



WINTER- 18 EXAMINATION

Subject Name: Applied Mathematics

Model Answer

Subject Code:

22210

**Important Instructions to Examiners:**

- 1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
- 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
- 3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills).
- 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
- 5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
- 6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
- 7) For programming language papers, credit may be given to any other program based on equivalent concept.

| Q. No. | Sub Q.N. | Answers   | Marking Scheme |
|--------|----------|---|----------------|
| 1.     |          | <b>Solve any FIVE of the following:</b>                 | <b>10</b>      |
|        | a)       | If $f(x) = 3x^2 - 5x + 7$ show that $f(-1) = 3f(1)$     | <b>02</b>      |
|        | Ans      | $f(-1) = 15$  | 1              |
|        |          | $f(1) = 5$  | ½              |
|        |          | $\therefore 3f(1) = 15$                                 | ½              |
|        |          | $\therefore f(-1) = 3f(1)$                              |                |
|        | b)       | Define odd and even function with suitable examples.    | <b>02</b>      |
|        | Ans      | If $f(-x) = -f(x)$ then the function is an odd function | ½              |
|        |          | e.g. $f(x) = x^3 + x$                                   | ½              |
|        |          | $f(-x) = (-x)^3 + (-x)$                                 |                |
|        |          | $\therefore f(-x) = -(x^3 + x)$                         |                |
|        |          | $\therefore f(-x) = -f(x)$                              |                |
|        |          | If $f(-x) = f(x)$ then the function is an even function | ½              |
|        |          | e.g. $f(x) = x^2 + 1$                                   | ½              |
|        |          | $\therefore f(-x) = (-x)^2 + 1$                         |                |
|        |          | $\therefore f(-x) = x^2 + 1$                            |                |
|        |          | $\therefore f(-x) = f(x)$                               |                |



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| 1.     | c)  | Find $\frac{dy}{dx}$ if $y = a^x + x^a + a^a + \sqrt{x}$   | 02             |
|        | Ans   | $y = a^x + x^a + a^a + \sqrt{x}$ $\therefore \frac{dy}{dx} = a^x \log a + ax^{a-1} + 0 + \frac{1}{2\sqrt{x}}$ $\therefore \frac{dy}{dx} = a^x \log a + ax^{a-1} + \frac{1}{2\sqrt{x}}$ | 2              |
|        | d)  | Evaluate $\int \frac{1}{x^2 + 4} dx$   | 02             |
|        | Ans   | $\int \frac{1}{x^2 + 4} dx$ $= \int \frac{1}{x^2 + (2)^2} dx$ $= \frac{1}{2} \tan^{-1} \left( \frac{x}{2} \right) + c$   | 1/2<br>1 1/2   |
| e)     | Evaluate $\int x.e^x dx$  | 02   |                |
| Ans    | $\int x.e^x dx$ $= x \left( \int e^x dx \right) - \int \left( \int e^x dx \frac{d}{dx}(x) \right) dx$ $= xe^x - \int e^x \cdot 1 dx$ $= xe^x - \int e^x dx$ $= xe^x - e^x + c$              | 1/2<br>1/2<br>1/2<br>1/2   |                |
| f)     | If $z_1 = 4 - 5i$ and $z_2 = 3 + 7i$ find $ z_1 + z_2 $   | 02   |                |
| Ans    | $z_1 + z_2 = 4 - 5i + 3 + 7i$ $\therefore z_1 + z_2 = 7 + 2i$ $\therefore  z_1 + z_2  = \sqrt{(7)^2 + (2)^2}$ $\therefore  z_1 + z_2  = \sqrt{49 + 4}$ $\therefore  z_1 + z_2  = \sqrt{53}$ | 1<br>1   |                |



