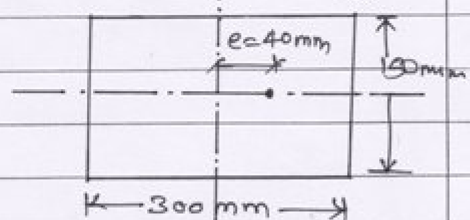




Note $\therefore F_{BF} = \frac{10000}{2.5} = 4000\ N$ (comp.) 1M

* Examiner shall consider either F_{BC} or F_{BF} for giving proportionate marks. If student finds any other member force, then also proportionate marks shall be given.

Q.NO	SOLUTION	MARKS
Q.2.	Attempt any FOUR of the following.	
a)	Given data.	
	$P = 60\ kN = 60 \times 10^3\ N$	
	$b = 300\ mm$	
	$d = 150\ mm$	
	$e = 40\ mm$	



To find :- σ_{max} & σ_{min}.

Cross-sectional area $= A = b \times d = 300 \times 150$
$= 45 \times 10^3\ mm^2$.

$\therefore$ Direct stress $= \sigma_0 = \frac{P}{A} = \frac{60 \times 10^3}{45 \times 10^3} = 1.33\ N/mm^2$ — $\frac{1}{2}$M

Bending stress $= \sigma_b = \frac{M}{Z_{yy}} = \frac{P.e}{I_{yy}} \cdot y_{max}$.

M.I. @ y-y axis, $I_{yy} = \frac{db^3}{12} = \frac{150 \times 300^3}{12}$
$= 337.50 \times 10^6\ mm^4$

$y_{max} = \frac{b}{2} = \frac{300}{2} = 150\ mm$.

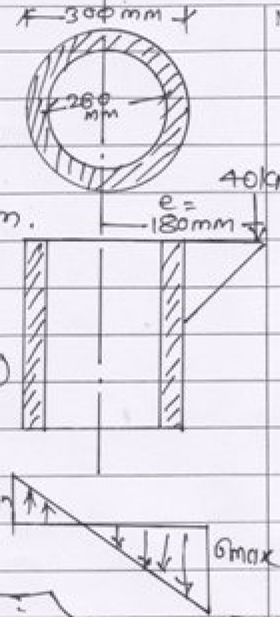
$\therefore \sigma_b = \frac{60 \times 10^3 \times 40}{337.50 \times 10^6} \times 150 = 1.067\ N/mm^2$ — 1M

$\sigma_{max} = \sigma_0 + \sigma_b = 1.33 + 1.067 = 2.39\ N/mm^2$ (comp.) — 1M

$\sigma_{min} = \sigma_0 - \sigma_b = 1.33 - 1.067 = 0.263\ N/mm^2$ (comp) — 1M

Bending stress distribution Dia.

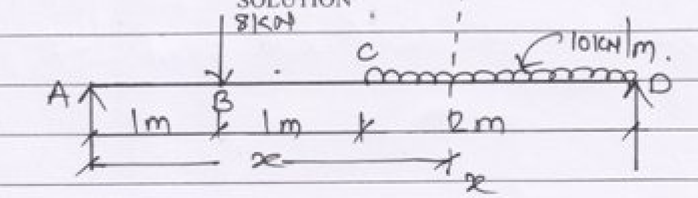
$\frac{1}{2}$M

Q.NO	SOLUTION	MARKS
2 b)	Given data.	
	$D = 300\,mm$, $t = 20\,mm$, $d = D - 2t$.	
	$d = 300 - 2\times 20 = 260\,mm$	
	$P = 40\,kN = 40\times10^3\,N$, $e = 180\,mm$.	
	To find, σ_{max} & σ_{min}.	
	Cross-sectional area	
	$A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(300^2 - 260^2)$	
	$= 17592.92\,mm^2$	
		
	$\therefore$ Direct stress $= \sigma_0 = \frac{P}{A} = \frac{40\times10^3}{17592.92}$	
	$= 2.27\,N/mm^2$.	1M (for σ_0)
	Bending stress $= \sigma_b = \frac{P.e}{I_{yy}} \times y_{max}$.	
	$\therefore$ M.I about any axis $= \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}(300^4 - 260^4)$	
	$= 173.29\times10^6\,mm^6$.	
	$y_{max} = \frac{300}{2} = 150\,mm$.	
	$\therefore \sigma_b = \frac{40\times10^3\times180}{173.29\times10^6}\times150$	1M
	$= 6.23\,N/mm^2$	
	$\therefore \sigma_{max} = \sigma_0 + \sigma_b = 2.27 + 6.23 = 8.50\,N/mm^2$ (Compressive)	1M
	$\sigma_{min} = \sigma_0 - \sigma_b = 2.27 - 6.23 = -3.96\,N/mm^2$ (Tensile)	1M

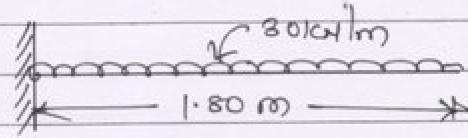
Q .NO	SOLUTION	MARKS
2 c)	Given data.	
	$P = 150\ kN = 150 \times 10^3\ N$	
	$b = 150\ mm,\ e = 5mm.$	
		
	[Note: ⇒ In this Problem Permissible tensile stress or depth is not given, Therefore Two solutions Prepared.	
	Solution I – considering depth d = 20mm.	
	Assumme $d = 20\ mm$	1M
	To find σ_{min}	
	$A = b \times d = 150 \times 20 = 3000\ mm^2$	
	$\sigma_o = P/A = \dfrac{150 \times 10^3}{3000} = 50\ N/mm^2$	1M
	$\sigma_b = \dfrac{P \cdot e}{I_{YY}} \times Y_{max}.$	
	$Y_{max} = 150/2 = 75\ mm$	
	$I_{YY} = \dfrac{db^3}{12} = \dfrac{20 \times 150^3}{12} = 5.625 \times 10^6\ mm^4$	
	$\sigma_b = \dfrac{150 \times 10^3 \times 5}{5.625 \times 10^6} \times 75 = 10\ N/mm^2$	1M
	$\sigma_{min} = \sigma_o - \sigma_b = 50 - 10 = 40\ N/mm^2$	1M

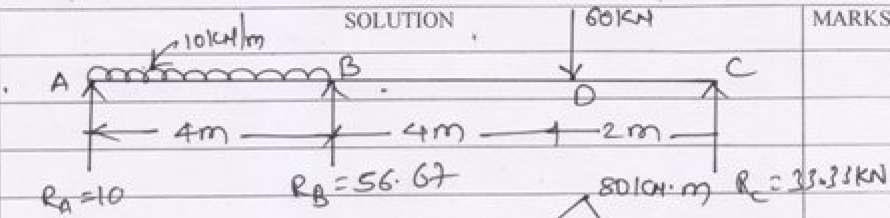
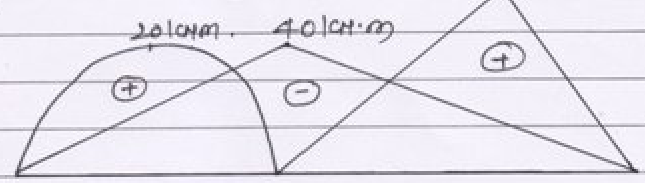
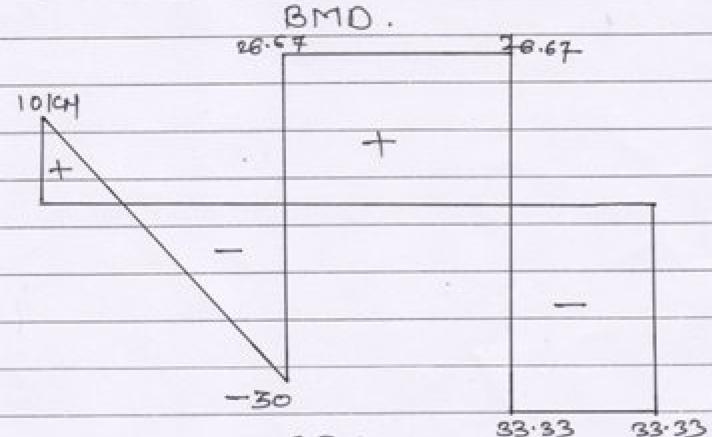
Q.NO	SOLUTION	MARKS
2(c) cont.--	solution II [OR] Assuming Max. Tensile stress = 100 N/mm^2.	
	Assume $\sigma_{max} = 100$ N/mm^2	
	To find σ_{min}.	
	Let d be the thickness about Y-Y axis	
	If the load is eccentric about Y-Y axis	
	$\sigma_{max} = \frac{P}{A} + \frac{M}{Z_{YY}}$	
	$100 = \frac{150 \times 10^3}{150 \times d} + \frac{150 \times 10^3 \times 5}{d \times 150^2/6}$	1M
	$d = 12$ mm.	
	$\sigma_{min} = \frac{P}{A} - \frac{M}{Z_{YY}}$	1M
	$= \frac{150 \times 10^3}{150 \times 12} - \frac{150 \times 10^3 \times 5}{12 \times 150^2/6}$	
	$= 83.33 - 16.67$	
	$\sigma_{min} = 66.67$ N/mm^2.	2M
*	NOTE :- Examiner can consider any one solution for giving proportionate marks.	*

* If student assumes any suitable value of σ_{max} or depth (d), examiner shall give proportionate marks.

