22480

24225

3 Hours / 70 Marks Seat No. | | | | | | | | |

Instructions : (1) All Questions are *compulsory.*

(2) Answer each next main Question on a new page.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Assume suitable data, if necessary.

(6) Use of Non-programmable Electronic Pocket Calculator is permissible.

(7) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Attempt any FIVE of the following : **10**

(a) If $u = \log (x^2 + y^2)$, then find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.

(b) Find $\frac{\partial^2 f}{\partial x^2}$ if $f(x) = (x^2 + y^2)^2 - 2(x^2 - y^2)$.

(c) Find the eigen value of the matrix $A = \begin{bmatrix} 1 & 2 \\ 5 & 4 \end{bmatrix}$

(d) Find the magnitude of the vector $\bar{a} = i - 2j + 4k$.

(e) Find the rank of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 6 \end{bmatrix}$.

(f) If $\bar{a} = 3i + 4j - 5k$ and $\bar{b} = 3i - 4j - 5k$, find the projection of $\bar{a}$ in the direction of $\bar{b}$.

(g) Construct difference table for the following data :

X	4	6	8	10
Y = *f*(X)	1	3	8	20

2. Attempt any THREE of the following : **12**

(a) Examine $f(x, y) = x^3 - y^2 - 3x$ for maximum and minimum value.

(b) Examine consistency of system of linear equations and solve it if consistent.

$x + 3y + 3z = 12$

$x + 4y + 4z = 15$

$x + 3y + 4z = 13$

(c) Evaluate $\int_0^1 x^3 dx$, by using trapezoidal rule.

Given that

x	0	0.2	0.4	0.6	0.8	1.0
***f*(*x*)**	0	0.008	0.064	0.216	0.512	1.000

(d) A furniture dealer deals in only two items, viz table and chairs. He has ₹ 11,000 to invest and a space to store at most 40 pieces. A table costs him ₹ 500 and a chair ₹ 200. He can sell a table at a profit of ₹ 50 and a chair at a profit of ₹ 15. Assume that he can sell all the items that he buys. Formulate this problem as an LPP so that he can maximize the profit.

3. Attempt any THREE of the following : **12**

(a) Find the inverse of matrix $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ by elementary transformation.

(b) Write out $a_{rs} x^s = br$ (r, s = 1, 2, 3, ... n) in full.

(c) Using Newton's backward difference formula find $f(4)$ for the following :

x	0	1	2	3
$f(x)$	1	2	1	10

(d) Find the maximum value of $Z = 7x + 11y$

Subject to constraints

$3x + 5y \leq 26$

$5x + 3y \leq 30$

$x \geq 0, y \geq 0$

4. Attempt any THREE of the following : **12**

(a) Test for consistency and solve $5x + 3y + 7z = 4$; $3x + 26y + 2z = 9$

$7x + 2y + 10z = 5$

(b) The matrix A is defined as, $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$. Find the Eigen value of matrix A.

(c) Find the cross product $\bar{a} \times \bar{b}$ for the vector $\bar{a} = \hat{i} + 3\hat{j} - 2\hat{k}$, $\bar{b} = -\hat{i} + 5\hat{k}$.

(d) Find the angle between the vector $\bar{a} = 2i + 6j + 3k$ & $\bar{b} = 12i - 4j + 3k$.

(e) Find the scalar triple product $\bar{a} \cdot (\bar{b} \times \bar{c})$ for the vector

$\bar{a} = i + 2j + 3k$

$\bar{b} = -i + j + 2k$

$\bar{c} = 2i + j + 4k$

5. Attempt any TWO of the following : **12**

(a) (i) Discuss the maximum and minimum of $f(x, y) = x^2 + y^2 + 6x + 12$

(ii) Construct a forward difference table for the following data :

x	0	10	20	30
y	0	0.174	0.347	0.518

(b) Using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule to evaluate $\int_1^3 (3x^2 + 3x)\,dx$.

where n = 4. Given that

x	1	1.5	2.0	2.5	3.6
$f(x)$	5	9.75	16	23.75	33

(c) Find the Eigen values and the corresponding Eigen vectors for the matrix.

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$$

6. Attempt any TWO of the following : **12**

(a) (i) Find the absolute maximum and minimum values of $f(x, y) = 2 + 2x + 2y - x^2 - y^2$ on triangular plate in the first quadrant, bounded by the lines $x = 0$, $y = 0$ and $y = g - x$.

(ii) Write out all the tensor in $S = a_{ij}\, x^i\, x^j$ taking n = 3 and write the tensor contained in x^{pq}, x_{qr} if n= 2.

(b) Find the value of K such that the system of equations

$x + ky + 3z = 0$

$4x + 3y + kz = 0$

$2x + y + 2z = 0$

has non-trivial solution.

(c) Using simplex method solve the LPP

Minimize $Z = 7x + y$

Subject to $5x + y \geq 5$

$x + y \geq 3$

$x \geq 0, y \geq 0$