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| 1.     | g)       | Find the area bounded by the curve $y = 3x^2$ , the lines $x = 1$ , $x = 3$ and $x$ -axis   | 02                                  |
|        | Ans      | $\text{Area } A = \int_a^b y \, dx$ $= \int_1^3 3x^2 \, dx$ $= 3 \left[ \frac{x^3}{3} \right]_1^3$ $= 3 \left( \frac{3^3}{3} - \frac{1^3}{3} \right)$ $= 26$  | 1<br>1                              |
| 2.     |          | <b>Solve any <u>THREE</u> of the following:</b>   | 12                                  |
|        | a)       | Find $\frac{dy}{dx}$ if $x^3 + y^3 = 3axy$  | 04                                  |
|        | Ans      | $x^3 + y^3 = 3axy$ $\therefore 3x^2 + 3y^2 \frac{dy}{dx} = 3a \left( x \frac{dy}{dx} + y \cdot 1 \right)$ $\therefore 3x^2 + 3y^2 \frac{dy}{dx} = 3ax \frac{dy}{dx} + 3ay$ $\therefore (3y^2 - 3ax) \frac{dy}{dx} = 3ay - 3x^2$ $\therefore \frac{dy}{dx} = \frac{3ay - 3x^2}{3y^2 - 3ax}$ $\therefore \frac{dy}{dx} = \frac{ay - x^2}{y^2 - ax}$ | 2<br>$\frac{1}{2}$<br>$\frac{1}{2}$ |
|        | b)       | Find $\frac{dy}{dx}$ if $x = \frac{1}{t}$ and $y = 1 - \frac{1}{t}$   | 04                                  |
|        | Ans      | $x = \frac{1}{t} \text{ and } y = 1 - \frac{1}{t}$ $\therefore \frac{dx}{dt} = -\frac{1}{t^2} \text{ and } \frac{dy}{dt} = \frac{1}{t^2}$ $\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$  | 1+1                                 |



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| 2.     | b)        | $\therefore \frac{dy}{dx} = \frac{1/t^2}{-1/t^2}$ $\therefore \frac{dy}{dx} = -1$   | 1<br>1                          |
|        | c)        | <p>A bullet is fired into a mud bank and penetrates <math>(120t - 3600t^2)</math> m. in 't' sec. after impact. Calculate maximum depth of penetration.</p> <p>Ans Let <math>s = 120t - 3600t^2</math></p> $\therefore \frac{ds}{dt} = 120 - 7200t$ $\therefore \frac{d^2s}{dt^2} = -7200$ <p>Consider <math>\frac{ds}{dt} = 0</math></p> $\therefore 120 - 7200t = 0$ $\therefore 120 = 7200t$ $\therefore t = \frac{1}{60}$ <p>at <math>t = \frac{1}{60}</math></p> $\therefore \frac{d^2s}{dt^2} = -7200 < 0$ <p><math>\therefore</math> The maximum depth of penetration is,</p> $s = 120\left(\frac{1}{60}\right) - 3600\left(\frac{1}{60}\right)^2$ $\therefore s = 1 \text{ meter}$ | 04<br>1<br>1<br>1<br>1/2<br>1/2 |
|        | d)        | <p>Find radius of curvature to the curve <math>y = x^3</math> at <math>(2,8)</math></p> <p>Ans <math>y = x^3</math></p> $\therefore \frac{dy}{dx} = 3x^2$ $\therefore \frac{d^2y}{dx^2} = 6x$   | 04<br>1<br>1                    |



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| 2.     | d)       | <p>at (2,8)</p> $\frac{dy}{dx} = 3(2)^2 = 12$ $\frac{d^2y}{dx^2} = 6(2) = 12$ $\therefore \text{Radius of curvature is } \rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ $\therefore \rho = \frac{\left[1 + (12)^2\right]^{\frac{3}{2}}}{12}$ $\therefore \rho = 145.50$  | <p>1/2</p> <p>1/2</p>                   |
| 3.     |          | <p>Solve any <b>THREE</b> of the following:</p>   | 12                                      |
|        | a)       | <p>Find the equation of the tangent to the curve <math>4x^2 + 9y^2 = 40</math> at (3, 2)</p>  | 04                                      |
|        | Ans      | $4x^2 + 9y^2 = 40$ $8x + 18y \frac{dy}{dx} = 0$ $\therefore \frac{dy}{dx} = \frac{-8x}{18y}$ <p>at (3, 2)</p> $\therefore \frac{dy}{dx} = \frac{-8(3)}{18(2)}$ $\therefore \frac{dy}{dx} = \frac{-2}{3}$ <p><math>\therefore</math> slope of tangent, <math>m = \frac{-2}{3}</math></p> <p>Equation of tangent at (3, 2) is</p> $y - 2 = \frac{-2}{3}(x - 3)$ $\therefore 3y - 6 = -2x + 6$ $\therefore 2x + 3y - 12 = 0$ | <p>1</p> <p>1/2</p> <p>1</p> <p>1/2</p> |