Q .NO	SOLUTION	MARKS
2 d)		
	a) Support reaction.	
	To find support reaction apply conditions of equilibrium., $\Sigma fx = 0$, $\Sigma fy = 0$, $\Sigma M_O = 0$	
	$\therefore \Sigma fy = 0$,	
	$R_A + R_D - 8 - 10 \times 2 = 0$; $R_A + R_D = 28$ — eq^n (1)	
	$\Sigma M_O = 0$ Taking @ moment @ A $\Sigma M_A = 0$	
	$R_D \times 4 - 10 \times 2 \times (2 + 2/2) - 8 \times 1 = 0$	
	$R_D = 17$ KN.	
	$\therefore$ $R_A = 11$ KN.	
	Consider a section x-x at a distance x from A in portion CD.	
	$\therefore$ Moment at x.	
	$M_x = 11x \mid - 8(x-1) \mid - \frac{10(x-2)^2}{2}$	$\frac{1}{2}$ M
	Consider differential Eq^n.	
	$EI \frac{d^2y}{dx^2} = M_x$	
	$EI \frac{d^2y}{dx^2} = 11x \mid -8(x-1) \mid -\frac{10(x-2)^2}{2}$	
	Integrate w.r. to x we get.	

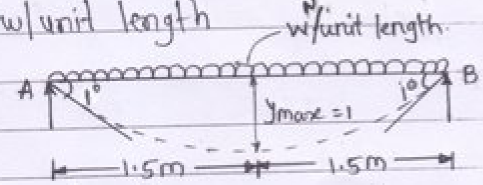
Q.NO	SOLUTION	MARKS
2 (d)	$EI\frac{dy}{dx} = \frac{11x^2}{2} + C1 \;\vert - \frac{8(x-1)^2}{2} \;\vert - \frac{10(x-2)^3}{6}$	
cont---	Again Integrate w.r.to x.	
	$EIy = \frac{11x^3}{6} + C1x + C2 \;\vert - \frac{8(x-1)^3}{6} \;\vert - \frac{10(x-2)^4}{24}$	$\frac{1}{2}$M
	To find Integrating constant C1 & C2.	
	Apply boundary conditions.	
	∴ To find C2, at support A, $x=0, y=0$	
	$C2 = 0$.	
	To find C1 at support B, $x=4, y=0$.	
	$\therefore 0 = \frac{11\times4^3}{6} + C1\times4 + 0 - \frac{8(4-1)^3}{6} - \frac{10(4-2)^4}{24}$	
	$C1 = -18.67$.	
	Slope eq^n $EI\frac{dy}{dx} = \frac{11x^2}{2} - 18.67 \;\vert - \frac{8(x-1)^2}{2} \;\vert - \frac{10(x-2)^3}{6}$	1M
	Deflection eq^n.	
	$EIy = \frac{11x^3}{6} - 18.67x \;\vert - \frac{8(x-1)^3}{6} \;\vert - \frac{10(x-2)^4}{24}$	
	∴ Deflection at B. $x = 1m$.	
	$EIy = \frac{11\times1^3}{6} - 18.67\times1$	
	$\therefore y = 16.83/EI$ mm	1M
	∴ Slope at D., $x = 4m$	
	$EI\theta_D = \frac{11(4)^2}{2} - 18.67 - \frac{8(4-1)^2}{2} - \frac{10(4-2)^3}{6}$	
	$\theta_B = \frac{19.996}{EI} \simeq \frac{20}{EI}$	1M

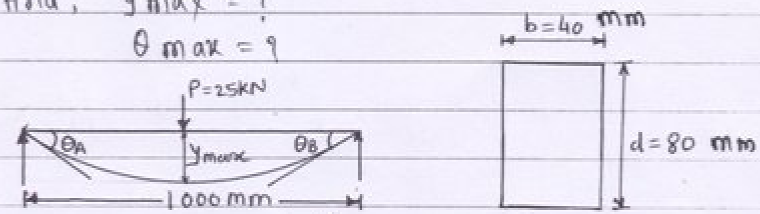
Q.NO	SOLUTION	MARKS
2 e)		
	Given data.	
	$\omega = 30\,kN/m$	
	$L = 1.80\,m.$	
	$y_B = 25\,mm = 0.025\,m.$	
	$I = 1.3 \times 10^{8}\,mm^4 = 1.3 \times 10^{-4}\,m^4$	
	Maximum Deflection at free End.	
	$y_B = \dfrac{\omega L^4}{8EI}$	1M
	$0.025 = \dfrac{30 \times (1.80)^4}{8 \times E \times 1.3 \times 10^{-4}}$	1M
	$E = \dfrac{30 \times (1.80)^4}{8 \times 0.025 \times 1.3 \times 10^{-4}}$	
	$\boxed{E = 12.11 \times 10^{6}\ kN/m^2}$	2M

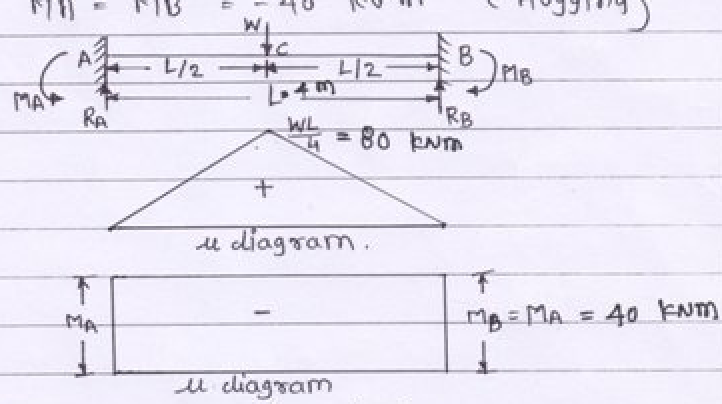
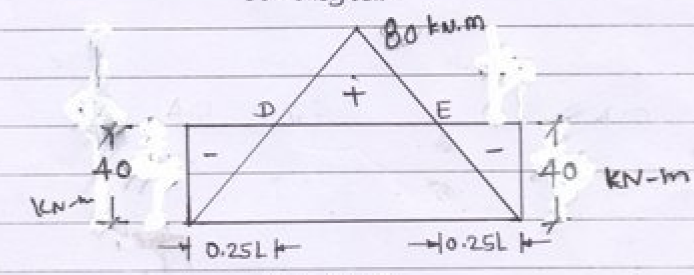
Q .NO	SOLUTION	MARKS
2 (f)		
	 BMD.	1M
	 SFD.	$\frac{1M}{2}$
	step① free Bending moment for span AB $= \frac{wl^2}{8} = \frac{10 \times 4^2}{8} = 20$KNm. for Span BC $= \frac{Wab}{L} = \frac{60 \times 4 \times 2}{6} = 80$KN.m	

