

Important Instruction to Examiners:-

- 1) The answers should be examined by key words & not as word to word as given in the model answers scheme.
- 2) The model answers & answers written by the candidate may vary but the examiner may try to access the understanding level of the candidate.
- 3) The language errors such as grammatical, spelling errors should not be given more importance.
- 4) While assessing figures, examiners, may give credit for principle components indicated in the figure.
- 5) The figures drawn by candidate & model answer may vary. The examiner may give credit for any equivalent figure drawn.
- 5) Credit may be given step wise for numerical problems. In some cases, the assumed contact values may vary and there may be some difference in the candidate's answers and model answer.
- 6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidates understanding.
- 7) For programming language papers, credit may be given to any other programme based on equivalent concept.

Important notes to examiner

Q-1 (a) i) Define moment of inertia state its value for semi circle about its centroid.

→ a) moment of inertia of section is defined as the product of area of a section and the square of the distance between the centroid of section and reference axis. it is also called as second moment of area.

0.1M

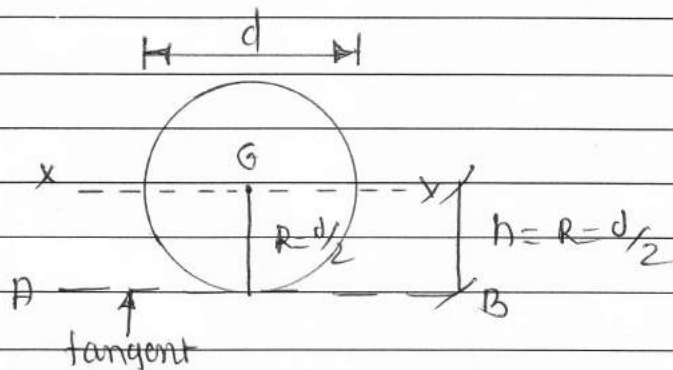
b) moment of inertia value for semi circle

$$I_{xx} = 0.11 R^4$$

$$I_{yy} = \frac{\pi R^4}{8}$$

 $\frac{1}{2}M$ $\frac{1}{2}M$

ii) find moment of inertia of solid circular lamina of diameter 'd' about its tangent.



let AB be the tangent to the circular lamina.
By applying parallel axis theorem.

$$I_{AB} = I_{\text{tangent}} = I_G + Ah^2 = \frac{\pi d^4}{64} + \left(\frac{\pi d^2}{4} \times \left(\frac{d}{2} \right)^2 \right)$$

1M

$$I_{AB} = \frac{\pi d^4 + 4\pi d^4}{64}$$

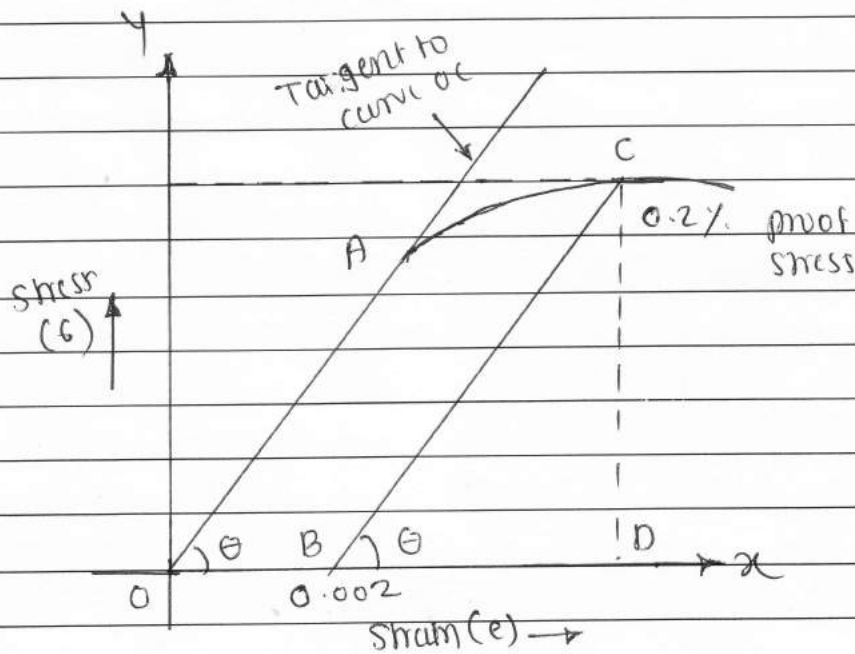
$$I_{AB} = \frac{5\pi d^4}{64}$$

$$I_{AB} = 0.245 d^4$$

1M

iii) Draw stress-strain curve for HYSID bar.

2M



2M

iv) Define proof stress & ultimate stress.

0.2M

a) proof stress —:

offset yield point (proof stress) when a yield point is not easily defined based on the shape of the stress-strain curve an offset yield point is arbitrary defined the value for this is commonly set at 0.1% or 0.2% plastic strain

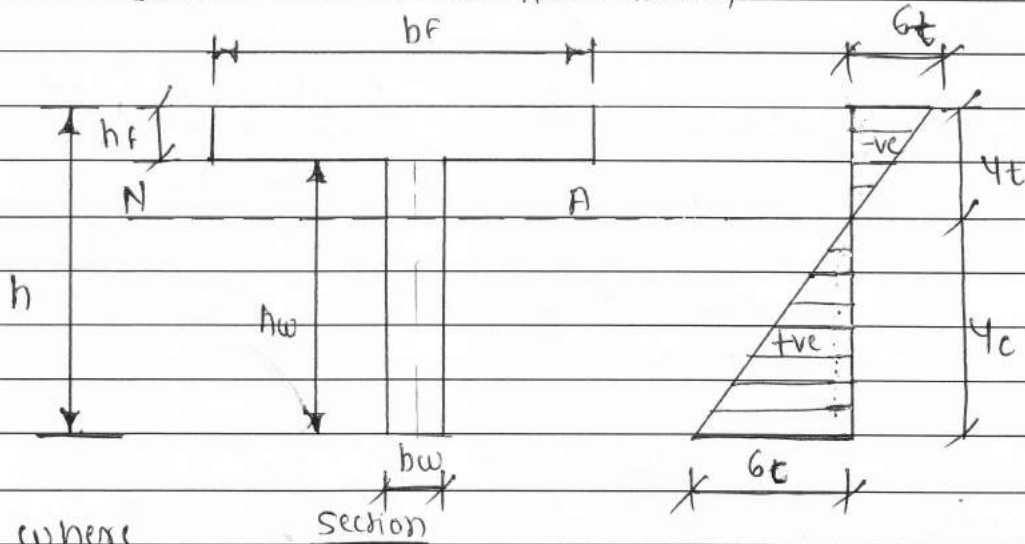
0.1M

b) ultimate stress

ultimate tensile strength, often shortened to tensile strength or ultimate strength, is the capacity of a material or structure to with stand loads tending to elongate, as opposed to compressive strength, which with stands loads tending to reduce size.