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| 3.     | b)       | Find $\frac{dy}{dx}$ if $y = \sec^{-1} \left[ \frac{1}{4x^3 - 3x} \right]$  | <b>04</b>      |
|        | Ans      | $y = \sec^{-1} \left[ \frac{1}{4x^3 - 3x} \right]$ <p>Put <math>x = \cos \theta \Rightarrow \theta = \cos^{-1} x</math></p> $\therefore y = \sec^{-1} \left[ \frac{1}{4 \cos^3 \theta - 3 \cos \theta} \right]$ $\therefore y = \sec^{-1} \left[ \frac{1}{\cos 3\theta} \right]$ $\therefore y = \sec^{-1} [\sec 3\theta]$ $\therefore y = 3\theta$ $\therefore y = 3 \cos^{-1} x$ $\therefore \frac{dy}{dx} = 3 \left( \frac{-1}{\sqrt{1-x^2}} \right)$ $\therefore \frac{dy}{dx} = \frac{-3}{\sqrt{1-x^2}}$ |                |
|        | c)       | If $y^x = e^y$ prove that $\frac{dy}{dx} = \frac{(\log y)^2}{\log y - 1}$   | <b>04</b>      |
|        | Ans      | $y^x = e^y$ <p>taking log on both sides,</p> $\therefore \log y^x = \log e^y$ $\therefore x \log y = y \log e$ $\therefore x \log y = y$ $\therefore x = \frac{y}{\log y}$ <p>diff.w.r.t.y</p> $\therefore \frac{dx}{dy} = \frac{\log y(1) - y \frac{1}{y}}{(\log y)^2}$ $\therefore \frac{dx}{dy} = \frac{\log y - 1}{(\log y)^2}$   |                |



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| 3.     | c)       | $\therefore \frac{dy}{dx} = \frac{(\log y)^2}{\log y - 1}$ <p><b>OR</b></p> $y^x = e^y$ <p>taking log on both sides,</p> $\therefore \log y^x = \log e^y$ $\therefore x \log y = y \log e$ $\therefore x \log y = y \quad \text{-----}(i)$ <p>diff.w.r.t.x</p> $\therefore x \frac{1}{y} \frac{dy}{dx} + \log y(1) = \frac{dy}{dx}$ $\therefore \log y = \left(1 - \frac{x}{y}\right) \frac{dy}{dx}$ $\therefore \frac{\log y}{\left(1 - \frac{x}{y}\right)} = \frac{dy}{dx}$ $\therefore \frac{\log y}{\left(1 - \frac{1}{\log y}\right)} = \frac{dy}{dx} \quad \text{-----}(\because eq^n.(i))$ $\therefore \frac{(\log y)^2}{(\log y - 1)} = \frac{dy}{dx}$ $\therefore \frac{dy}{dx} = \frac{(\log y)^2}{\log y - 1}$ | <p>1/2</p> <p>1</p> <p>1/2</p> <p>1</p> <p>1/2</p> <p>1/2</p> |
|        | d)       | <p>Evaluate <math>\int \frac{(x-1)e^x}{x^2 \sin^2\left(\frac{e^x}{x}\right)} dx</math></p>  | 04  |
|        | Ans      | $\int \frac{(x-1)e^x}{x^2 \sin^2\left(\frac{e^x}{x}\right)} dx$   |   |
|        |          | <p>Put <math>\frac{e^x}{x} = t</math></p>   | 1/2   |



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| 3.     | d)       | $\therefore \frac{xe^x - e^x(1)}{x^2} dx = dt$ $\therefore \frac{(x-1)e^x}{x^2} dx = dt$ $\therefore \int \frac{1}{\sin^2 t} dt$ $= \int \operatorname{cosec}^2 t dt$ $= -\cot t + c$ $= -\cot \left( \frac{e^x}{x} \right) + c$   | <p>½</p> <p>½</p> <p>1</p> <p>1</p> <p>½</p>                              |
| 4.     |          | <p>Solve any <b>THREE</b> of the following:</p> <p>a) Evaluate <math>\int \frac{dx}{4\cos^2 x + 9\sin^2 x}</math></p> <p>Ans <math>\int \frac{dx}{4\cos^2 x + 9\sin^2 x}</math></p> $= \int \frac{\frac{dx}{\cos^2 x}}{4\cos^2 x + 9\sin^2 x}$ $= \int \frac{\sec^2 x dx}{4 + 9\tan^2 x}$ <p>Put <math>\tan x = t</math></p> $\sec^2 x dx = dt$ $= \int \frac{dt}{4 + 9t^2}$ $= \int \frac{dt}{\left(\frac{2}{3}\right)^2 + (3t)^2} \quad \text{or} \quad = \frac{1}{9} \int \frac{dt}{\left(\frac{2}{3}\right)^2 + t^2}$ $= \frac{1}{2} \frac{\tan^{-1}\left(\frac{3t}{2}\right)}{3} + c \quad \text{or} \quad = \frac{1}{9\left(\frac{2}{3}\right)} \tan^{-1}\left(\frac{t}{\frac{2}{3}}\right) + c$ | <p>12</p> <p>04</p> <p>½</p> <p>½</p> <p>½</p> <p>½</p> <p>½</p> <p>1</p> |