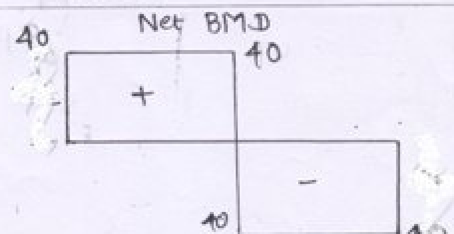
Q.NO	SOLUTION	MARKS
2 (F)	Step② Determination of $\frac{6a\bar{x}}{L}$	
* cont---	For Span AB = $\frac{6a_1\bar{x}_1}{L_1}$	
	$a_1 = \frac{2}{3} \times 4 \times 20 = 53.33$, $\bar{x}_1 = 4/2 = 2m$	
	$\frac{6a_1\bar{x}_1}{L_1} = \frac{6 \times 53.33 \times 2}{4} = 160$.	
	For span BC $\frac{6a_2\bar{x}_2}{L_2}$	
	$a_2 = \frac{1}{2} \times 6 \times 80 = 240$; $\bar{x}_2 = \frac{L+b}{3} = 2.67m$	
	$\frac{6a_2\bar{x}_2}{L_2} = \frac{6 \times 240 \times 2.67}{6} = 640.8$	
	Step③ End/Support moment @ B.	
	Apply three moment theorem to support/ span ABC	
	$M_AL_1 + 2M_B(L_1+L_2) + M_CL_2 = -\left[\frac{6a_1\bar{x}_1}{L_1} + \frac{6a_2\bar{x}_2}{L_2}\right]$	$\frac{1}{2}$M
	As support A & C are simply supported	
	$M_A = M_C = 0$	
	$0 + 2M_B(4+6) = -[160 + 640.8]$	
	$20 M_B = -800.8$	
	$M_B = -40 kNm$.	1M
	Step④ To find support reaction.	
	$\Sigma f_y = 0$ $R_A + R_B + R_C = 100 kN$ —— eq^{n}①	
	$\Sigma M_0 = 0$ Taking moment @ B.	
	consider left side	

Q .NO	SOLUTION	MARKS
2 (f)	$10 \times 4 \times 4/2 - RA \times 4 - 40 = 0$	
cont--	$RA = 10 kN$.	
	Consider Right side	
	$RC \times 6 + 40 - 60 \times 4 = 0$	
	$RC = 33.33 kN$.	
	Substitute RA & RC in eq^n (1)	
	$10 + RB + 33.33 = 100$.	1M.
	$RB = 56.67 kN$.	

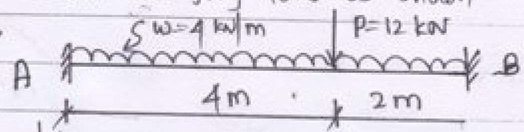
Q. NO	SOLUTION	MARKS
3. a]	Given	
	$L = m$; θ = slope at end = 1^0	
	u.d.l = w/unit length	
		
	θ = slope at end = $1^0 = \left(1 \times \frac{\pi}{180}\right)$ rad	
	$= 0.017$ rad	
	but $\theta = \frac{wL^3}{24EI}$	1M
	$0.017 = \left(\frac{w}{EI}\right)\frac{L^3}{24}$	
	$\therefore \left(\frac{w}{EI}\right) = \frac{0.017 \times 24}{L3} = \frac{0.017 \times 24}{3^3} = 0.0151$	
	$\boxed{\therefore \frac{W}{EI} = 0.0151}$	1M
	To find max. deflection (y_{max})	
	$y_{max} = \frac{-5wL^4}{384EI} = -\frac{5L^4}{384} \times \left(\frac{w}{EI}\right)$	1M
	$= -\frac{5 \times 3^4}{384} \times 0.0151$	
	$= -0.0159 m = -15.92 mm$	
	$\boxed{\therefore y_{max} = -15.92 mm}$	
	–ve sign indicate downward deflection	1M

Q.NO	SOLUTION	MARKS
3 b]	Given: $L = 1m = 1000\ mm$	
	$P = 25\ kN = 25 \times 10^3\ N$	
	$b = 40\ mm$; $d = 80\ mm$	
	$E = 100\ kN/mm^2 = 1 \times 10^5\ N/mm^2$	
	Find; $y_{max} = ?$	
	$\theta_{max} = ?$	
		
	$\rightarrow I = I_{xx} = \frac{bd^3}{12} = \frac{40 \times 80^3}{12}$	
	$= 1.707 \times 10^6\ mm^4$	
	i) $y_{max} = \frac{PL^3}{48EI} = \frac{25 \times 10^3 \times 1000^3}{48 \times 1 \times 10^5 \times 1.707 \times 10^6}$	1M
	$= 3.051\ mm$	
	$\therefore y_{max} = 3.051\ mm$	1M
	ii) $\theta_{max} = \frac{PL^2}{16EI} = \frac{25 \times 10^3 \times 1000^2}{16 \times 1 \times 10^5 \times 1.707 \times 10^6}$	1M
	$= 9.153 \times 10^{-3}$ rad	
	$\therefore \theta_{max} = 9.153 \times 10^{-3}$ rad	1M

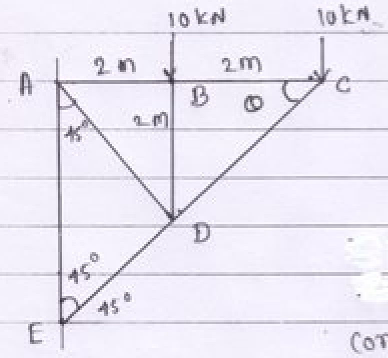
Q.NO	SOLUTION	MARKS
3c	Given $L = 4\,m$; $P = 80\,kN$	
	→ Reaction of simply supported beam = Reaction of fixed beam	
Q 3 (C)	$\because R_A = R_B = \frac{W}{2} = \frac{80}{2} = 40\,kN$	
	Free B.M. at C $= \frac{WL}{4} = \frac{80 \times 4}{4} = 80\,kN{\cdot}m$ (sagging)	1M
	* Area of μ diagrame = area of μ' diagram	
	$\frac{1}{2} \times 4 \times 80 = -M_A \times 4$	
	$\therefore M_A = -40\,kN{\cdot}m$	
	$\because M_A = M_B = -40\,kN{\cdot}m$ (Hogging)	1M
		
		1M



S.F.D. 1 M

Q. NO	SOLUTION	MARKS
3 d]	Given : fixed beam carrying load as shown in fig.	
		
	→ For pt. load	
	12 kN , $a = 4\ m$; $b = 2\ m$	
	$M_A = -\frac{wL^2}{12} - \frac{Pab^2}{L^2} = -\frac{4\times6^2}{12} - \frac{12\times4\times2^2}{6^2}$	1 M
	$= -17.33\ kN{\cdot}m$	
	$\boxed{\therefore M_A = -17.33\ kN{\cdot}m}$	1M
	$M_B = -\frac{wL^2}{12} - \frac{Pba^2}{L^2} = -\frac{4\times6^2}{12} - \frac{12\times2\times4^2}{6^2} = -22.67\ kN{\cdot}m$	1M
	$\boxed{\therefore M_B = -22.67\ kN{\cdot}m}$	1M
3 e)	Assumption made in the analysis of simple frame. →	
	1) The joints of a truss are assumed to be pin connected and frictionless so cannot resist moments	
	2) The truss is loaded at the joints only	
	3) The truss is a perfect truss	1 M
	4) Weight of members are neglected	for Each

Q.NO	SOLUTION	MARKS
3 f	Given	



Solution

Let $\angle BCD = \theta$

$\therefore \tan\theta = \frac{BD}{BC} = \frac{2}{2} = 1$

$\therefore \theta = \tan^{-1}(1)$

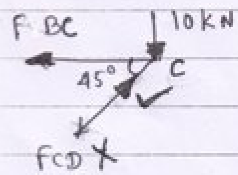
$\therefore \theta = 45^\circ$

Consider FCD and FBC Tensile

Joint C

$F_{CD} \sin 45^\circ = 10$

$F_{CD} = 14.14$ kN (comp.)



$\overset{+}{\rightleftarrows}$ $\Sigma F_x = -F_{BC} - F_{CD} \cos 45$

$0 = -F_{BC} - F_{CD} \cos 45$ ... (i)

$\overset{+}{\uparrow\downarrow}$ $\Sigma F_y = -10 - F_{CD} \sin 45$

$0 = -10 - F_{CD} \sin 45$

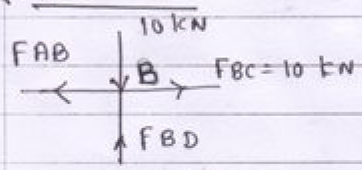
$\because F_{CD} = \frac{-10}{\sin 45} = -14.14$ kN (comp.) (Assumed direction is wrong)

Put in eq^n (i) $F_{CD} = 14.14$ kN (comp)

$0 = -F_{BC} + 14.14 \cos 45$

$\boxed{\because F_{BC} = 10 \text{ kN (Tensile)}}$ 1M.