0.1M

v) Draw a bending stress distribution diagram for a T-section used as cantilever beam



1M
section

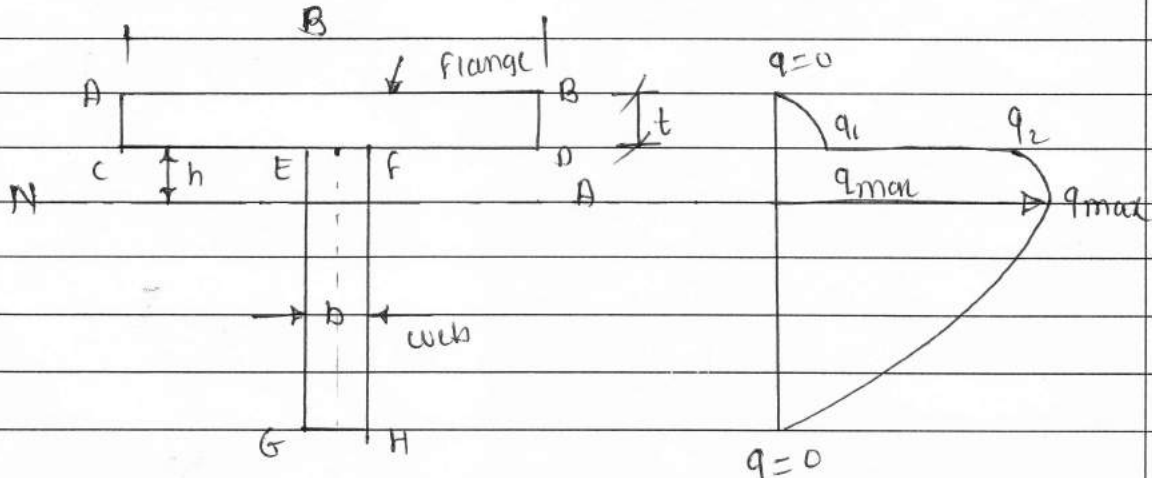
1M
Bending
stress
diagram

where

- h = height of the beam
- b_w = width of the web
- b_f = flange width
- h_f = flange thickness
- h_w = web height

Bending stress
 variation diagram
 $G_c \neq G_t$

vi) Draw shear stress distribution diagram for a T-section used as SSB



1M
section

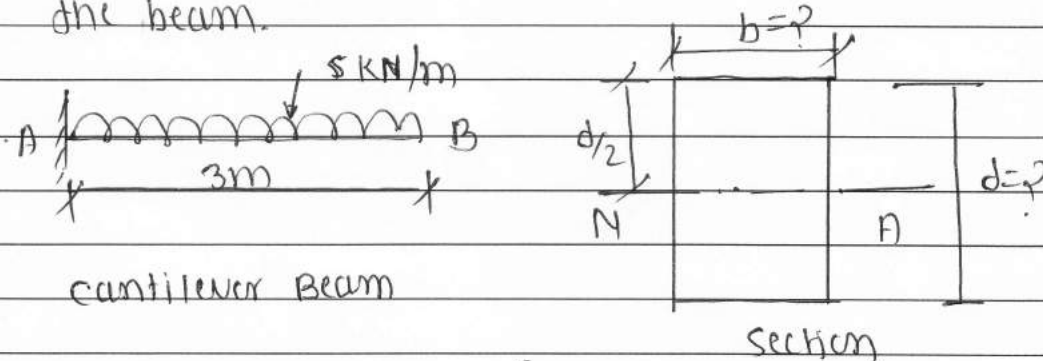
1M
shear
stress
diagram

section

shear stress distribution
 diagram

	vii) Define radius of gyration and slenderness ratio	
	<p>a) radius of gyration</p> <p>radius of gyration or gyration refers to the distribution of the component of an object around an axis in terms of mass moment of inertia, it is the perpendicular distance from the axis of rotation to a point mass (of mass m) that gives an equivalent inertia to the original object (S).</p>	0.1M
	<p>b) slenderness ratio</p> <p>slenderness ratio is the ratio of the length of a column and the least radius of gyration</p>	0.1M
	<p>viii) Assumptions in Euler's column Theory</p> <p>i) the material of the column is perfectly homogeneous and isotropic</p> <p>ii) The column is initially straight and of uniform lateral dimensions</p> <p>iii) the load on the column is exactly axial.</p> <p>iv) the column is long and fails due to buckling or bending</p> <p>v) the self weight of the column is neglected.</p> <p>vi) The column is stressed up to the limit of proportionality</p>	1M each

- b) i) A cantilever beam of rectangular section supports UDL of 5 kN/m . The span of the beam is 3 m . If the maximum bending stress is 100 N/mm^2 and the depth of the beam is 1.5 times the width, determine the size of the beam.



$$i) \text{ B.M.}_{\text{max}} = (M) = \frac{wL^2}{2}$$

$$M = \frac{5 \times 3^2}{2} = 22.5 \text{ kN}\cdot\text{m} = 22.5 \times 10^6 \text{ N}\cdot\text{mm} \quad \frac{1}{2} \text{ M}$$

$$y = \frac{d}{2} = \frac{1.5d}{2}$$

$$I = I_{NA} = \frac{bd^3}{12} = \frac{b \times (1.5b)^3}{12} = \frac{(1.5)^3 \times b \times b^3}{12} = \frac{3.375b^4}{12} \quad \frac{1}{2} \text{ M}$$

- ii) To find the size of the beam

$$\frac{M}{I} = \frac{\sigma}{y} \quad \frac{22.5 \times 10^6}{\frac{3.375b^4}{12}} = \frac{100}{1.5b/2} \quad 1 \text{ M}$$

$$\therefore \frac{12 \times 22.5 \times 10^6}{3.375b^4} = \frac{2 \times 100}{1.5b} \quad \frac{1}{2} \text{ M}$$

$$\frac{12 \times 22.5 \times 10^6 \times 1.5}{3.375 \times 2 \times 100} = \frac{b^4}{b}$$

$$b^3 = 600 \times 10^3$$

$$b = 84.34 \text{ mm} \quad \boxed{b = 85 \text{ mm}} \quad \frac{1}{2} \text{ M}$$

$$d = 1.5b = 1.5 \times 85 = 127.5 \text{ mm} \quad \text{say} = 128 \text{ mm} \quad \frac{1}{2} \text{ M}$$

$$b \times d = 85 \text{ mm} \times 128 \text{ mm} \quad \dots \text{ Ans}$$

ii) for applying Euler's formula, find the minimum value of slenderness ratio for mild steel strut with both ends fixed. Take yield stress as 315 MPa and $E = 210 \text{ GPa}$		
$\sigma = \frac{\pi^2 E}{\left(\frac{Le}{R}\right)^2} = \frac{\pi^2 E}{\lambda^2}$		1M
$\therefore \lambda^2 = \frac{\pi^2 E}{\sigma}$		1M
$\therefore \lambda = \sqrt{\frac{\pi^2 E}{\sigma}}$		
$\lambda = \sqrt{\frac{\pi^2 \times 210 \times 10^3}{315}}$		1M
$\lambda = 81.11$		1M
iii) Differentiate gradual load and sudden load		4M
gradual load	sudden load	
i) The type of loading starting from zero and slowly increasing uniformly.	i) when a load as a whole is applied instantly on a body, then it is called as suddenly applied load.	1M each
ii) UTM is used for gradual load	ii) Load acting on weighing machine.	
iii) Example: tension test on mild steel	iii) Load of person while entering into lift.	
iv) Formula for stress	iv) Formula for stress	
$\sigma = \frac{P}{A}$	$\sigma = \frac{2P}{A}$	