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|--------|--|--|--|
| 4.     | a)   | $= \frac{1}{6} \tan^{-1} \left( \frac{3 \tan x}{2} \right) + c$  | ½  |
|        | b)   | Evaluate: $\int \frac{\log x}{x[2 + \log x][3 + \log x]} dx$   | <b>04</b>                                      |
|        | Ans  | $\int \frac{\log x}{x[2 + \log x][3 + \log x]} dx$ <p>Put <math>\log x = t</math></p> $\therefore \frac{1}{x} dx = dt$ $\int \frac{t}{(2+t)(3+t)} dt$ <p>consider <math>\frac{t}{(2+t)(3+t)} = \frac{A}{2+t} + \frac{B}{3+t}</math></p> $\therefore t = A(3+t) + B(2+t)$ <p>Put <math>t = -2</math></p> $A = -2$ <p>Put <math>t = -3</math></p> $B = 3$ $\therefore \frac{t}{(2+t)(3+t)} = \frac{-2}{2+t} + \frac{3}{3+t}$ $\therefore \int \frac{t}{(2+t)(3+t)} dt = \int \left( \frac{-2}{2+t} + \frac{3}{3+t} \right) dt$ $= -2 \log(2+t) + 3 \log(3+t) + c$ $= -2 \log(2 + \log x) + 3 \log(3 + \log x) + c$ | ½<br><br>½<br><br>½<br><br>½<br><br>1<br><br>½ |
| c)     | Evaluate $\int_2^5 \frac{\sqrt{x}}{\sqrt{7-x} + \sqrt{x}} dx$  | <b>04</b>  |  |
| Ans    | $I = \int_2^5 \frac{\sqrt{x}}{\sqrt{7-x} + \sqrt{x}} dx \text{-----(1)}$ $\therefore I = \int_2^5 \frac{\sqrt{(2+5-x)}}{\sqrt{7-(2+5-x)} + \sqrt{(2+5-x)}} dx$ $\therefore I = \int_2^5 \frac{\sqrt{7-x}}{\sqrt{x} + \sqrt{7-x}} dx \text{-----(2)}$ | 1<br><br>½   |  |



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| 4.     | c)  | <p>add (1) and (2)</p> $I + I = \int_2^5 \frac{\sqrt{x}}{\sqrt{7-x} + \sqrt{x}} dx + \int_2^5 \frac{\sqrt{7-x}}{\sqrt{x} + \sqrt{7-x}} dx$ $\therefore 2I = \int_2^5 \frac{\sqrt{x} + \sqrt{7-x}}{\sqrt{7-x} + \sqrt{x}} dx$ $\therefore 2I = \int_2^5 1 dx$ $\therefore 2I = [x]_2^5$ $\therefore 2I = 5 - 2$ $\therefore 2I = 3$ $\therefore I = \frac{3}{2}$  | <p>1/2</p> <p>1</p> <p>1</p>                                  |
|        | d)  | <p>Evaluate <math>\int x \cdot \tan^{-1} x dx</math></p>   | 04  |
|        | Ans   | $\int \tan^{-1} x \cdot x dx$ $= \tan^{-1} x \int x dx - \int \left( \int x dx \frac{d}{dx} (\tan^{-1} x) \right) dx$ $= \tan^{-1} x \frac{x^2}{2} - \int \frac{x^2}{2} \frac{1}{1+x^2} dx$ $= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{x^2}{1+x^2} dx$ $= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \left( \frac{1+x^2-1}{1+x^2} \right) dx$ $= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \left( \frac{1+x^2}{1+x^2} - \frac{1}{1+x^2} \right) dx$ $= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \left( 1 - \frac{1}{1+x^2} \right) dx$ $= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} (x - \tan^{-1} x) + c$ | <p>1/2</p> <p>1</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1</p> |
| e)     | <p>Evaluate <math>\int \frac{x}{(x+1)(x+2)} dx</math></p> | 04   |   |