Joint B



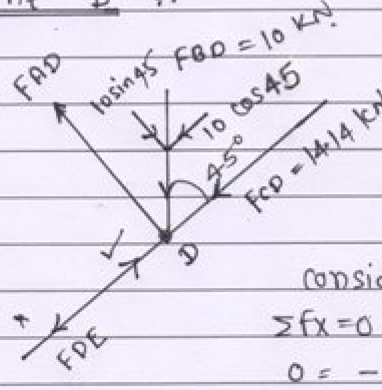
$\therefore \Sigma F_x = 10 - F_{AB}$

$\boxed{\because F_{AB} = 10 \text{ kN (Tensile)}}$ 1M

$\Sigma F_y = -10 - F_{BD}$

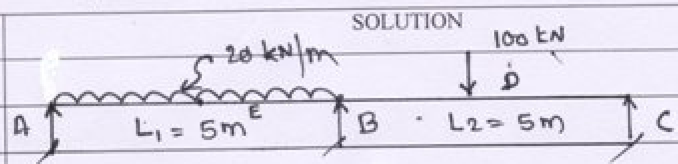
$0 = -10 - F_{BD}$

$\because F_{BD} = -10$ kN (comp.)

Q.NO	SOLUTION	MARKS
3 (f) cont.	Joint D :-	
		
	Assuming FDE and FAD Tensile	
	considering the equilibrium	
	$\Sigma fx = 0$	
	$0 = -FDE - 14.14 - 10\cos 45$	
	$\therefore FDE = -21.21$ kN	
	FDE = 21.21 kN (Comp.)	1M
	$\Sigma fy = 0$	
	$0 = FAD - 10\sin 45$	
	$\therefore$ FAD = 7.07 kN (Tensile)	1M
	Forces in Member 1) AB = 10 kN Tensile 2) AD = 7.07 kN Tensile 3) DE = 21.21 kN compressive 4) BC = 10 kN Tensile	

Q.NO	SOLUTION	MARKS

4 a)



Known moment $M_A = M_C = 0$

Assume the span AB and BD as simply supported and draw μ diagrame

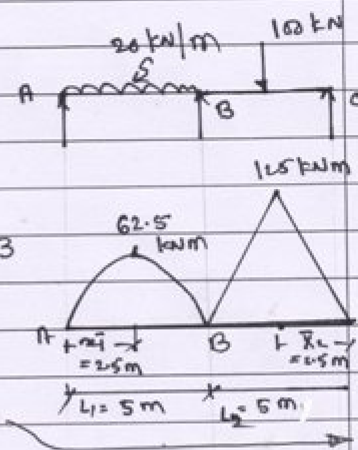
Free moment →

Span AB $\quad m_E = \frac{wL_1^2}{8} = \frac{20\times5^2}{8} = 62.5$ kNm

Span BC $\quad m_C = \frac{WL_2}{4} = \frac{100\times5}{4} = 125$ kN·m 1 M

$M_A = M_B = M_C = 0$

Evaluate $\frac{6a\bar{x}}{L}$ →



For span AB →

a_1 = Area of μ dig. for span AB

$= 2/3 \times 5 \times 62.5 = 208.33$

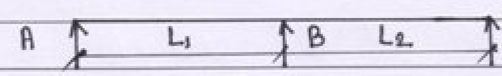
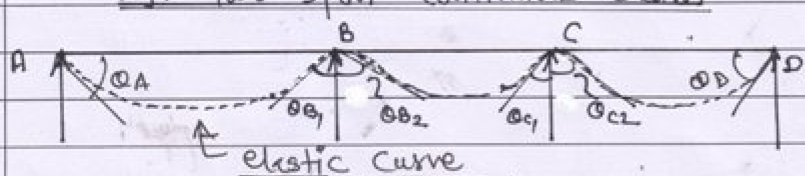
$\bar{x}_1 = 2.5$ m from A

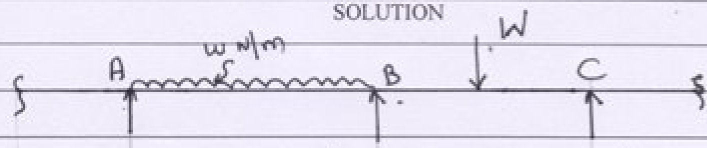
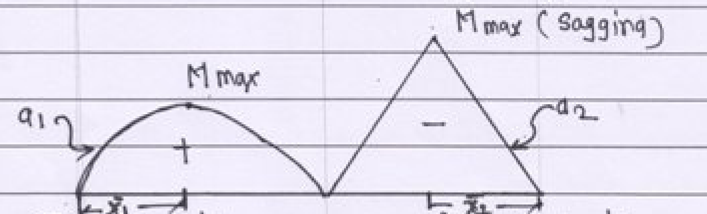
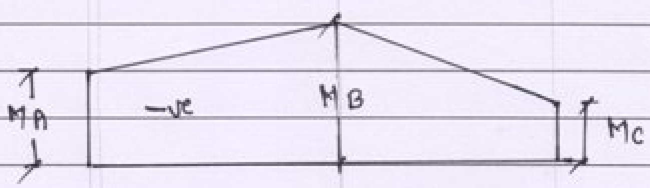
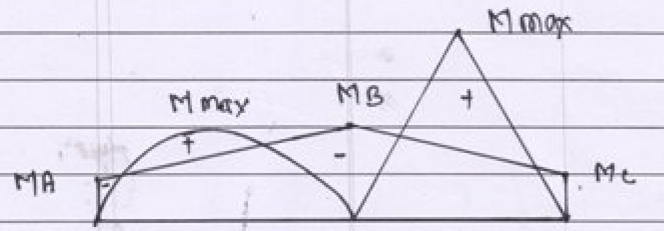
$\because \frac{6a_1\bar{x}_1}{L_1} = \frac{6\times208.33\times2.5}{5} = 625$ 1 M

For span BC →

a_2 = Area of μ dig. for span BC

$= \frac{1}{2}\times5\times125 = 312.5$

Q .NO	SOLUTION	MARKS
4. a cont.	$\bar{x}_2 = 2.5\,m$ from B	
	$\because \frac{6 a_2 \bar{x}_2}{L_2} = \frac{6 \times 312.5 \times 2.5}{5} = 937.5$	1M
	Apply clapeyron's theorem	
	since support 'A' and 'C' are simply supported $M_A = 0$ and $M_C = 0$	
	$M_A \times L_1 + 2 M_B (L_1 + L_2) + M_C \times L_2 = -\left[\frac{6 a_1 \bar{x}_1}{L_1} + \frac{6 a_2 \bar{x}_2}{L_2}\right]$	
	$0 + 2 M_B (5+5) + 0 = -[625 + 937.5]$	
	$20\,M_B = -1562.5$	
	$\boxed{\therefore M_B = 78.125\ kN.m}$	1M
4 b)	Continuous beam: A beam which is supported on more than two support is called continuous beam	2M
		
	Fig: Two span continuous beam	
		2M
	Fig: Deflected shape of three span continuous beam	

Q .NO	SOLUTION	MARKS
4 C		
	ⓐ Two consecutive span of continuous span	
		
	ⓑ μ - diagram or Free B.M. diagram	
		
	ⓒ μ' - diagram of support moments	1 M
		
	ⓓ Net B.M. D	

Q.NO	SOLUTION	MARKS

Q.4c cont.

Statement of clapeyron's theorem of three moment with uniform M.I:

$$\boxed{M_A L_1 + 2 M_B (L_1 + L_2) + M_C L_2 = -\left(\frac{6a_1\bar{x}_1}{L_1} + \frac{6a_2\bar{x}_2}{L_2}\right)}$$ 1M

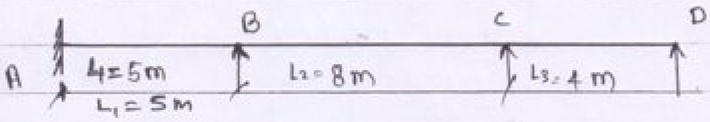
Where M_A = Support moment at A
M_B = Support moment at B
M_C = Support moment at C
L_1 = length of span AB
L_2 = length of span BC
a_1 = Area of μ-diagram for the span AB
a_2 = Area of μ-diagram for the span BC 1M
$\bar{x}_1$ = centroidal distance of μ diagram over the span AB from the left end A
$\bar{x}_2$ = centroidal distance of μ diagram over the span BC from the right end C

Statement of clapeyron's theorem of three moment with different MI.