- Q-2 (a) A hollow circular section of external diameter 100 mm has a uniform thickness of 10 mm calculate its moment of inertia with respect to
- Diameter
 - Tangent to bottom of circle
 - the axis parallel to and 20 mm below the tangent

i) To find M.I w.r.t diameter
let

$I_{dia} = M.I$ w.r.t diameter

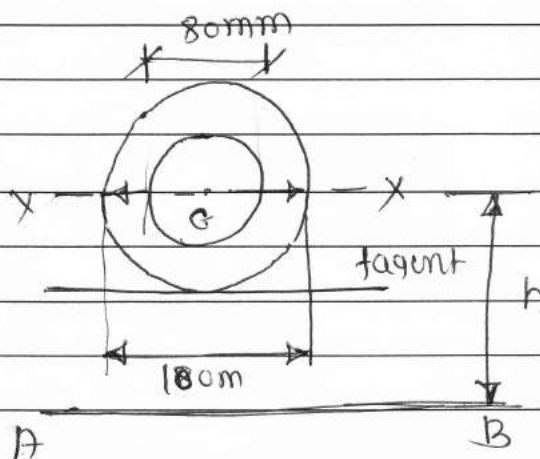
$$I_{dia} = I_{xx} = I_G = \frac{\pi}{64} (D^4 - d^4)$$

1M

$$I_{dia} = \frac{\pi}{64} (100^4 - 80^4)$$

$$I_{dia} = 2.898 \times 10^6 \text{ mm}^4$$

1M



ii) To find the M.I
w.r.t tangent to
the circle

$\frac{1}{2}$ M
for
fig

$$I_{tgt} = I_G + Ah^2$$

$\frac{1}{2}$ M

$$A = \frac{\pi}{4} (D^2 - d^2) = \frac{\pi}{4} (100^2 - 80^2)$$

$$A = 2827.33 \text{ mm}^2$$

$\frac{1}{2}$ M

$$h = \frac{D}{2} = \frac{100}{2} = 50 \text{ mm}$$

$\frac{1}{2}$ M

$$I_{tgt} = 2.898 \times 10^6 + 2827.33 \times 50^2$$

1M

$$I_{tgt} = 9.966 \times 10^6 \text{ mm}^4$$

iii) To find the M.I for the axis parallel to and 20mm below the tangent

Using parallel axis theorem we get

$$I_{AB} = I_G + Ah^2$$

 $\frac{1}{2}M$

$$h = \frac{D}{2} + \text{Dist}^n \text{ bet}^n \text{ two parallel axis}$$

 $\frac{1}{2}M$

$$h = \frac{100}{2} + 20 \quad \boxed{h = 70 \text{ mm}}$$

$$I_{AB} = 2.898 \times 10^6 + 2827.33 \times 70^2$$

$$\boxed{I_{AB} = 16.75 \times 10^6 \text{ mm}^4}$$

0.2M

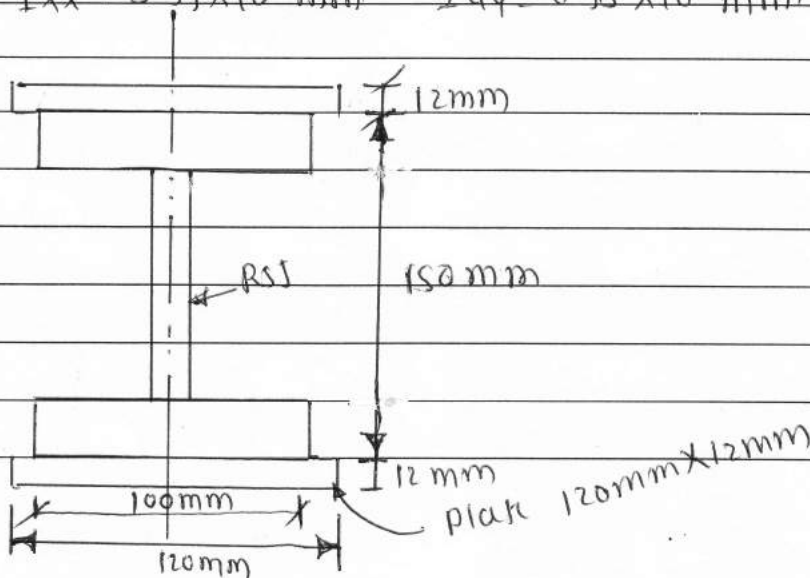
b) A steel stanchion is built up of 100mm x 150mm RSJ with one 120mm x 12mm plate riveted to each flange.

The overall depth of stanchion is 174mm.

Calculate moment of inertia about the centroidal axes

properties of RSJ are Area = 2162 mm²

$I_{xx} = 8.39 \times 10^6 \text{ mm}^4$ $I_{yy} = 0.98 \times 10^6 \text{ mm}^4$



1M

Given data

for RSJ

$$A = 2167 \text{ mm}^2$$

$$I_{xx} = 8.39 \times 10^6 \text{ mm}^4$$

$$I_{yy} = 0.98 \times 10^6 \text{ mm}^4$$

i) To find I_{xx} & I_{yy} of plate (120x12mm) from dia

$$b = 120 \text{ mm}, d = 12 \text{ mm}$$

$$I_{xx \text{ plate}} = 2 \left[\frac{bd^3}{12} + Ah^2 \right]$$

1M

$$A = 120 \times 12 = 1440 \text{ mm}^2$$

$$h = \left(\frac{150}{2} + \frac{12}{2} \right) = 75 + 6 = 81 \text{ mm}$$

1M

$$I_{xx} = 2 \left[\frac{120 \times 12^3}{12} + 1440 \times 81^2 \right]$$

$$I_{xx} = 18.930 \times 10^6 \text{ mm}^4$$

1M

Now

$$I_{xx \text{ total}} = I_{xx \text{ RSJ}} + I_{xx \text{ plate}}$$

$$= 8.39 \times 10^6 + 18.93 \times 10^6$$

1M

$$\boxed{I_{xx \text{ total}} = 27.32 \times 10^6 \text{ mm}^4}$$

similar

$$I_{yy \text{ plate}} = 2 \left[\frac{db^3}{12} \right] = 2 \left[\frac{12 \times 120^3}{12} \right]$$

1M

$$\boxed{I_{yy \text{ plate}} = 3.456 \times 10^6 \text{ mm}^4}$$

1M

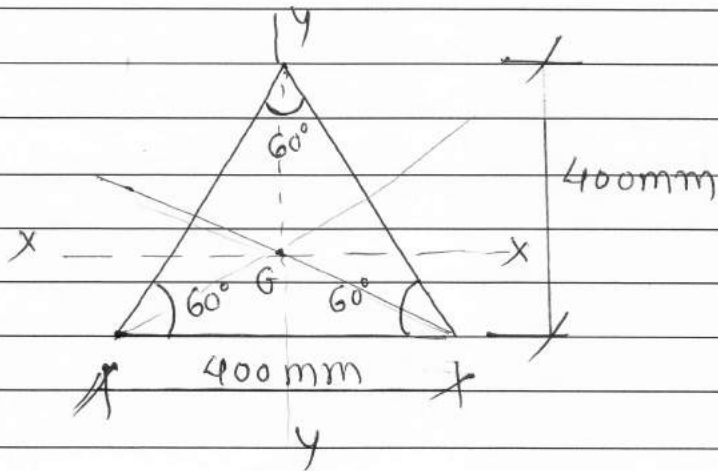
$$I_{yy} = I_{yy \text{ RSJ}} + I_{yy \text{ plate}}$$

$$= 0.98 \times 10^6 + 3.456 \times 10^6$$

$$\boxed{I_{yy \text{ total}} = 4.436 \times 10^6 \text{ mm}^4}$$

1M

- c) i) For an equilateral triangle of side 400 mm. Show that M.I about the horizontal and vertical centroidal axes are equal.



