314320

24225

3 Hours / 70 Marks Seat No. | | | | | | | | |

Instructions : (1) All Questions are *compulsory.*

(2) Answer each next main Question on a new page.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Use of Non-programmable Electronic Pocket Calculator is permissible.

(6) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Attempt any FIVE of the following : **10**

(a) If $f(x, y) = x^2 - 2xy$, find $\frac{\partial^2 f}{\partial y \cdot \partial x}$.

(b) Find the rank of the matrix $\begin{bmatrix} 3 & -2 \\ 6 & -4 \end{bmatrix}$.

(c) Find Eigen values of matrix A, where A = $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

(d) Show that the vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 3\hat{j} - 5\hat{k}$ are perpendicular to each other.

(e) If $\vec{a} = 3\hat{i} - 6\hat{j} + 2\hat{k}$, find $|\vec{a}|$.

(f) State Simpson's $1/3^{rd}$ rule of Numerical Integration.

(g) Given :

x	0	1	2	3	4	5
y	1	3	7	13	21	31

Find a backward difference table for above data.

2. Attempt any THREE of the following : **12**

(a) If $f(x, y) = 4x^3y^2 + 5x^2y^3$, find $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial y^2}$.

(b) Discuss the maxima & minima of the function $3x^2 - y^2 + x^3$.

(c) A rectangular box, open at the top, is to have a volume of 32 m^3. Find the dimensions of the box requiring least material for its construction.

(d) Use Lagrange's multiplier method to find extreme values of the function $x^2 + 2y^2$ subject to the constraint $x + y - 6 = 0$.

3. Attempt any THREE of the following : **12**

(a) Find the inverse of the matrix by elementary transformations :

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

(b) Solve the following system of equations :

$$x - y + 2z = 3,\ x - 2y + 3z = 5,\ 3x - 4y - 5z = -13$$

(c) Reduce the following matrix to normal form and hence find its rank :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

(d) Find the Eigen value of a matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

4. Attempt any THREE of the following : **12**

(a) Determine the value of λ for which the system of linear equations $3x + 2y + 4z = 3$, $x + y + z = \lambda$, $5x + 4y + 6z = 15$ are consistent and find corresponding solution.

(b) Find angle between the vectors $\vec{a} = 2\hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} + 6\hat{j} + 2\hat{k}$.

(c) A force $\vec{F} = 2\hat{i} + \hat{j} - \hat{k}$ acts at the point $3\hat{i} - \hat{j} - 2\hat{k}$. Find moment of the force about the point $4\hat{i} - \hat{j} + \hat{k}$.

(d) A particle is placed from a point whose position vector is $5\hat{i} - 5\hat{j} - 7\hat{k}$ to the point $6\hat{i} + 2\hat{j} - 2\hat{k}$ under the action of the forces $10\hat{i} - \hat{j} + 11\hat{k}$, $4\hat{i} + 5\hat{j} + 6\hat{k}$, $-2\hat{i} + \hat{j} - 9\hat{k}$. Find work done.

(e) Find area of parallelogram formed by two vectors $3\hat{i} + 2\hat{j}$ & $2\hat{j} + 4\hat{k}$.

5. Attempt any TWO of the following : **12**

(a) Find y'(0) & y''(0) from the following data :

x	0	1	2	3	4	5
y	4	8	15	7	6	2

(b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by Trapezoidal null by taking n = 4.

(c) Given :

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$y = e^{-x}$	1	0.6064	0.3676	0.2231	0.1353

Evaluate $\int_0^2 e^{-x}\,dx$ by using Simpson's 1/3rd rule.

6. Attempt any TWO of the following : **12**

(a) Find the Eigen value and Eigen vector of a matrix $A = \begin{bmatrix} 10 & -9 \\ 6 & -5 \end{bmatrix}$.

(b) A paper mill received three orders for paper rolls with the widths & length indicated in the following table :

Order No.	Width (meters)	Length (meters)
1	5	10,000
2	7	30,000
3	9	20,000

Rolls are produced in the mill in two standard widths 10 & 20 meters which are slit to the sizes specified by the orders. There is no limit on the lengths of the standard rolls. The objective is to determine the production schedule that minimizes the firm losses while satisfying the given demand.

(c) Solve the following L.P.P. problem by using graphical method :

Maximize $Z = 12x + 16y$

Subject to $10x + 20y \leq 120$

$8x + 8y \leq 80$

$x \geq 0, y \geq 0$