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| 4.     | e)       | <p>Consider <math>\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}</math></p> <p><math>\therefore x = A(x+2) + B(x+1)</math></p> <p>Put <math>x = -1</math></p> <p><math>\therefore -1 = A(-1+2)</math></p> <p><math>\therefore A = -1</math></p> <p>Put <math>x = -2</math></p> <p><math>\therefore -2 = B(-2+1)</math></p> <p><math>\therefore B = 2</math></p> <p><math>\frac{x}{(x+1)(x+2)} = \frac{-1}{x+1} + \frac{2}{x+2}</math></p> <p><math>\therefore \int \frac{x}{(x+1)(x+2)} dx = -\int \frac{1}{x+1} dx + 2\int \frac{1}{x+2} dx</math></p> <p><math>= -\log(x+1) + 2\log(x+2) + c</math></p> | <p>1/2</p> <p>1</p> <p>1</p> <p>1/2</p> <p>1</p> |
| 5.     |          | <p><b>Solve any TWO of the following:</b></p>   | 12   |
|        | a)       | <p>Find by integration the area between the curves <math>y = x^2 + 1</math> and line <math>y = 2x + 1</math></p>  | 06   |
|        | Ans      | <p><math>y = x^2 + 1</math> ----- (1)</p> <p><math>y = 2x + 1</math></p> <p><math>\therefore \text{eq}^n (1) \Rightarrow 2x + 1 = x^2 + 1</math></p> <p><math>\therefore 2x + 1 - x^2 - 1 = 0</math></p> <p><math>\therefore 2x - x^2 = 0</math></p> <p><math>\therefore x = 0, 2</math></p> <p>Area <math>A = \int_a^b (y_1 - y_2) dx</math></p> <p><math>\therefore A = \int_0^2 (x^2 + 1 - (2x + 1)) dx</math></p> <p><math>\therefore A = \int_0^2 (x^2 - 2x) dx</math></p> <p><math>\therefore A = \left[ \frac{x^3}{3} - x^2 \right]_0^2</math></p>   | <p>1</p> <p>1</p> <p>2</p>                       |



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| 5.     | a)       | $\therefore A = \left[ \frac{(2)^3}{3} - (2)^2 \right]$ $\therefore A = \frac{-4}{3}$ $\therefore \text{area } A = \frac{4}{3} \quad \text{or} \quad 1.333$  | 1              |
|        | b)       | Solve the following.   | 06             |
|        | (i)      | Verify that $y = \log x$ is a solution of differential equation $x \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$  | 03             |
|        | Ans      | $y = \log x$   | 1              |
|        |          | $\frac{dy}{dx} = \frac{1}{x}$  | 1              |
|        |          | $\frac{d^2y}{dx^2} = -\frac{1}{x^2}$   | 1              |
|        |          | $L.H.S. = x \frac{d^2y}{dx^2} + \frac{dy}{dx}$ $= x \left( -\frac{1}{x^2} \right) + \frac{1}{x}$ $= -\frac{1}{x} + \frac{1}{x}$ $= 0$ $= R.H.S.$   | 1/2            |
|        |          | <b>OR</b>  |                |
|        |          | $y = \log x$ $\therefore \frac{dy}{dx} = \frac{1}{x}$ $\therefore x \frac{dy}{dx} = 1$ $\therefore x \frac{d^2y}{dx^2} + \frac{dy}{dx} (1) = 0$ $\therefore x \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ | 1              |
|        |          |  | 1/2            |