$$i) \sin 60^\circ = \frac{h}{400} \quad \therefore h = 400 \sin 60^\circ$$

$$\boxed{h = 346.41 \text{ mm}}$$

01M

$$I_{xx} = I_G = \frac{bh^3}{36} = \frac{400 \times (346.41)^3}{36}$$

$$I_{xx} = 461.88 \times 10^6 \text{ mm}^4$$

01M

- ii) To find M.I about vertical centroidal axis i.e. Y-Y

$$I_{yy} = I_G = \frac{hb^3}{48} = \frac{346.41 \times 400^3}{48}$$

01M

$$I_{yy} = 461.88 \times 10^6 \text{ mm}^4$$

01M

cii) A bar of c/s area 200mm^2 is axially pulled by force P kN. if the maximum stress induced in the bar is 30Mpa . determine 'P' if elongation of 1.2mm is observed over a gauge length 3m . determine young's modulus

$$A = 200\text{mm}^2 \quad L = 3\text{m}$$

$$P = P \quad \delta L = 1.2\text{mm}$$

$$\sigma = 30\text{N/mm}^2$$

$$E = \frac{\sigma}{e}$$

$$\therefore e = \frac{\delta L}{L} = \frac{1.2}{3000} = 4 \times 10^{-4}$$

0.1M

$$\therefore E = \frac{\sigma}{e} = \frac{30}{4 \times 10^{-4}} \quad \therefore \boxed{E = 75 \times 10^3 \text{Mpa}}$$

0.1M

$$\delta L = \frac{P \cdot L}{A \cdot E}$$

$$\therefore 1.2 = \frac{P \times 3000}{200 \times 75 \times 10^3}$$

0.1M

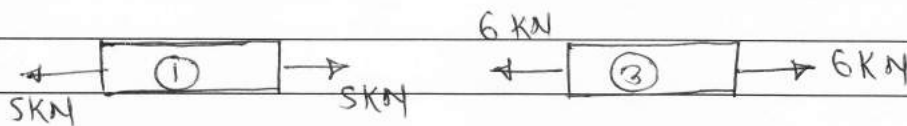
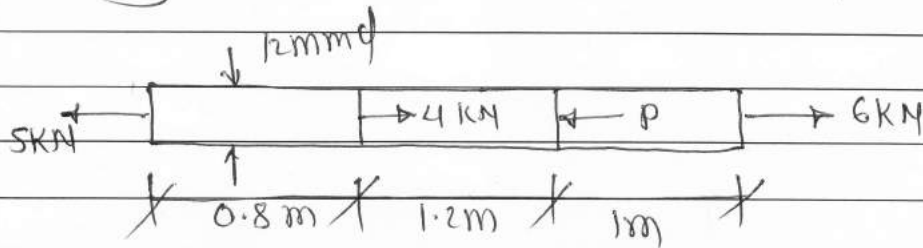
$$\therefore 1.2 \times 200 \times 75 \times 10^3 = P \times 3000$$

$$\therefore P = \frac{1.2 \times 200 \times 10^3 \times 75}{3000}$$

$$\therefore \boxed{P = 6 \text{KN}}$$

0.1M

Q-3 (a) Determine load P and total elongation in the bar shown in fig having 12mm diameter $E = 2 \times 10^5 \text{ N/mm}^2$



1M

$$E = 200 \text{ GPa}$$

$$L_1 = 0.8 \text{ m} = 800 \text{ mm}$$

$$L_2 = 1.2 \text{ m} = 1200 \text{ mm}$$

$$L_3 = 1.0 \text{ m} = 1000 \text{ mm}$$

$$\sum F_x = 6 - P + 4 - 5$$

$$\sum F_x = 0$$

$$\therefore 6 - P + 4 - 5 = 0$$

$$\boxed{P = 5 \text{ kN}}$$

2M

$$\therefore A = \frac{\pi}{4} \times 12^2 = 113.09 \text{ mm}^2$$

$$\delta L = \delta L_1 + \delta L_2 + \delta L_3$$

1M

$$\delta L = \frac{P_1 L_1}{AE} + \frac{P_2 L_2}{AE} + \frac{P_3 L_3}{AE} = \frac{1}{AE} (P_1 L_1 + P_2 L_2 + P_3 L_3)$$

0.2M

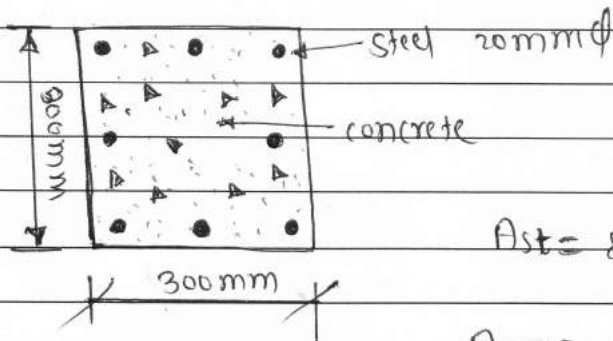
$$\delta L = \frac{1}{113.09 \times 200} [(5 \times 800) + (1 \times 1200) + (6 \times 1000)]$$

0.1M

$$\boxed{\delta L = 0.495 \text{ mm}}$$

0.1M

- b) A reinforced concrete Column is $300 \times 300 \text{ mm}$ in section reinforced with 8 bars of 20 mm dia. the column carries a load of 360 kN . Find the stresses in concrete and steel bars
Take $E_s = 2.1 \times 10^5 \text{ N/mm}^2$ $E_c = 1.4 \times 10^4 \text{ N/mm}^2$



σ_{st} = stress in steel
 σ_{con} = stress in concrete

$$A_{st} = 8 \times \frac{\pi}{4} \times 20^2 = 2513.29 \text{ mm}^2$$

$$A_{con} = 300^2 = 2513.29$$

$$A_{con} = 87486.726 \text{ mm}^2$$

0.1M

we know

$$\sigma_{st} = m \sigma_{con}$$

0.1M

$$\sigma_{st} = 15 \times \sigma_{con}$$

$$P = P_{st} + P_{con}$$

$$\therefore P = \sigma_{st} A_{st} + \sigma_{con} A_{con}$$

0.1M

$$360 \times 10^3 = \sigma_{st} \times 2513.29 + \sigma_{con} \times 87486.726$$

$$360 \times 10^3 = 15 \sigma_{con} \times 2513.29 + \sigma_{con} \times 87486.726$$

0.1M

$$360 \times 10^3 = \sigma_{con} (15 \times 2513.29 + 87486.726)$$

$$360 \times 10^3 = \sigma_{con} (125.185 \times 10^3)$$

$$\sigma_{con} = \frac{360 \times 10^3}{125.185 \times 10^3}$$

0.2M

$$\boxed{\sigma_{con} = 2.87 \text{ N/mm}^2}$$

0.1M

$$\sigma_{st} = 15 \times \sigma_{con}$$

$$\sigma_{st} = 15 \times 2.87$$

$$\boxed{\sigma_{st} = 43.05 \text{ N/mm}^2}$$

0.1M

Q-3 (c) A steel rod 4m long and 20mm diameter is subjected to an axial tensile load of 45 kN. Find the change in length and diameter of the rod.
 $E_s = 2 \times 10^5 \text{ N/mm}^2$. Poisson's ratio $\left(\frac{1}{m}\right) = \frac{1}{4}$

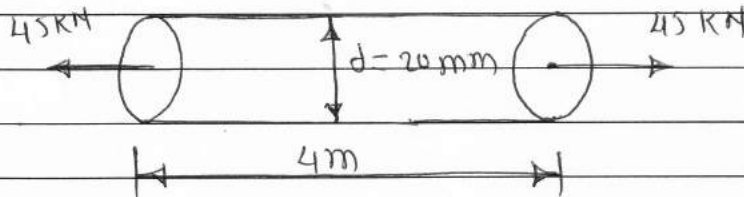
$$L = 4 \text{ m} = 4000 \text{ mm}$$

$$d = 20 \text{ mm}$$

$$P = 45 \text{ kN}$$

$$E_s = 2 \times 10^5 \text{ N/mm}^2$$

$$\frac{1}{m} = \frac{1}{4} = \mu = 0.25$$



i) To find change in length (δL)