$$\boxed{M_A \frac{L_1}{I_1} + 2 M_B \left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_C \frac{L_2}{I_2} = -\left(\frac{6a_1\bar{x}_1}{L_1 I_1} + \frac{6a_2\bar{x}_2}{L_2 I_2}\right)}$$ 1M

where I_1 = moment of inertia for the span AB
I_2 = moment of inertia for the span BC

Q.NO	SOLUTION	MARKS
4d]		



Stiffness factors

Joint B $\quad k_{BA} = \frac{4EI}{L_1} = \frac{4EI}{5} = 0.8\,EI$

$k_{BC} = \frac{4EI}{L_2} = \frac{4EI}{8} = 0.5\,EI$ 1M

$\Sigma k = k_{BA} + k_{BC} = 0.8\,EI + 0.5\,EI = 1.3\,EI$

Joint C $\quad k_{CB} = \frac{4EI}{L_2} = \frac{4}{8}EI = 0.5\,EI$

$k_{CD} = \frac{3EI}{L_3} = \frac{3}{4}EI = 0.75\,EI$ 1M

$\Sigma k = k_{CB} + k_{CD} = 0.5\,EI + 0.75\,EI = 1.25\,EI$

Distribution factors

Joint B

$DF_{BA} = \frac{k_{BA}}{\Sigma k} = \frac{0.8EI}{1.3EI} = 0.62$ 1M

$DF_{BC} = \frac{k_{BC}}{\Sigma k} = \frac{0.5EI}{1.3EI} = 0.38$

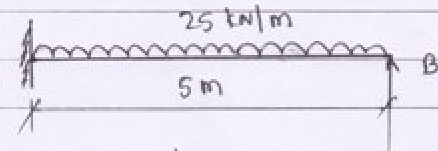
Joint C

$DF_{CB} = \frac{k_{CB}}{\Sigma k} = \frac{0.5EI}{1.25EI} = 0.4$ 1m

$DF_{CD} = \frac{k_{CD}}{\Sigma k} = \frac{0.75EI}{1.25EI} = 0.6$

Q.NO	SOLUTION	MARKS

4 e]

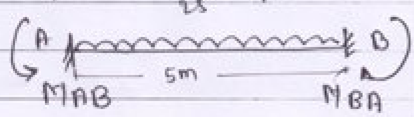


Date : A propped cantilever as shown in fig

Known moment $M_B = 0$

To find M_A,

Step I] Fixed end moment



$M_{AB} = -\frac{wL^2}{12} = -\frac{25 \times 5^2}{12}$

$= -52.08$ kN·m — 2M

$M_{BA} = +\frac{wL^2}{12} = \frac{25 \times 5^2}{12} = 52.08$ kN·m

Step II] Moment distribution table

Joint	A	B
Member	AB	BA
DF		
FEM	−52.08	+52.08
Balance B		−52.08
Carry over to A	−26.04	
Final moment	−78.12	0

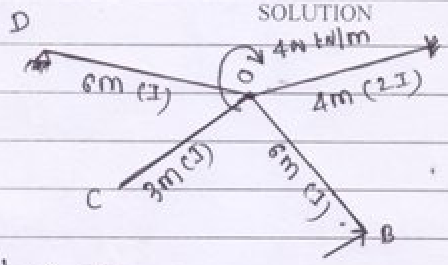
2M

Answer fixed end moment

$\boxed{M_A = -78.12 \text{ kNm}}$

Q.NO	SOLUTION	MARKS

4 f]



→ Joint o →

stiffness factor

$k_{OA} = \frac{4E \times 2I}{4} = 2EI$

$k_{OB} = \frac{3EI}{6} = 0.5\ EI$

$k_{OC} = 0$ (As end C is free)

$k_{OD} = \frac{3EI}{6} = 0.5\ EI$

$\Sigma = k_{OA} + k_{OB} + k_{OC} + k_{OD}$

$= 2EI + 0.5EI + 0 + 0.5EI$

$= 3EI$ — 1M

distribution factor →

$DF_{OA} = \frac{k_{OA}}{\Sigma K} = \frac{2EI}{3EI} = 0.66$

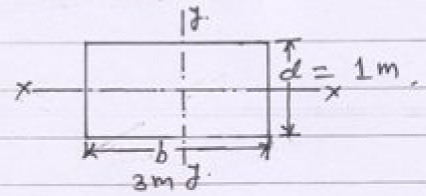
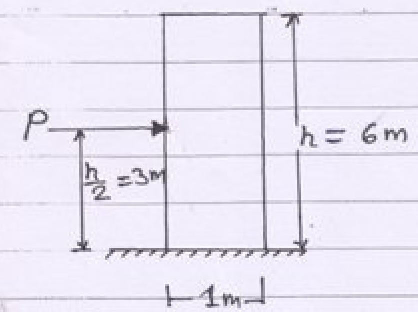
$DF_{OB} = \frac{k_{OB}}{\Sigma K} = \frac{0.5EI}{3EI} = 0.17$ — 1M

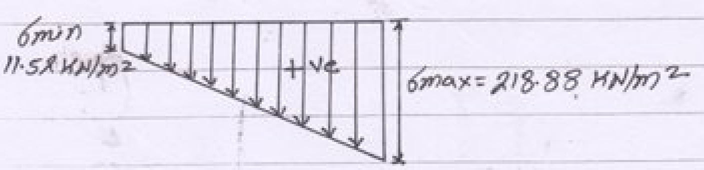
$DF_{OC} = 0$

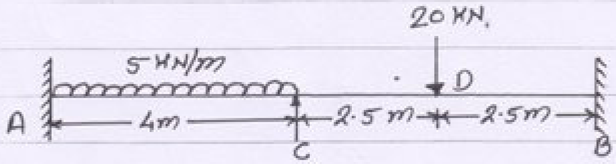
$DF_{OD} = \frac{k_{OD}}{\Sigma K} = \frac{0.5EI}{3EI} = 0.17$

Sr No	Member	Moment
1	OA	0.66 × 400 = 264 kN/m
2	OB	0.17 × 400 = 68 kN/m
3	OC	0 × 400 = 0
4	OD	0.17 × 400 = 68 kN/m

* ½ m for (each moment)

Q.NO.	SOLUTION	MARKS
5a	Given $h = 6m$; $b = 3m$; $d = 1m$; $p = 960\ N/m^2$	
	$\rho = 19.2\ KN/m^2$	
		
		
	1) Maximum stress (σ_{max})	
	σ_{max} = Direct stress (σ_0) + Bending stress (σ_b)	
	$\sigma_{max} = \frac{W}{A} + \frac{M}{Z}$	
	Direct stress $\sigma_0 = \frac{W}{A} = \frac{\rho \cdot A \cdot h}{A} = \rho \cdot h$	1
	$\therefore \sigma_0 = 19.2 \times 6 = 115.2\ KN/m^2$	1
	Bending stress $\sigma_b = \frac{M}{Z} = \frac{P \times \frac{h}{2}}{Z}$	

Q NO	SOLUTION	MARKS
Q5 a) Cont...	Total wind force $P = C \times p \times$ Area exposed to wind	
	$P = 1 \times 960 \times 3 \times 6$	
	$= 17280\ N$	
	$P = 17.28\ KN$	1M
	Moment $M = P \times \frac{h}{2} = 17.28 \times \frac{6}{2} = 51.84\ KN.m$	
	Section Modulus $Z = \frac{bd^2}{6} = \frac{3 \times 1^2}{6} = 0.5\ m^3$	1M
	$\therefore$ Bending stress $\sigma_b = \frac{M}{Z} = \frac{51.84}{0.5} = 103.68\ KN/m^2$	1M
	$\therefore \sigma_{max} = \sigma_0 + \sigma_b = 115.2 + 103.68$	
	$\sigma_{max} = 218.88\ KN/m^2$	1M
	σ_{min} = Direct stress (σ_0) - Bending stress (σ_b)	
	$= 115.2 - 103.68$	
	$\sigma_{min} = 11.52\ KN/m^2$	1M
		1M.

