23242

3 Hours / 70 Marks   Seat No.

Instructions –   (1) All Questions are *Compulsory.*

(2) Answer each next main Question on a new page.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Assume suitable data, if necessary.

(6) Use of Non-programmable Electronic Pocket Calculator is permissible.

(7) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1.   **Solve any FIVE of the following:**   **10**

a)   If $f(x) = x^2 + 6x + 10$, find $f(2) + f(-2)$

b)   State whether the function $f(x) = \dfrac{e^x + e^{-x}}{2}$ is even or odd.

c)   Find $\dfrac{dy}{dx}$, if $y = x \cdot \log x$

d)   Evaluate: $\int e^{2\log_e x} \, dx$

e)   Evaluate: $\int \sin^3 x \, dx$

f)   Find the area under the curve $y = x^2$ from $x = 0$ to $x = 3$ with x-axis

g)   If $Z_1 = -2 + 4i$, $Z_2 = 1 - 3i$, then find $Z_1 \cdot Z_2$

Marks

2.   **Solve any THREE of the following:**   **12**

   a) If $x^2 + y^2 = 4xy$, find $\dfrac{dy}{dx}$ at $(2, -1)$

   b) If $x = a\,(2\theta - \sin2\theta\,)$, $y = a(1 - \cos2\theta)$ find $\dfrac{dy}{dx}$ at $\theta = \pi/4$

   c) A telegraph wire hang in the form of a curve $y = a\log\left[\sec\left(\frac{x}{a}\right)\right]$, where '$a$' is constant. Show that the curvature at any point is $\dfrac{1}{a}\cos\left(\frac{x}{a}\right)$.

   d) A metal wire 36 cm long is bent to form a rectangle. Find its dimensions when its area is maximum.

3.   **Solve any THREE of the following:**   **12**

   a) Find the equation of the tangent and normal to the curve $y = x\,(2-x)$ at point $(2, 0)$.

   b) If $y = x^x + (\sin x)^x$, find $\dfrac{dy}{dx}$

   c) If $y = log\,[x + \sqrt{x^2 + a^2}\,]$, find $\dfrac{dy}{dx}$

   d) Evaluate: $\displaystyle\int \dfrac{\sin(\sqrt{x}\,)}{\sqrt{x}}\,dx$

4.   **Solve any THREE of the following:**   **12**

   a) Evaluate: $\displaystyle\int \dfrac{1}{x^2 + 4x + 5}\,dx$

   b) Evaluate: $\displaystyle\int \dfrac{1}{5 + 4\cos x}\,dx$

   c) Evaluate: $\displaystyle\int x\cdot\tan^{-1}x\;dx$

   d) Evaluate: $\displaystyle\int \dfrac{1}{(x + 1)\;(x + 2)\;(x + 3)}\,dx$

   e) Evaluate: $\displaystyle\int_{0}^{\pi/2} \dfrac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx$

Marks

5. **Solve any TWO of the following:** 12

a) Find the area between the parabolas $y^2 = 4x$ and $x^2 = 4y$.

b) Attempt the following:

i) Find the order and degree of the differential equation

$$\sqrt[3]{\frac{dy}{dx} + y} = \sqrt[4]{\frac{d^2y}{dx^2}}$$

ii) Solve $\frac{dy}{dx} + y\cdot\tan x = \cos^2 x$

c) A resistance of 100Ω and inductance of 0.1 henry are connected in series with a battery of 20 volts satisfy the differential equation $L\frac{di}{dt} + Ri = E$. Find the current in the circuit at any instant.

6. **Solve any TWO of the following:** 12

a) Attempt the following:

i) Express $z = -2-2\sqrt{3}i$ into polar form

ii) Find the Laplace transform of: $(e^{-2t}\cdot t^3)$

b) Find the inverse Laplace transform of: $\dfrac{2S+1}{(S+1)(S^2+1)}$

c) Solve the differential equation using Laplace transform:

$$\frac{dq}{dt} + \frac{q}{RC} = \frac{E}{R}; \quad q(0) = 0$$

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