



SUMMER – 19 EXAMINATION

Subject Name: Theory of Machines

Model Answer

Subject Code: **17412**

Important Instructions to examiners:

- 1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
- 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
- 3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills).
- 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
- 5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
- 6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
- 7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
Q.1 A	a	Four inversions of single slider chain Pendulum pump Rotary engine Whitworth quick return mechanism Crank & slotted lever mechanism	½ M each
	b	List 4 types of followers Knife edge follower Roller follower Flat faced or mushroom follower Spherical faced follower	½ M each
	c	Materials for Belt Leather belts. Cotton or fabric belts Rubber belt. Balata belts.	½ M each



d	<p>The different types of chains used in power transmission are,</p> <p>Roller chain. Silent chain. Leaf Chain. Flat-top Chain. Engineering Steel Chain.</p>	<p>Any two types 1 M each</p>
e	<p>A flywheel used in machines serves as a reservoir, which stores energy during the period when the supply of energy is more than the requirement, and releases it during the period when the requirement of energy is more than the supply.</p>	<p>2 M</p>
f	<p>Sensitivity</p> <p>The sensitiveness is defined as the ratio of the difference between the maximum and minimum equilibrium speeds to the mean equilibrium speed.</p>	<p>2 M</p>
g	<p>Classification of Dynamometers</p> <p>Following are the two types of dynamometers, used for measuring the brake power of an engine.</p> <ol style="list-style-type: none">1. Absorption dynamometers, and2. Transmission dynamometers. <p>Classification of Absorption Dynamometers</p> <ol style="list-style-type: none">1. Prony brake dynamometer, and 2. Rope brake dynamometer.	<p>Broad classification 2 M</p>
h	<p>Define Balancing & state its necessity</p> <p>The process of providing the second mass in order to counteract the effect of the centrifugal force of the first mass is called balancing of rotating masses.</p> <p>The high speed of engines and other machines is a common phenomenon now-a-days. It is, therefore, very essential that all the rotating and reciprocating parts should be completely balanced as far as possible. If these parts are not properly balanced, the dynamic forces are set up. These forces not only increase the loads on bearings and stresses in the various members, but also produce unpleasant and even dangerous vibrations. Thus, balancing is very necessary.</p>	<p>1 M Each</p>

1B

a

Identify basic kinematic chain

Oldham's coupling: Double Slider Chain

Whitworth's Quick Return Mechanism: Single Slider Chain

Pantograph: Four Bar Chain

Elliptical Trammel: Double Slider Chain

1 M Each

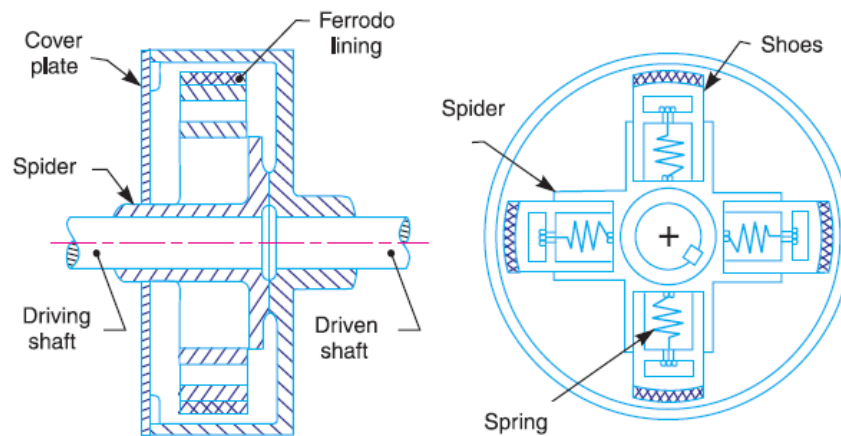
b

The centrifugal clutches are usually incorporated into the motor pulleys. It consists of a number of shoes on the inside of a rim of the pulley, as shown in Fig. The outer surfaces of the shoes are covered with a friction material. These shoes, which can move radially in guides, are held against the boss (or spider) on the driving shaft by means of springs. The springs exert a radially inward force which is assumed constant. The mass of the shoe, when revolving, causes it to exert a radially outward force (*i.e.* centrifugal force).