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| 5.     | b)(ii)   | The velocity of a particle is given by $v = t^2 - 6t + 7$ . Find distance covered in 3 second, initially $x = 0$ when $t = 0$   | <b>03</b>   |
|        | Ans      | $v = t^2 - 6t + 7$<br>$\therefore \frac{dx}{dt} = t^2 - 6t + 7$<br>$\therefore dx = (t^2 - 6t + 7) dt$<br>$\therefore \int dx = \int (t^2 - 6t + 7) dt$<br>$\therefore x = \frac{t^3}{3} - 3t^2 + 7t + c$<br>Initially $x = 0$ when $t = 0$<br>$\therefore c = 0$<br>$\therefore x = \frac{t^3}{3} - 3t^2 + 7t$<br>Distance covered in 3 sec,<br>$\therefore x = \frac{(3)^3}{3} - 3(3)^2 + 7(3)$<br>$\therefore x = 3$ | $\frac{1}{2}$<br>$\frac{1}{2}$<br>1<br>$\frac{1}{2}$<br>$\frac{1}{2}$ |
|        | c)       | Solve the following.  | <b>06</b>   |
|        | (i)      | Solve $(1+x^2)dy - (1+y^2)dx = 0$   | <b>03</b>   |
|        | Ans      | $(1+x^2)dy - (1+y^2)dx = 0$<br>$\therefore (1+x^2)dy = (1+y^2)dx \quad \therefore \frac{dy}{1+y^2} = \frac{dx}{1+x^2}$<br>$\therefore \int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$<br>$\therefore \tan^{-1} y = \tan^{-1} x + c$  | 1<br>1<br>1   |
|        | c)(ii)   | Solve $\frac{dy}{dx} + y \cot x = \cos x$   | <b>03</b>   |
|        | Ans      | $\frac{dy}{dx} + y \cot x = \cos x$<br>$\therefore$ Comparing with $\frac{dy}{dx} + Py = Q$   |   |



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| 5.     | c)(ii)   | $P = \cot x$ , $Q = \cos x$<br>Integrating factor $IF = e^{\int \cot x dx} = e^{\log(\sin x)} = \sin x$<br>$y \cdot IF = \int Q \cdot IF dx + c$<br>$\therefore y \sin x = \int \cos x \cdot \sin x dx$<br>$\therefore y \sin x = \frac{1}{2} \int 2 \cos x \cdot \sin x dx$<br>$\therefore y \sin x = \frac{1}{2} \int \sin 2x dx$<br>$\therefore y \sin x = \frac{1}{2} \left( \frac{-\cos 2x}{2} \right) + c$<br>$\therefore y \sin x = \frac{-\cos 2x}{4} + c$<br><br><u>OR</u><br>$\frac{dy}{dx} + y \cot x = \cos x$<br>$\therefore$ Comparing with $\frac{dy}{dx} + Py = Q$<br>$P = \cot x$ , $Q = \cos x$<br>Integrating factor $IF = e^{\int \cot x dx} = e^{\log(\sin x)} = \sin x$<br>$y \cdot IF = \int Q \cdot IF dx + c$<br>$\therefore y \sin x = \int \cos x \cdot \sin x dx$<br>Put $\sin x = t$<br>$\therefore \cos x dx = dt$<br>$= \int t dt$<br>$= \frac{t^2}{2} + c$<br>$\therefore y \sin x = \frac{(\sin x)^2}{2} + c$ | <p>1</p> <p>½</p> <p>½</p> <p>1</p> <p>1</p> <p>½</p> <p>½</p> <p>1</p> |
| 6.     |          | <p>Solve any <u>TWO</u> of the following:</p> <p>a) If <math>\omega_1 = \frac{-1}{2} + i \frac{\sqrt{3}}{2}</math> , <math>\omega_2 = \frac{-1}{2} - i \frac{\sqrt{3}}{2}</math> show that <math>\omega_1^2 = \omega_2</math></p>  | <p>12</p> <p>06</p>   |



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| 6.     | a)       | $\omega_1^2 = \left( \frac{-1}{2} + i \frac{\sqrt{3}}{2} \right)^2$ $= \frac{1}{4} - 2 \left( \frac{1}{2} \right) \left( i \frac{\sqrt{3}}{2} \right) + i^2 \left( \frac{\sqrt{3}}{2} \right)^2$ $= \frac{1}{4} - i \frac{\sqrt{3}}{2} - \frac{3}{4}$ $= \frac{-1}{2} - i \frac{\sqrt{3}}{2}$ $= \omega_2$ $\therefore \omega_1^2 = \omega_2$ | 2                     |
|        | Ans      |   | 2                     |
|        |          |   | 2                     |
|        | b)       | Find $L\{e^3 t(t^2 + t)\}$  | <b>06</b>             |
|        | Ans      | $L\{e^3 t(t^2 + t)\}$ $= L\{e^3 (t^3 + t^2)\}$ $= e^3 L\{t^3 + t^2\}$ $= e^3 \{L(t^3) + L(t^2)\}$ $= e^3 \left( \frac{3!}{s^{3+1}} + \frac{2!}{s^{2+1}} \right)$ $= e^3 \left( \frac{6}{s^4} + \frac{2}{s^3} \right)$   | 1<br>1<br>1<br>2<br>1 |
|        | c)       | Find $L^{-1} \left\{ \frac{2s^2 - 4}{(s+1)(s-2)(s-3)} \right\}$   | <b>06</b>             |
|        | Ans      | <p>Let</p> $\frac{2s^2 - 4}{(s+1)(s-2)(s-3)} = \frac{A}{s+1} + \frac{B}{s-2} + \frac{C}{s-3}$ $2s^2 - 4 = (s-2)(s-3)A + (s+1)(s-3)B + (s+1)(s-2)C$ <p>Put <math>s = -1</math></p> $\therefore 2(-1)^2 - 4 = (-1-2)(-1-3)A$  |                       |