We know

$$\delta L = \frac{PL}{AE} = \frac{45 \times 10^3 \times 4000}{\frac{\pi}{4} \times 20^2 \times 2 \times 10^5} \quad 0.1 \text{ M}$$

$$\delta L = 2.86 \text{ mm} \quad (\text{increase due to tensile load}) \quad 0.1 \text{ M}$$

ii) To find change in diameter (δd)

$$\text{Linear strain } (e) = \frac{\delta L}{L} \quad 0.1 \text{ M}$$

$$e = \frac{2.86}{4000}$$

$$e = 7.15 \times 10^{-4} \quad 0.1 \text{ M}$$

$$\text{Lateral strain } (e_{td}) = -\mu \cdot e$$

$$= -0.25 \times 7.15 \times 10^{-4}$$

$$= -1.7875 \times 10^{-4}$$

01M

01M

$$\text{But lateral strain} = \frac{\delta d}{d}$$

$$-1.7875 \times 10^{-4} = \frac{\delta d}{d}$$

01M

$$\therefore \delta d = -1.7875 \times 10^{-4} \times d$$

$$\delta d = -1.7875 \times 10^{-4} \times 20$$

$$\delta d = -3.575 \times 10^{-3} \text{ mm}$$

01M

Q4a) Given

$$\begin{array}{lll}
 L = 200 \text{ mm} & \sigma_x = 120 \text{ MPa} & \delta V = 140 \text{ mm}^3 \\
 b = 40 \text{ mm} & \sigma_y = 40 \text{ MPa} & E = 200 \text{ GPa} \\
 t = 40 \text{ mm} & \sigma_z = 40 \text{ MPa} &
 \end{array}$$

→ Volumetric strain of a rectangular body subjected to tri-axial loading is given by.

$$\frac{\delta V}{V} = \left[\frac{\sigma_x + \sigma_y + \sigma_z}{E} \right] (1 - 2\mu) \quad \text{--- (i)} \quad 3M$$

$$\begin{aligned}
 V &= b \times t \times L = 40 \times 40 \times 200 = 320 \times 10^3 \text{ mm}^3 \\
 \delta V &= 140 \text{ mm}^3 \\
 \sigma_x &= 120 \text{ MPa} \\
 \sigma_y &= \sigma_z = 40 \text{ MPa} \\
 E &= 200 \text{ GPa} = 200 \times 10^3 \text{ N/mm}^2
 \end{aligned}$$

$$\therefore \frac{140}{320 \times 10^3} = \left[\frac{120 + 40 + 40}{200 \times 10^3} \right] (1 - 2\mu) \quad 1M$$

$$4.375 \times 10^{-4} = 1 \times 10^{-3} (1 - 2\mu) \quad 1M$$

$$\frac{4.375 \times 10^{-4}}{1 \times 10^{-3}} = 1 - 2\mu$$

$$0.4375 = 1 - 2\mu$$

$$0.4375 - 1 = -2\mu \quad 1M$$

$$-0.5625 = -2\mu$$

$$\therefore \mu = \frac{0.5625}{2}$$

$$\therefore \mu = 0.28125 \approx 0.28$$

$$\therefore \text{Poisson's ratio } \mu = 0.28 \quad 2M$$

Q4b) Given

$$d = 20 \text{ mm } \phi$$

$$\delta L = 0.32 \text{ mm}$$

$$L = 200 \text{ mm}$$

$$\delta d = 0.0085 \text{ mm}$$

$$P = 100 \text{ kN}$$

$$1) \text{ Stress } \sigma = \frac{P}{A} = \frac{100 \times 10^3}{\frac{\pi}{4} \times 20^2} = 318.309 \text{ N/mm}^2 \quad 1M$$

$$2) \text{ Strain } e = \frac{\delta L}{L} = \frac{0.32}{200} = 1.6 \times 10^{-3} \quad 1M$$

$$3) \text{ Young's modulus } E = \frac{\sigma}{e} = \frac{318.309}{1.6 \times 10^{-3}} = 198.94 \text{ N/mm}^2 \times 10^3$$

$$E = 198.94 \times 10^3 \text{ N/mm}^2 \quad 1M$$

$$4) \text{ Poisson's ratio } \mu = \frac{\text{lateral strain } e_L}{\text{linear strain } e}$$

$$\text{lateral strain } e_L = \frac{\delta d}{d} = \frac{0.0085}{20} = 4.25 \times 10^{-4} \quad 1M$$

$$\therefore \mu = \frac{4.25 \times 10^{-4}}{1.6 \times 10^{-3}} = 0.2656 \cong 0.266$$

$$\mu = 0.266 \quad 1M$$

5) Modulus of rigidity G

$$E = 2G(1 + \mu) \quad \frac{1}{2}M$$

$$\therefore G = \frac{E}{2(1 + \mu)} = \frac{198.94 \times 10^3}{2(1 + 0.266)}$$

$$\therefore G = 78.57 \times 10^3 \text{ N/mm}^2 \quad 1M$$

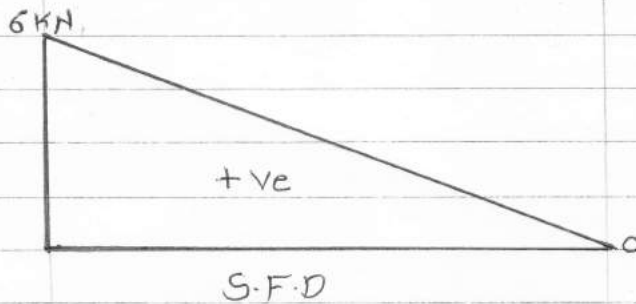
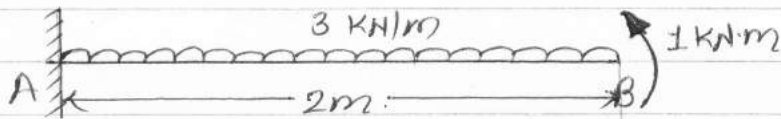
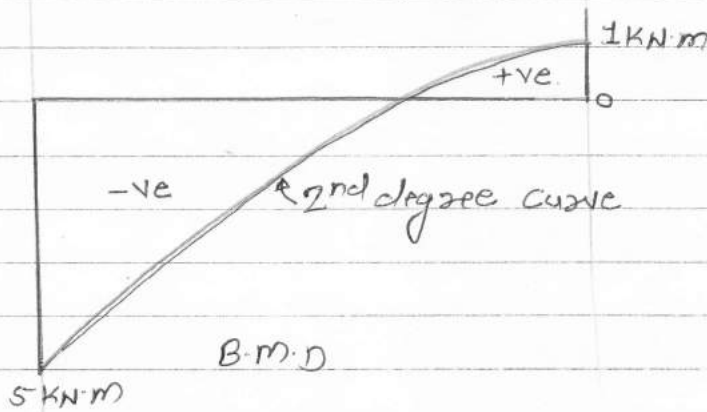
6) Bulk Modulus K

$$E = 3K(1 - 2\mu) \quad \frac{1}{2}M$$

$$K = \frac{E}{3(1 - 2\mu)} = \frac{198.94 \times 10^3}{3(1 - 2 \times 0.266)}$$

$$\therefore K = 141.69 \times 10^3 \text{ N/mm}^2 \quad 1M$$

Q4C)