Q NO.	SOLUTION	MARKS
Q5 b)		
	i) Fixed end moments	
	$M_{AC} = M_{CA} = \frac{wl^2}{12} = \frac{5\times4^2}{12} = 6.67\ KN\cdot m$	
	$M_{CB} = M_{BC} = \frac{WL}{8} = \frac{20\times5}{8} = 12.5\ KN\cdot m$	
	ii) Stiffness factor	
	$K_{CA} = \frac{4EI}{L} = \frac{4EI}{4} = EI$	
	$K_{CB} = \frac{4EI}{L} = \frac{4EI}{5} = \frac{4EI}{5}$	$\frac{1}{2}$ M
	$\Sigma K_C = K_{CA} + K_{CB} = 1EI + 0.8EI = 1.8EI$	
	iii) Distribution factor	
	$DF_{CA} = \frac{K_{CA}}{\Sigma K} = \frac{1EI}{1.8EI} = 0.56$	
	$DF_{CB} = \frac{K_{CB}}{\Sigma K} = \frac{0.8EI}{1.8EI} = 0.44$	$\frac{1}{2}$
	$\Sigma D.F = 0.56 + 0.44 = 1 \ ..\ ok$	

Q. NO.	SOLUTION	MARKS
Q5 b) Cont...	iv) Moment Distribution Table	

Joint	A	C		B
Member	AC	CA	CB	BC
D.F	–	0.56	0.44	–
F.E.M	-6.67	6.67	-12.5	12.5
1st Distribution		3.265	2.565	
Carry over	1.632			1.282
Final moments	-5.04	9.935	-9.935	13.782

2M

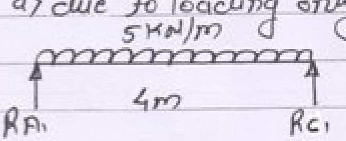
$M_A = -5.04$ KN.m

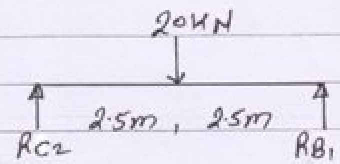
$M_C = 9.935$ KN.m

$M_B = 13.782$ KN.m

v) Support reaction's

a) due to loading only





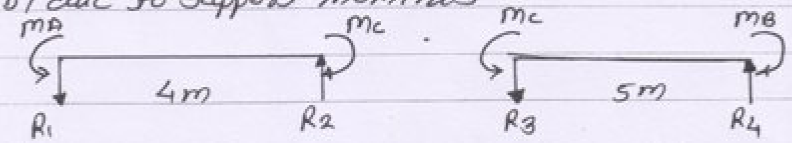
$R_{A1} = R_{C1} = \frac{wl}{2} = \frac{5 \times 4}{2}$

$= 10$ KN

$R_{C2} = R_{B1} = \frac{W}{2} = \frac{20}{2}$

$R_{C2} = R_{B1} = 10$ KN

Q NO	SOLUTION	MARKS
Q5b Cont...	b) due to support moments	



$R_1 = R_2 = \pm\left(\frac{M_A - M_C}{4}\right)$ $R_3 = R_4 = \pm\left(\frac{M_C - M_B}{5}\right)$

$= \frac{-5.04 + 9.935}{4}$ $= \frac{-9.935 + 13.782}{5}$

$R_1 = R_2 = \pm 1.22\ KN$ $R_3 = R_4 = \pm 0.77\ KN$

Final support reactions

$R_A = R_{A1} - R_1 = 10 - 1.22 = 8.78\ KN$

$R_C = R_{C1} + R_{C2} + R_2 - R_3 = 10 + 10 + 1.22 - 0.77 = 20.45\ KN$ 1M

$R_B = R_{B1} + R_4 = 10 + 0.77 = 10.77\ KN$

vi) S.F. Calculations

S.F at just left of A = 0

S.F at just right of A = R_A = 8.78 KN

S.F at just left of C = $8.78 - (5 \times 4) = -11.22\ KN$

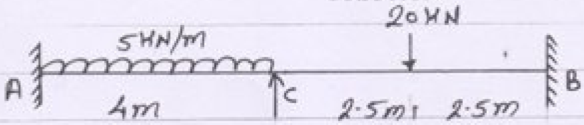
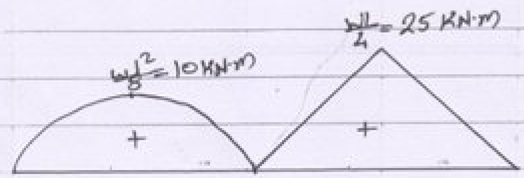
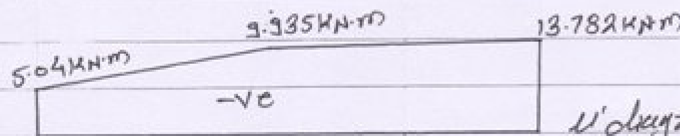
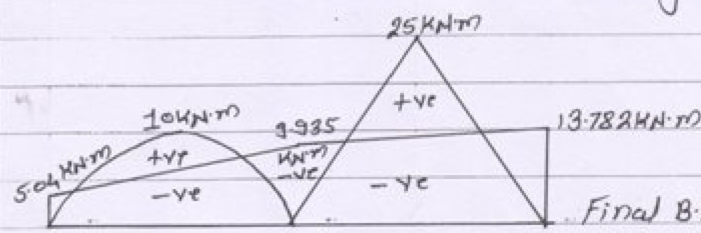
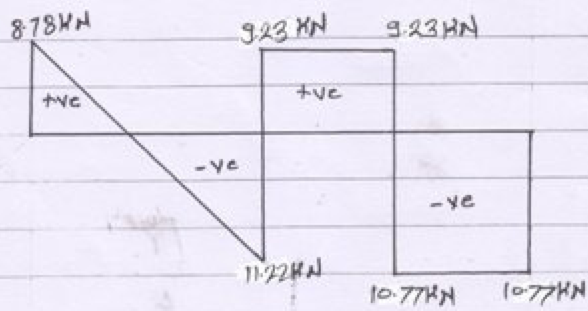
S.F at just right of C = $-11.22 + R_C = -11.22 + 20.45 = 9.23\ KN$

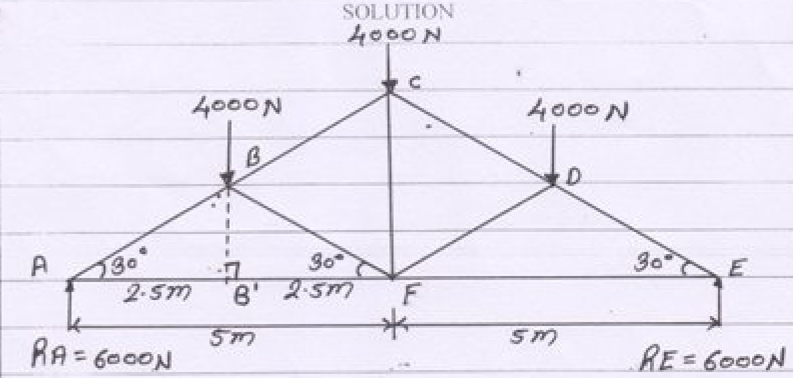
S.F at just left of D = 9.23 KN

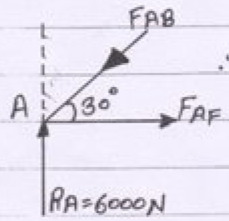
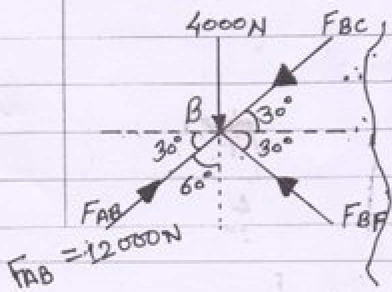
S.F at just right of D = $9.23 - 20 = -10.77\ KN$

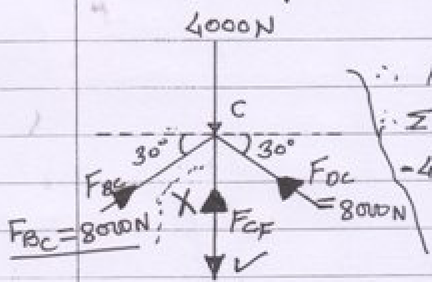
S.F at just left of B = $-10.77\ KN$

S.F. at just right of B = $-10.77 + R_B = 0\ KN$

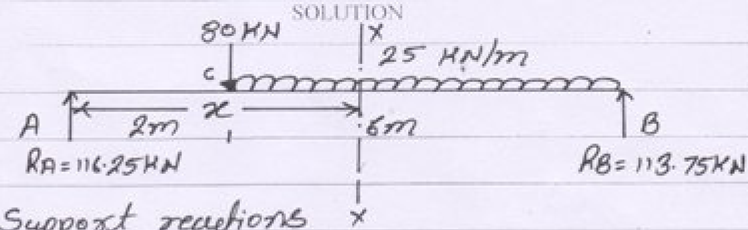
Q.NO	SOLUTION	MARKS
Q5b Cont...		
	 M diagram	
	 M' diagram.	
	 Final B.M.D.	2M.
	 S.F.D	2M

