23242

3 Hours / 70 Marks   Seat No.

Instructions – (1) All Questions are *Compulsory.*

(2) Answer each next main Question on a new page.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Use of Non-programmable Electronic Pocket Calculator is permissible.

(6) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1.   **Solve any FIVE of the following:**   **10**

a) If $f(x) = 16^x + \log_2 x$, find $f\left(\dfrac{1}{4}\right)$.

b) State whether the function $f(x) = \left(\dfrac{a^x - a^{-x}}{2}\right)$ is odd or even?

c) Find $\dfrac{dy}{dx}$, if $y = \dfrac{1-x}{1+x}$.

d) Evaluate $\displaystyle\int \sin^3 x \; dx$

e) Evaluate $\displaystyle\int \dfrac{2x+3}{2x-1} \; dx$

f) Find order and degree of the differential equation
$\dfrac{d^2 y}{dx^2} = \left(y + \dfrac{dy}{dx}\right)^{3/2}$.

g) Find the area enclosed by the curve $y = x^3$, x–axis and the ordinates $x = 1$ and $x = 3$.

Marks

2. **Solve any THREE of the following:** 12

a) Differentiate $\tan^{-1}\left(\dfrac{5x}{1-6x^2}\right)$ w. r. to x

b) If $x^2 + y^2 = 4xy$, find $\dfrac{dy}{dx}$ at $(2, -1)$

c) If $x = a\ (\cos\theta + \theta\sin\theta)$ and $y = a\ (\sin\theta - \theta\cos\theta)$,
   find $\dfrac{dy}{dx}$ at $\theta - \pi/4$

d) Find the equation of the tangent and normal to the curve $y = 2x - x^2$ at point $(2, 0)$.

3. **Solve any THREE of the following:** 12

a) Differentiate $\tan x^{\cot x}$ w.r.to x.

b) Find the maximum and minimum values of $x^3 - 18x^2 + 96x$

c) Show that the radius of curvature to the curve $y = a\ \log\ \sec(x/a)$ at any point is a $\sec(x/a)$.

d) Evaluate $\displaystyle\int \dfrac{e^x(x+1)}{\sin^2(xe^x)}\,dx$

4. **Solve any THREE of the following:** 12

a) Evaluate $\displaystyle\int \dfrac{1}{3-2\sin x}\,dx$

b) Evaluate $\displaystyle\int x\cdot\sin^{-1}x\ dx$

c) Evaluate $\displaystyle\int \dfrac{\sec^2 x}{(1-\tan x)\ (2+\tan x)}\ dx$

d) Evaluate $\displaystyle\int_2^5 \dfrac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\ dx$

e) Evaluate $\displaystyle\int_0^{\pi/2} \dfrac{\tan x}{1+\tan x}\,dx$

Marks

5.   **Solve any <u>TWO</u> of the following:**   **12**

    a) Find the area enclosed by the parabola $y^2 = 4x$ and the line
        $2x - y + 4 = 0$

    b) Attempt the following:

        i)   Solve $(1+x^2)\ dy - (1+y^2)\ dx = 0$

        ii)   Solve $\dfrac{dy}{dx} + y \cot x = \cos x$

    c) The acceleration of a particle is given by $\dfrac{d^2x}{dt^2} = 3t^2 - 6t + 8$.
        Find the distance covered in 2 seconds. given that $v = 0$,
        $x = 0$ at $t = 0$.

6.   **Solve any <u>TWO</u> of the following:**   **12**

    a) Attempt the following:

        i)   Compute $\displaystyle\int_0^2 (1+x^3)\ dx$, using Trapezoidal rule. Divide the
            interval $[0, 2]$ into four sub-intervals.

        ii)   Evaluate $\displaystyle\int_2^7 \dfrac{dx}{x}$ using Trapezoidal rule, taking n = 5.

    b) Evaluate $\displaystyle\int_0^4 e^x\ dx$, using Simpson's $\dfrac{1}{3}$ rd rule by dividing the
        interval $[0, 4]$ into four equal parts.

    c) Evaluate $\displaystyle\int_0^6 \dfrac{1}{1+x^2}\ dx$ by using Simpson's $\dfrac{3}{8}$ th rule.

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