22480

12425

3 Hours / 70 Marks Seat No. | | | | | | | | |

Instructions : (1) All Questions are *compulsory.*

(2) Illustrate your answers with neat sketches wherever necessary.

(3) Figures to the right indicate full marks.

(4) Assume suitable data, if necessary.

(5) Use of Non-programmable Electronic Pocket Calculator is permissible.

Marks

1. Attempt any FIVE of the following : **10**

(a) If $f(x, y) = 2x + 3y$ find $\frac{\partial f}{\partial x}$.

(b) If $f(x, y) = \sin(x^2 + y^2)$ find $\frac{\partial^2 f}{\partial x \cdot \partial y}$.

(c) Find the Eigen value of matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

(d) Find rank of matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{bmatrix}$.

(e) Find value of 'p' if the vectors $\bar{a} = 3\bar{i} - 4\bar{j} + \bar{k}$ & $2\bar{i} + 5\bar{j} + p\bar{k}$ are perpendicular to each other.

(f) Find the projection of $\bar{a} = 2\bar{i} + \bar{j} + \bar{k}$ on $\bar{b} = \bar{i} + 3\bar{j} + \bar{k}$.

(g) Construct forward difference for the following data :

x_1	0	5	10	15	20	25
f(*x*)	7	11	14	18	24	32

2. Attempt any THREE of the following : **12**

(a) Examine $f(x, y) = x^3 - y^2 - 3x$ for maximum & minimum value.

(b) Find the angle between the vectors $\bar{a} = 2\bar{i} + 3\bar{j} + \bar{k}$ & $\bar{b} = \bar{i} - 3\bar{j} - \bar{k}$.

(c) Examine the following linear system of equation for consistency & solve it if consistent :

$$4x - 2y + 6z = 8,\ x + y - 3z = -1,\ 15x - 3y + 9z = 21$$

(d) Reduce the following matrix to normal form and hence find its rank :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

3. Attempt any THREE of the following : **12**

(a) Find the maximum & minimum values of function $3x + 4y$ subject to constraint $x^2 + y^2 = 100$.

(b) Find the inverse of following matrix by elementary transformation :

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 1 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

(c) Determine the value of λ for which the system of equations is consistent :

$$x + y + 4z = 1,\ x + 2y - 2z = 1,\ \lambda x + y + z = 1$$

(d) If $f(x, y) = x^3y + e^{xy^2}$ show that

$$\frac{\partial^2 f}{\partial x \cdot \partial y} = \frac{\partial^2 f}{\partial y \cdot \partial x}$$

4. Attempt any THREE of the following : **12**

(a) Find eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 10 & -9 \\ 6 & -5 \end{bmatrix}$.

(b) Show that the equations $2x + 6y = -11$, $6x + 20y - 6z = -3$, $6y - 18z = -1$ are not consistent.

(c) Find a vector of magnitude $\sqrt{3}$ units & perpendicular to the vectors

$\bar{a} = 2\bar{i} + \bar{j} - 3\bar{k}$ & $\bar{b} = \bar{i} - 2\bar{j} + \bar{k}$.

(d) If $\bar{a} = 2\bar{i} + 3\bar{j} + 4\bar{k}$, $\bar{b} = \bar{i} + 2\bar{j} - 3\bar{k}$ and $\bar{c} = 3\bar{i} + 4\bar{j} - \bar{k}$, then find $\bar{a} \times (\bar{b} \times \bar{c})$.

(e) Find the work done by the force $\bar{F} = 6\bar{i} + 10\bar{j} - 3\bar{k}$ when its point of applications moves from (2, 3, 1) to (2, 4, 2).

5. Attempt any TWO of the following : **12**

(a) Given :

x	5	10	15	20
f(x)	50	70	100	145

Estimate f(8) using Newton's forward difference interpolation formula.

(b) Find y'(0) & y''(0) from following data. Find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ at $x = 0$.

x	0	1	2	3	4	5
y	4	8	15	7	6	2

(c) Solve the following linear programming problem graphically to find optimal solution :

Maximize $z = 2x + 5y$

subject to $x + 2y \leq 16$,

$5x + 3y \leq 45$

$x, y \geq 0$

6. Attempt any TWO of the following : **12**

(a) Given the cube of integers in following data. Find the value of $(12)^3$ using extrapolation :

x	2	5	8	11
y	8	125	512	1331

(b) (i) Evaluate $\int_2^7 \frac{1}{x^2} dx$ using trapezoidal rule & by dividing the interval [2, 7]. into five equal sub-interval.

(ii) Evaluate $\int_1^3 \frac{dx}{x}$ taking h = 0.5 by Simpson's one third rule.

(c) Solve the following linear programming problem using simplex method to find optimal solutions :

Maximize $Z = 6x + 10y$

subject to $2x + y \leq 104$

$x + 2y \leq 76$

$x \geq 0, y \geq 0.$