Q NO	SOLUTION	MARKS
Q5c		
	i> Support reactions	
	Due to Symmetrical loading	
	$R_A = R_E = \frac{\text{Total load}}{2} = \frac{4000 \times 3}{2}$	
	$\therefore R_A = R_E = 6000\ N$	1M.
	ii> Geometrical Properties	
	Length of member CF	
	$CF = AF \times \tan 30 = 5 \times \tan 30 = 2.88 m$	
	Length of member AC	
	$AC = \sqrt{AF^2 + CF^2} = \sqrt{5^2 + 2.88^2} = 5.77 m$	
	$\therefore AB = BC = \frac{AC}{2} = \frac{5.77}{2} = 2.88 m$	
	Length of perpendicular BB'	
	$BB' = AB \times \sin 30 = 2.88 \times \sin 30$	
	$BB' = 1.44 m$	

Q.NO	SOLUTION	MARKS
Q5 c) cont...	Length of $AB' = AB \cdot \cos 30$	
	$= 2.88 \cdot \cos 30$	
	$AB' = 2.50\,m$	
	$\therefore \angle BFA = \angle BAF = 30^\circ$	
	* Assume the directions of forces as shown in FBD	
	iii) Consider joint A	
		
	$\Sigma F_y = 0$	
	$\therefore 6000 - F_{AB} \sin 30 = 0$	
	$\therefore F_{AB} = \frac{-6000}{\sin 30} = 12000\,N$ (comp.)	2M
	-ve sign indicate force is Compressive in member	
	iv) Consider joint B	
		
	$\therefore \Sigma F_x = 0$	
	$\therefore F_{AB} \cos 30 - F_{BC} \cos 30 - F_{BF} \cos 30 = 0$	
	$\therefore 12000 \cos 30 - 0.866 F_{BC} - 0.866 F_{BF} = 0$ ----	
	$\boxed{F_{BC} + F_{BF} = 12000\,N}$ ---(i)	

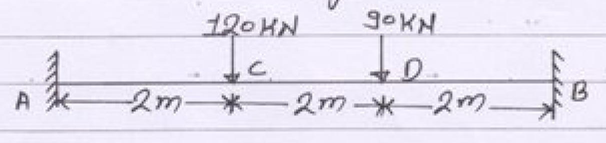
Q NO	SOLUTION	MARKS
Q5 c) Cont...	$\Sigma F_y = 0$	
	$-4000 + F_{AB}\sin 30 + F_{BF}\sin 30 - F_{BC}\sin 30 = 0$	
	$-4000 + 12000 \cdot \sin 30 + 0.5 F_{BF} - 0.50 F_{BC} = 0$	
	$-0.50 F_{BC} + 0.5 F_{BF} = -2000$ ----	
	$\therefore F_{BC} - F_{BF} = 4000$ ------- (ii)	
	Solving eq^n (i) & eq^n (ii) we get	
	$F_{BC} = 8000$ N (comp.)	2M
	$F_{BF} = 4000$ N (comp.)	
	from symmetry the force in member FD	
	$F_{FD} = F_{BF} = 4000$ N (Comp)	1M
*	Assume the directions of the forces as shown FBD *	
	v) Consider joint C	
		
	$\therefore F_{BC} = F_{DC} = 8000$ N	
	$\therefore \Sigma F_y = 0$	
	$-4000 + F_{BC}\sin 30 + F_{DC}\sin 30 + F_{CF} = 0$	
	$-4000 + 8000 \cdot \sin 30 + 8000 \cdot \sin 30 + F_{CF} = 0$	
	$\therefore F_{CF} = -4000$ N	
	− sign indicates the force in F_{CF} is Tensile	

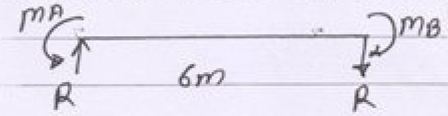
$\therefore F_{CF} = 4000$ N (Tensile) 2M

Q NO.	SOLUTION	MARKS
Q6 a)		
	i) Support reactions	
	$\Sigma F_y = 0$	
	$R_A + R_B - 80 - (25 \times 6) = 0$	
	$R_A + R_B = 230$ KN	
	$\Sigma M_A = 0$	
	$(80 \times 2) + (25 \times 6 \times (2 + \frac{6}{2})) - 8R_B = 0$	
	$\therefore R_B = 113.75$ KN	
	$\therefore R_A = 116.25$ KN	1
	Consider a section x-x at a distance x from A.	
	$M_x = R_A \cdot x - 80(x-2) - 25\frac{(x-2)^2}{2}$	
	Now, $EI\frac{d^2y}{dx^2} = M_x$	
	$EI\frac{d^2y}{dx^2} = 116.25x - 80(x-2) - 25\frac{(x-2)^2}{2}$ ... (A)	1
	Integrate the above w.r.to. x	
	$EI\frac{dy}{dx} = 116.25\frac{x^2}{2} + C_1 - 80\frac{(x-2)^2}{2} - 25\frac{(x-2)^3}{6}$ ... (B)	

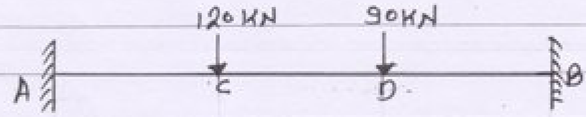
Q.NO	SOLUTION	MARKS
	Again integrate above eq^n w.r.to x	
	$EI \cdot y = 116.25\frac{x^3}{6} + C_1 x + C_2 - 80\frac{(x-2)^3}{6} - 25\frac{(x-2)^4}{24}$ ...(c)	1
	Apply Boundary Condition	
	At A, $x=0$, $y=0$ put in eq^n(C)	
	$EI(0) = 0 + 0 + C_2$	
	$C_2 = 0$	
	At, B, $x = 8$, $y=0$	
	$EI(0) = 116.25\frac{(8)^3}{6} + 8C_1 + 0 - 80\frac{(8-2)^3}{6} - 25\frac{(8-2)^4}{24}$	
	$0 = 9920 + 8C_1 - 2880 - 1350$	
	$0 = 9920 + 8C_1 - 4230$	
	$C_1 = -711.25$	1
	slope equation	
	$EI\frac{dy}{dx} = 116.25\frac{x^2}{2} - 711.25 - 80\frac{(x-2)^2}{2} - 25\frac{(x-2)^3}{6}$ ...	
	Deflection equation	
	$EI\ y = 116.25\frac{x^3}{6} - 711.25x - 80\frac{(x-2)^3}{6} - 25\frac{(x-2)^4}{24}$	1

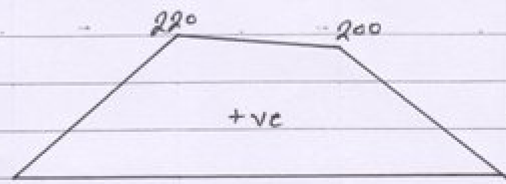
Q. NO.	SOLUTION	MARKS
	Slope at A, put $x = 0$ in slope equation	
	$EI\left(\frac{dy}{dx}\right)_A = 116.25\frac{(0)^2}{2} - 711.25 - 80\frac{(\cancel{0-2})^2}{2} - 25\frac{(\cancel{0-2})^3}{6}$	
	$EI\left(\frac{dy}{dx}\right)_A = 0 - 711.25 - 0 - 0$	
	$\left(\frac{dy}{dx}\right)_A = -\frac{711.25}{EI}$	
	$E = 2\times10^5\ N/mm^2 \rightarrow 2\times10^8\ KN/m^2$	
	$I = 12\times10^6\ mm^4 \rightarrow 12\times10^{-6}\ m^4$	
	$EI = 2\times10^8 \times 12\times10^{-6} = 2400\ KN\cdot m^2$	
	$\therefore\ \theta_A = \frac{-711.25}{2400} = -0.2963$ rad	1
	Slope at B, put $x = 8m$ in slope equation	
	$EI\left(\frac{dy}{dx}\right)_B = 116.25\frac{(8)^2}{2} - 711.25 - 80\frac{(8-2)^2}{2} - 25\frac{(8-2)^3}{6}$	
	$EI\left(\frac{dy}{dx}\right)_B = 3720 - 711.25 - 1440 - 900$	
	$\theta_B = \frac{668.75}{EI} = \frac{668.75}{2400} = 0.2786$ rad	1
	Deflection at C, put $x = 2$ in Deflection eq^n	
	$EI(y) = 116.25\frac{(2)^3}{6} - 711.25(2)$	