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| 6.     | c)       | $\therefore A = -\frac{1}{6}$ <p>Put <math>s = 2</math></p> $2(2)^2 - 4 = (2+1)(2-3)B$ $\therefore B = \frac{-4}{3}$ <p>Put <math>s = 3</math></p> $2(3)^2 - 4 = (3+1)(3-2)C$ $\therefore C = \frac{7}{2}$ $\therefore \frac{2s^2 - 4}{(s+1)(s-2)(s-3)} = \frac{-\frac{1}{6}}{s+1} + \frac{-\frac{4}{3}}{s-2} + \frac{\frac{7}{2}}{s-3}$ $\therefore L^{-1} \left\{ \frac{2s^2 - 4}{(s+1)(s-2)(s-3)} \right\} = -\frac{1}{6} L^{-1} \left\{ \frac{1}{s+1} \right\} - \frac{4}{3} L^{-1} \left\{ \frac{1}{s-2} \right\} + \frac{7}{2} L^{-1} \left\{ \frac{1}{s-3} \right\}$ $= -\frac{1}{6} e^{-t} - \frac{4}{3} e^{2t} + \frac{7}{2} e^{3t}$ | <p>1</p> <p>1</p> <p>1</p> <p>1+1+1</p> |
|        | d)       | <p>Solve differential equation using Laplace Transform.</p> $\frac{dy}{dt} + 2y = e^{-t}, \text{ given } y(0) = 2$ <p>Ans <math>\frac{dy}{dt} + 2y = e^{-t}</math></p> <p>Apply Laplace Transform on both sides,</p> $\therefore L \left\{ \frac{dy}{dt} + 2y \right\} = L \{ e^{-t} \}$ $\therefore sL(y) - y(0) + 2L(y) = \frac{1}{s+1}$ $\therefore sL(y) - 2 + 2L(y) = \frac{1}{s+1}$ $\therefore (s+2)L(y) - 2 = \frac{1}{s+1}$ $\therefore (s+2)L(y) = \frac{1}{s+1} + 2$ $\therefore (s+2)L(y) = \frac{2s+3}{s+1}$   | <p>06</p> <p>1</p> <p>½</p> <p>½</p>    |





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| 6.     | d)       | $\therefore (s+2)L(y) = \frac{2s+3}{s+1}$ $\therefore L(y) = \frac{2s+3}{(s+1)(s+2)}$ $\therefore y = L^{-1} \left\{ \frac{2s+3}{(s+1)(s+2)} \right\}$ $\therefore \frac{2s+3}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$ $\therefore 2s+3 = A(s+2) + B(s+1)$ <p>Put <math>s = -1</math></p> $\therefore A = 1$ <p>Put <math>s = -2</math></p> $\therefore B = 1$ $\therefore \frac{2s+3}{(s+1)(s+2)} = \frac{1}{s+1} + \frac{1}{s+2}$ $\therefore L^{-1} \left\{ \frac{2s+3}{(s+1)(s+2)} \right\} = L^{-1} \left\{ \frac{1}{s+1} + \frac{1}{s+2} \right\}$ $= L^{-1} \left\{ \frac{1}{s+1} \right\} + L^{-1} \left\{ \frac{1}{s+2} \right\}$ $= e^{-t} + e^{-2t}$ | <p>1/2</p> <p>1</p> <p>1</p> <p>1/2</p> <p>1</p> |
|        |          | <p><b><u>Important Note</u></b></p> <p><i>In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.</i></p>  |  |