 $1 \frac{1}{2} m$  $1 \frac{1}{2} m$

1) Support Reaction

$$\sum F_y = 0 \quad R_A - (3 \times 2) = 0 \quad \therefore R_A = 6 \text{ kN.}$$

1m

2) S.F. Calculation.

$$\text{S.F. at just left of A} = 0 \text{ kN.}$$

$$\text{S.F. at just right of A} = R_A = 6 \text{ kN}$$

2m

$$\text{S.F. at B} = R_A - (3 \times 2) = 6 - (3 \times 2) = 0 \text{ kN}$$

3) B.M. Calculation

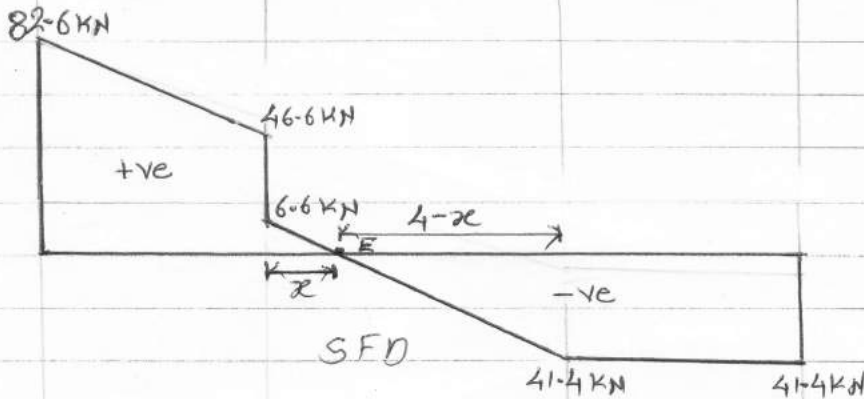
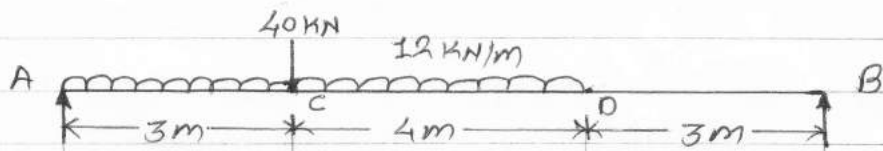
$$\text{B.M. at just right of B} = 0 \text{ kN}\cdot\text{m.}$$

$$\text{B.M. at just left of B} = 1 \text{ kN}\cdot\text{m.}$$

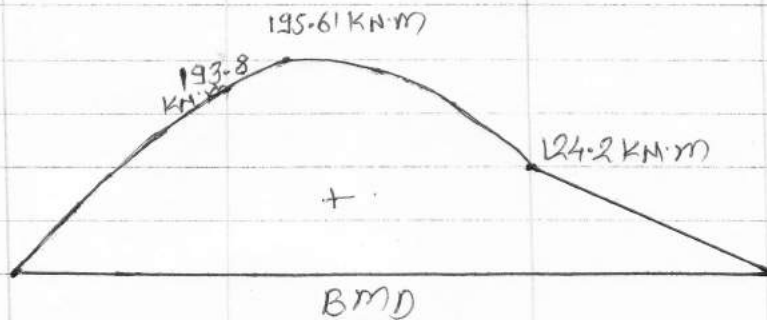
2m

$$\text{B.M. at A} = 1 - (3 \times 2 \times 1) = -5 \text{ kN}\cdot\text{m.}$$

Q5a)



$1 \frac{1}{2} m$



$1 \frac{1}{2} m$

1) Support reactions

i) $\sum F_y = 0$; $R_A + R_B - 40 - (12 \times 7) = 0 \quad \therefore R_A + R_B = 124 \text{ kN}$

ii) $\sum M @ A = 0$; $(40 \times 3) + (12 \times 7 \times 3.5) - R_B \times 10 = 0$

$\therefore R_B = 41.4 \text{ kN} \quad \therefore R_A = 82.6 \text{ kN}$

1m

2) S.F. Calculation.

S.F at just left of A = 0 kN

S.F at just right of A = $R_A = 82.6 \text{ kN}$

S.F at just left of C = $82.6 - (12 \times 3) = 46.6 \text{ kN}$

S.F at just right of C = $46.6 - 40 = 6.6 \text{ kN}$

S.F at D = $6.6 - (12 \times 4) = -41.4 \text{ kN}$

2m

S.F at just left of B = -41.4 kN

S.F at just right of B = $-41.4 + R_B = 0 \text{ kN}$.

To locate point E i.e. Point of Contra Shear

$$\frac{6.6}{x} = \frac{41.4}{4-x}$$

Q5a)

cont...

$$6.6(4-x) = 41.4x$$

$$26.4 - 6.6x = 41.4x$$

$$26.4 = 48x$$

$$\therefore x = 0.55 \text{ m from C point.}$$

3) B.M. calculation.

$$\text{B.M at A} = \text{B.M at B} = 0 \text{ KN}\cdot\text{m} \dots \text{s.s ends}$$

$$\begin{aligned} \text{B.M at C} &= R_A \times 3 - (12 \times 3 \times 1.5) = (82.6 \times 3) - (12 \times 3 \times 1.5) \\ &= 193.8 \text{ KN}\cdot\text{m} \end{aligned}$$

$$\text{B.M at D} = R_A \times 7 - (12 \times 7 \times 3.5) - (40 \times 4) \quad 2 \text{ m}$$

$$= 82.6 \times 7 - (12 \times 7 \times 3.5) - (40 \times 4)$$

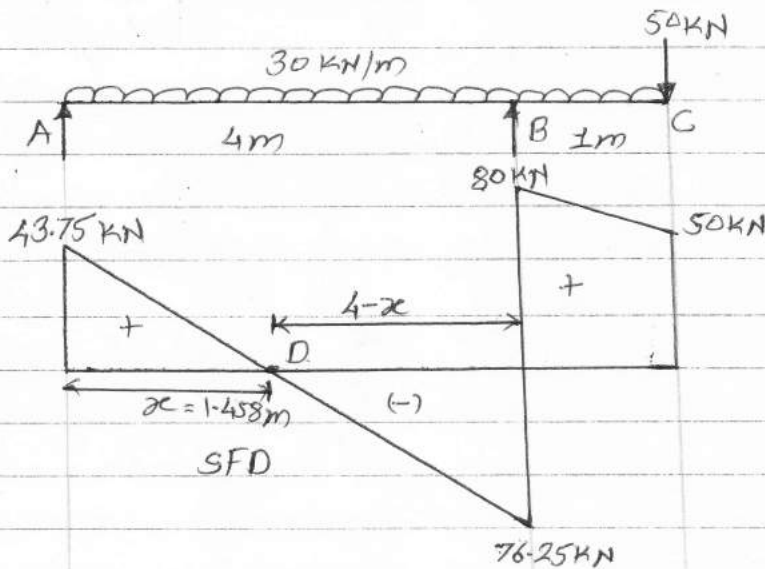
$$= 124.2 \text{ KN}\cdot\text{m}$$

$$\text{Maximum B.M at E} = (R_A \times 3.55) - (12 \times 3.55 \times \frac{3.55}{2}) - (40 \times 0.55)$$