Q.NO	SOLUTION	MARKS
	$EI(y)_c = 155 - 1422.5$	
	$y_c = \frac{-1267.5}{EI} = \frac{-1267.5}{2400} = -0.528\ m$	
	$y_c = -528\ mm$ -ve sign indicate downward deflection.	1
Q6 b>		
	i) Support Moments	
	$M_A = -\left[\frac{W_1 a_1 b_1^2}{L^2} + \frac{W_2 a_2 b_2^2}{L^2}\right]$	
	$= -\left[\frac{120 \times 2 \times 4^2}{6^2} + \frac{90 \times 4 \times 2^2}{6^2}\right]$	
	$M_A = -146.67\ KN\cdot m$	1M
	$M_B = -\left[\frac{W_1 a_1^2 b_1}{L^2} + \frac{W_2 a_2^2 b_2}{L^2}\right]$	
	$= -\left[\frac{120 \times 2^2 \times 4}{6^2} + \frac{90 \times 4^2 \times 2}{6^2}\right]$	
	$M_B = -133.33\ KN\cdot m$	1M
	ii) Support reactions	
	a) due to loading only.	

Q.NO	SOLUTION	MARKS
Q6 b Cont...	$\Sigma F_y = 0 \quad R_A + R_B = 120 + 90 = 210\ KN$	
	$\Sigma M@A = 0$	
	$(120 \times 2) + (90 \times 4) - 6R_B = 0$	
	$\therefore R_B = 100\ KN$	
	$\therefore R_A = 110\ KN$	1 M
	b) Reactions due to end moments only.	
		
	$R = \frac{M_A - M_B}{L} = \frac{146.67 - 133.33}{6} = 2.22\ KN$	
	c) final reaction	
	$\therefore R_A = 110 + 2.22 = 112.22\ KN$	
	$\therefore R_B = 100 - 2.22 = 97.78\ KN$	1 M
	iii) S.F. Calculation.	
	S.F at A = 112.22 KN	
	S.F at just left of C = 112.22	
	S.F at just right of C = 112.22 − 120 = −7.78 KN	
	S.F at just left of D = −7.78 KN	
	S.F at just right of D = −7.78 − 90 = −97.78 KN	
	S.F at just left of B = −97.78 KN	
	S.F at just right of B = $-97.78 + R_B = 0$ KN.	

Q NO.	SOLUTION	MARKS
Q6 b Cont...	Free B.M.	
	$M_C = R_A \times 2 = 110 \times 2 = 220$ KN.m	
	$M_D = R_A \times 4 - (120 \times 2) = (110 \times 4) - (240) = 200$ KN.m	





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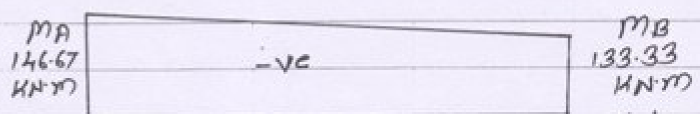

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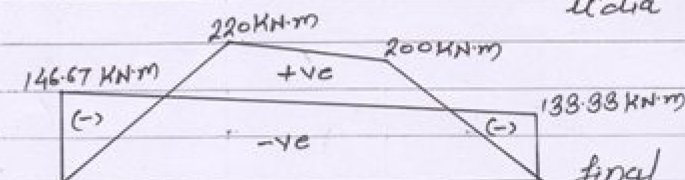
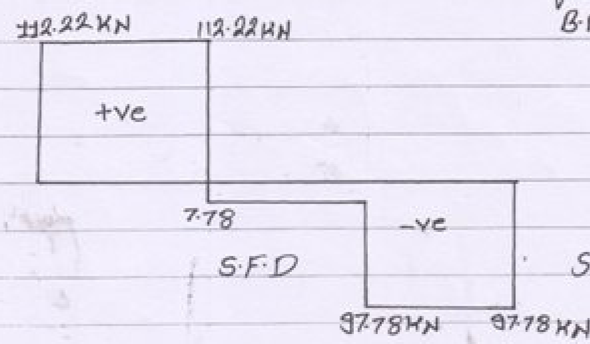
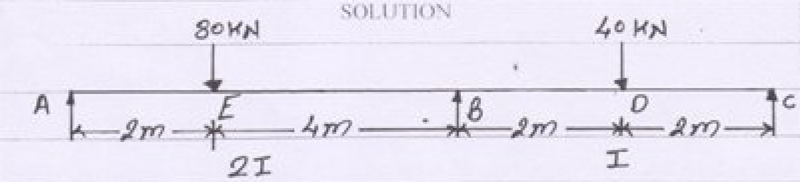


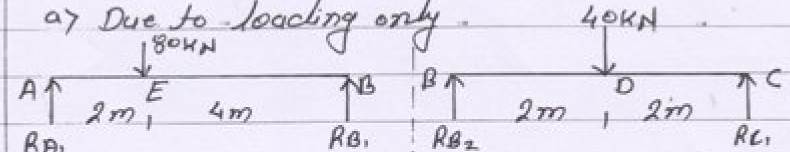
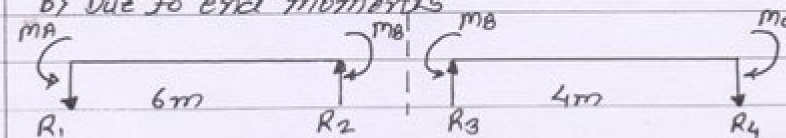
final B.M.D 2M



S.F.D 2M

S.F.D

Q NO	SOLUTION	MARKS
Q6 c)		
	i) Assume the spans AB & BC as simply Supported and draw μ diagram.	
	Bending moment under 80 KN, $M_E = \frac{Wab}{L} = \frac{80\times2\times4}{6} = 106.67$ KN·m	
	Bending moment under 40 KN, $M_D = \frac{WL}{4} = \frac{40\times4}{4} = 40$ KN·m.	1M
	ii) Applying clapeyron's theorem to spans AB & BC	
	$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1}+\frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right) = -\left[\frac{6a_1x_1}{L_1I_1} + \frac{6a_2x_2}{L_2I_2}\right]$	1M
	$M_A = M_C = 0$ KN·m --- s.s. ends	
	$a_1 = \frac{1}{2}\times6\times106.67 = 320$ | $a_2 = \frac{1}{2}\times4\times40$, $a_2 = 80$	
	$x_1 = \frac{L+a}{3} = \frac{6+2}{3} = 2.67$ | $x_2 = \frac{L_2}{2} = \frac{4}{2} = 2m$	1M
	$L_1 = 6m$ | $L_2 = 4m$	
	$I_1 = 2$ | $I_2 = 1$	
	$0 + 2M_B\left[\frac{6}{2}+\frac{4}{1}\right] = -\left[\frac{6\times320\times2.67}{6\times2} + \frac{6\times80\times2}{4\times1}\right]$	

Q NO	SOLUTION	MARKS
Q6 c Cont...	$14 M_B = -[427.2 + 240]$	
	$\therefore M_B = 47.65$ KN·m Hogging.	2M
	$M_A = M_C = 0$	
	iii) Support reactions	
	a) Due to loading only 	
	$R_{A_1} = \frac{80 \times 4}{6}$ $= \frac{80 \times 2}{6}$ $R_{B_2} = R_{C_1} = \frac{W}{2} = \frac{40}{2}$	1 M
	$R_{A_1} = 53.33$ KN $R_{B_1} = 26.67$ KN $R_{B_2} = R_{C_1} = 20$ KN	
	b) Due to end moments 	
	$R_1 = R_2 = \pm \frac{M_A - M_B}{L}$ $R_3 = R_4 = \pm \frac{M_B - M_C}{L}$	
	$= \frac{0 - 47.65}{6}$ $= \frac{47.65 - 0}{4}$	1 M
	$R_1 = R_2 = \pm 7.94$ KN. $R_3 = R_4 = \pm 11.91$ KN	
	Final Support reactions	
	$R_A = R_{A_1} - R_1 = 53.33 - 7.94 = 45.39$ KN	1M
	$R_B = R_{B_1} + R_{B_2} + R_2 + R_3 = 26.67 + 20 + 7.94 + 11.91 = 66.52$ KN	
	$R_C = R_{C_1} - R_4 = 20 - 11.91 = 8.09$ KN	