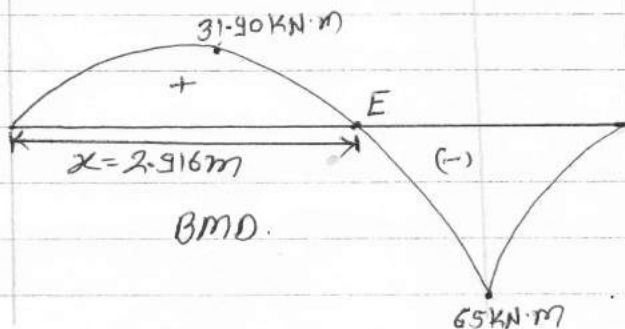
$$= (82.6 \times 3.55) - (12 \times 3.55 \times \frac{3.55}{2}) - (40 \times 0.55)$$

$$= 195.67 \text{ KN}\cdot\text{m}$$

Q5b



$1 \frac{1}{2} \text{ m}$



$1 \frac{1}{2} \text{ m}$

Q5b)

Contn.

i) Support reaction

$$i) \sum F_y = 0; R_A + R_B - (30 \times 5) - 50 = 0 \quad \therefore R_A + R_B = 200 \text{ KN}$$

$$ii) \sum M @ A = 0; (30 \times 5 \times 2.5) + (50 \times 5) - R_B \times 4 = 0$$

$$625 = 4R_B$$

$$\therefore R_B = 156.25 \text{ KN}$$

$$\therefore R_A = 200 - 156.25 = 43.75 \text{ KN}$$

1M

2) S.F. Calculation

$$\text{SF at just left of A} = 0 \text{ KN}$$

$$\text{SF at just right of A} = R_A = 43.75 \text{ KN}$$

$$\text{SF at just left of B} = 43.75 - (30 \times 4) = -76.25 \text{ KN}$$

$$\text{SF at just right of B} = -76.25 + 156.25 = 80 \text{ KN}$$

$$\text{SF at just left of C} = 80 - (30 \times 1) = 50 \text{ KN}$$

$$\text{SF at just right of C} = 50 - 50 = 0 \text{ KN.}$$

1M

To locate point D i.e. point of Contraflexure/zero shear

$$\frac{43.75}{x} = \frac{76.25}{4-x}$$

$$43.75(4-x) = 76.25x$$

$$\therefore x = 1.458 \text{ m from A.}$$

3) B.M. Calculation

$$\text{B.m at A} = 0 \text{ KN}\cdot\text{m} \quad \text{S.S. end}$$

$$\text{B.m at C} = 0 \text{ KN}\cdot\text{m} \quad \text{free end of overhang.}$$

$$\text{B.m at B} = R_A \times 4 - 30 \times 4 \times 2 = 43.75 \times 4 - 30 \times 4 \times 2 = -65 \text{ KN}\cdot\text{m}$$

$$\text{B.m at D} = (43.75 \times 1.458) - (30 \times 1.458 \times 1.458 / 2) = 31.90 \text{ KN}\cdot\text{m} \quad 2M$$

*To locate point E i.e. point of Contraflexure.

Take section xx at x from A in position AB

$$M_x = R_A \cdot x - 30 \cdot x \cdot \frac{x}{2} = 43.75x - \frac{30x^2}{2}$$

But at point E, $M_x = 0$.

$$\therefore 0 = 43.75x - \frac{30x^2}{2}$$

$$30x^2 = 2 \times 43.75x$$

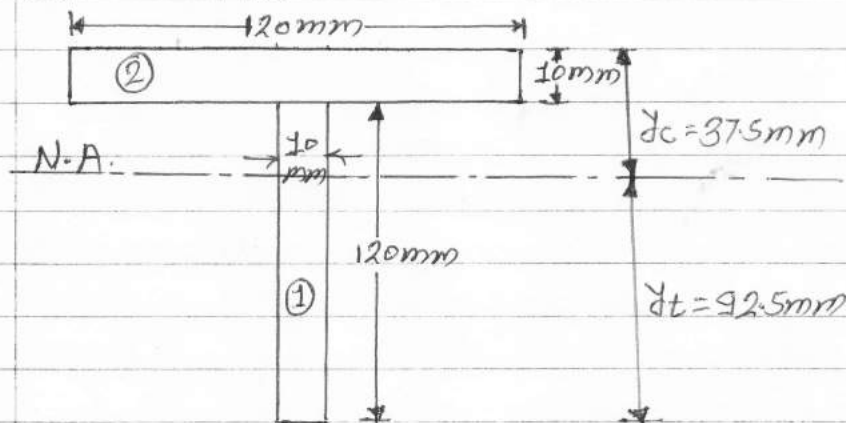
$$\therefore x = 2.916 \text{ m from A.}$$

1M

Q5c)

$$\rightarrow \sigma_{bt} = 160 \text{ MPa} \quad E = 210 \text{ GPa}$$

$$\sigma_{bc} = 100 \text{ MPa}$$



As the beam is simply supported beam, the sagging B.M will produced in beam & there will be Comp. at top.

1) Position of N.A

$$\bar{x} = \frac{120}{2} = 60 \text{ mm}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(120 \times 10 \times 60) + (120 \times 10 \times 125)}{(120 \times 10) + (120 \times 10)}$$

$$\bar{y} = 92.5 \text{ mm from bottom}$$

$$\therefore y_t = 92.5 \text{ mm}$$

1M

$$y_c = 37.5 \text{ mm}$$

2) M.I about N.A.

$$I_{xx} = I_{xx1} + I_{xx2}$$

$$= [I_{G1} + A_1 h_1^2] + [I_{G2} + A_2 h_2^2]$$

$$= \left[\frac{10 \times 120^3}{12} + 1200 (92.5 - 60)^2 \right] + \left[\frac{120 \times 10^3}{12} + 1200 (92.5 - 125)^2 \right]$$

$$= 2.70 \times 10^6 + 1.277 \times 10^6$$

$$I_{xx} = 3.977 \times 10^6 \text{ mm}^4$$

2M

Q5C 3) Moment of Resistance.

Contn

Using relation.

$$\frac{M_x}{I} = \frac{\sigma_b}{y}$$

1m

$$\sigma_b = \frac{M_x}{I} \times y$$

$$\therefore M_x = \frac{\sigma_b}{y} \times I$$

$$\therefore M_{xc} = \frac{\sigma_{bc}}{y_c} \times I \quad \text{and} \quad M_{xt} = \frac{\sigma_{bt}}{y_t} \times I \quad 1m$$

$$\therefore M_{xc} = \frac{100}{37.5} \times 3.977 \times 10^6 = 10.60 \times 10^6 \text{ N}\cdot\text{mm} \quad 1m$$

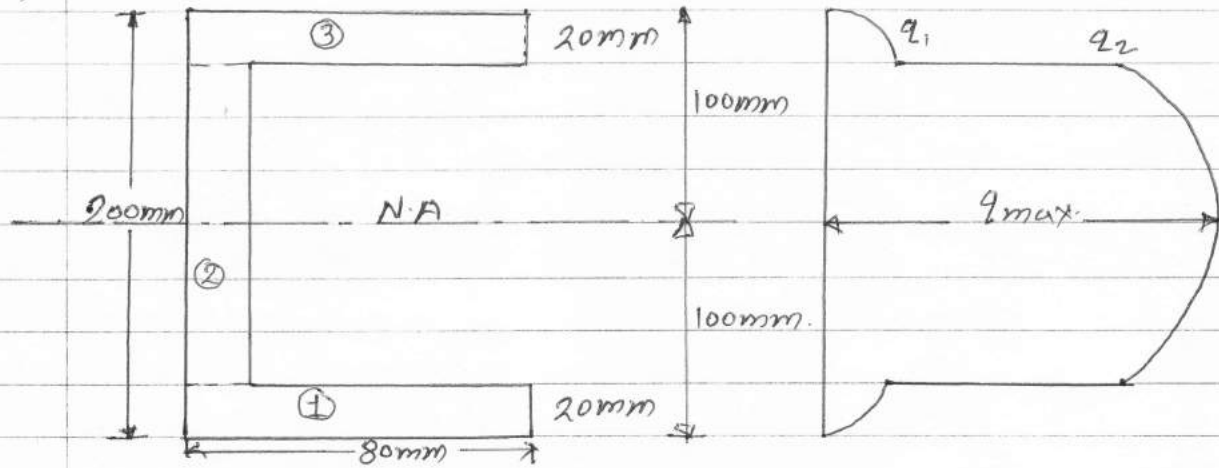
$$M_{xt} = \frac{160}{92.5} \times 3.977 \times 10^6 = 6.87 \times 10^6 \text{ N}\cdot\text{mm} \quad 1m$$

Consider least of above two values

Hence, Moment of resistance = $6.87 \times 10^6 \text{ N}\cdot\text{mm}$
OR 1m

$$M_x = 6.87 \text{ KN}\cdot\text{m}.$$

Q6a)



1) Since section is symmetrical about x -axis.

$$\therefore \bar{Y} = \frac{200}{2} = 100 \text{ mm}$$

1M

2) M.I. about N.A.

$$\begin{aligned} I_{xx} &= I_{xx1} + I_{xx2} + I_{xx3} \\ &= [I_{G1} + A_1 h_1^2] + [I_{G2} + A_2 h_2^2] + [I_{G3} + A_3 h_3^2] \\ &= \left[\frac{80 \times 20^3}{12} + 1600 (100 - 10)^2 \right] + \left[\frac{20 \times 160^3}{12} + 3200 (100 - 100)^2 \right] \\ &\quad + \left[\frac{80 \times 20^3}{12} + 1600 (100 - 190)^2 \right] \end{aligned}$$

$$= 13.01 \times 10^6 + 6.82 \times 10^6 + 13.01 \times 10^6$$

$$I_{xx} = 32.846 \times 10^6 \text{ mm}^4$$

2M

OR

OR

$$I_{xx} = \frac{BD^3}{12} - \frac{bd^3}{12} = \frac{80 \times 200^3}{12} - \frac{60 \times 160^3}{12}$$

$$I_{xx} = 32.85 \times 10^6 \text{ mm}^4$$

2M

3) Shear stress;

Let us consider the position of the beam above N.A.

96a) $q_1 =$ Shear stress at junction of flange & web by
 Considering width of flange, $b = 80 \text{ mm}$.

$$q_1 = \frac{S\bar{A}\bar{Y}}{bI} = \frac{150 \times 10^3 \times 80 \times 20 \times (100 - 5)}{80 \times 32.85 \times 10^6}$$

$$q_1 = 8.675 \text{ N/mm}^2$$

1M

$q_2 =$ Shear stress at junction of flange & web by
 Considering width of web, $b = 20 \text{ mm}$.

$$q_2 = \frac{S\bar{A}\bar{Y}}{bI} = \frac{150 \times 10^3 \times 80 \times 20 \times (100 - 5)}{20 \times 32.85 \times 10^6}$$

$$q_2 = 34.70 \text{ N/mm}^2$$

1M

$$q_{\max} = q_{NA} = q_2 + q_{\text{additional}}$$

$q_{\text{additional}} =$ Additional shear stress due to web area
 above NA taking $b = 20 \text{ mm}$.

$$= \frac{S\bar{A}\bar{Y}}{bI} = \frac{150 \times 10^3 \times 80 \times 20 \times \left(\frac{80}{2}\right)}{20 \times 32.85 \times 10^6}$$

$$q_{\text{addi}} = 14.61 \text{ N/mm}^2$$

$$\therefore q_{\max} = 34.70 + 14.61 = 49.31 \text{ N/mm}^2$$

1M

4) Average shear stresses.

$$q_{\text{avg}} = \frac{S}{A} = \frac{150 \times 10^3}{(2 \times 80 \times 20) + (160 \times 20)} = 23.43 \text{ N/mm}^2$$

1M

5) Ratio of q_{avg} to q_{\max}

$$\frac{q_{\text{avg}}}{q_{\max}} = \frac{23.43}{49.31} = 0.475$$

1M

Q6b)

→ $L = 2.5\text{m}$ with both end hinged.

$$D = 40\text{mm} \quad \alpha = \frac{1}{7500}$$

$$d = 30\text{mm}$$

$$E = 200\text{GPa} \quad \sigma_c = 320\text{MPa}$$

1) Geometrical Properties

i) Area $A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(40^2 - 30^2) = 549.778\text{mm}^2$

ii) M.I $I = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}(40^4 - 30^4) = 85.90 \times 10^3\text{mm}^4$ 1M

iii) Radius of Gyration $r_g = \sqrt{\frac{I}{A}} = \sqrt{\frac{85.90 \times 10^3}{549.778}} = 12.50\text{mm}$ 1M

iv) Effective length $L_e = L = 2500\text{mm}$. 1M

2) Crimping load by Euler's formula.

$$P_e = \frac{\pi^2 EI}{(L_e)^2} \quad 1\text{M}$$

$$= \frac{\pi^2 \times 200 \times 10^3 \times 85.90 \times 10^3}{(2500)^2}$$

$$P_e = 27.129 \times 10^3\text{N}$$

$$P_e = 27.129\text{KN}$$

1M

3) Crimping load by Rankine's formula.

$$P_R = \frac{\sigma_c \cdot A}{1 + \alpha \left(\frac{L_e}{r_g}\right)^2} \quad 1\text{M}$$

$$= \frac{320 \times 549.778}{1 + \frac{1}{7500} \left(\frac{2500}{12.50}\right)^2}$$

$$P_R = 27.778 \times 10^3\text{N}$$

$$P_R = 27.778\text{KN}$$

1M

The crippling load given by Euler's & Rankine's is approximately equal.

1M

$$\frac{P_e}{P_R} = \frac{27.129}{27.778} = 0.976.$$

Q6c

→ Given

$$P = 1000 \text{ N}$$

$$L = 5 \text{ m} = 5000 \text{ mm}$$

$$h = 200 \text{ mm}$$

$$\sigma = 80 \text{ MPa}$$

$$E = 210 \text{ GPa.}$$

1) In case of Impact load, the stress produced is given by.

$$\sigma = \frac{P}{A} + \sqrt{\left(\frac{P}{A}\right)^2 + \frac{2PhE}{AL}}$$

2M

$$\therefore \sigma = \frac{P}{A} + \sqrt{\left(\frac{P}{A}\right)^2 + \frac{2hE}{L} \left(\frac{P}{A}\right)}$$

$$\text{Let } \frac{P}{A} = x$$

1M

$$\therefore \sigma = x + \sqrt{x^2 + \frac{2hE}{L} x}$$

$$80 = x + \sqrt{x^2 + \frac{2 \times 200 \times 210 \times 10^3}{5000} x}$$

$$(80 - x) = \sqrt{x^2 + 16800x}$$

$$(80 - x)^2 = x^2 + 16800x$$

$$6400 - 160x + x^2 = x^2 + 16800x$$

1M

$$6400 = 16800x + 160x$$

$$6400 = 16960x$$

$$\therefore x = 0.3773$$

1M

96c) But $z = \frac{P}{A}$
Cont...

$$\therefore 0.3773 = \frac{1000}{A}$$

$$\therefore A = 2650.41 \text{ mm}^2$$

1M

But Area of Circular section $A = \frac{\pi d^2}{4}$

$$\therefore \frac{\pi d^2}{4} = 2650.41$$

1M

$$d^2 = \frac{4 \times 2650.41}{\pi} = 3374.60$$

$$\therefore d = \sqrt{3374.60}$$

$$\therefore d = 58.09 \text{ mm.}$$

1M