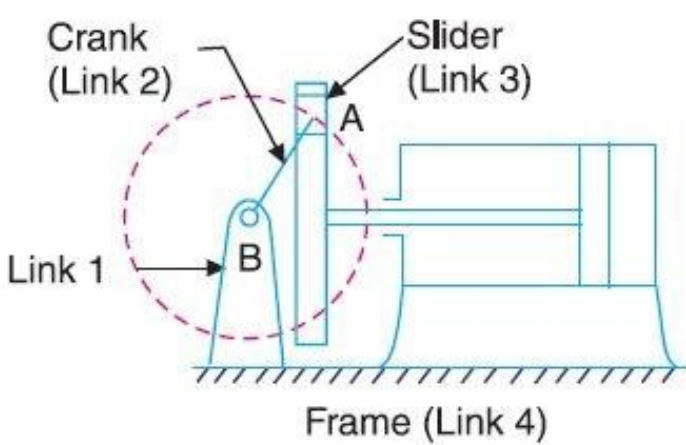
The magnitude of this centrifugal force depends upon the speed at which the shoe is revolving. A little consideration will show that when the centrifugal force is less than the spring force, the shoe remains in the same position as when the driving shaft was stationary, but when the centrifugal force is equal to the spring force, the shoe is just floating. When the centrifugal force exceeds the spring force, the shoe moves outward and comes into contact with the driven member and presses against it. The force with which the shoe presses against the driven member is the difference of the centrifugal force and the spring force.

2M Fig

2M
Explanation



Centrifugal clutch.

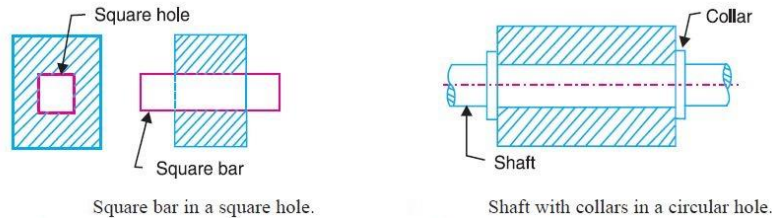
2	c	<p>Module. It is the ratio of the pitch circle diameter in millimeters to the number of teeth. It is usually denoted by m. Mathematically, Module, $m = D / T$</p> <p>Diametral Pitch. It is the ratio of number of teeth to the pitch circle diameter in millimeters. It is denoted by p_d so, Diametral Pitch = T/PCD</p> <p>Circular pitch. It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by p_c. Mathematically, Circular pitch, $p_c = \pi D/T$ where D = Diameter of the pitch circle, and T = Number of teeth on the wheel.</p> <p>Pitch point. It is a common point of contact between two pitch circles.</p>	1M Each
	a	<p>Scotch Yoke Mechanism:</p>  <p style="text-align: center;">Scotch yoke mechanism.</p> <p>Working:- This mechanism is used for converting rotary motion into a reciprocating motion. The inversion is obtained by fixing either the link 1 or link 3. In Fig., link 1 is fixed. In this mechanism, when the link 2 (which corresponds to crank) rotates about B as centre, the link 4 (which corresponds to a frame) reciprocates. The fixed link 1 guides the frame.</p>	2M Fig 2M Explanation

b

1. Completely constrained motion:- When the motion between a pair is limited to a definite direction irrespective of the direction of force applied, then the motion is said to be a completely constrained motion.

2M Each

Any one diagram

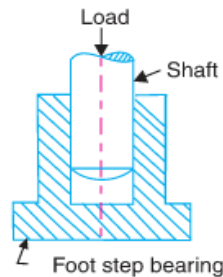


Examples:

The motion of a square bar in a square hole

The motion of a shaft with collars at each end in a circular hole,

2. Successfully constrained motion:- When the motion between the elements, forming a pair, is such that the constrained motion is not completed by itself, but by some other means, then the motion is said to be successfully constrained motion. Consider a shaft in a foot-step bearing as shown in Fig. The shaft may rotate in a bearing or it may move upwards. This is a case of incompletely constrained motion. But if the load is placed on the shaft to prevent axial upward movement of the shaft, then the motion of the pair is said to be successfully constrained motion.



Shaft in a foot step bearing.

- Examples:
1. The motion of an I.C. engine valve (these are kept on their seat by a spring)
 2. The piston reciprocating inside an engine cylinder
 3. Shaft in a foot step bearing

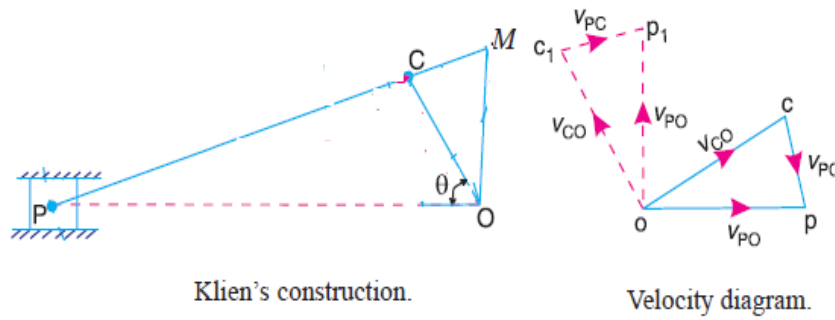
c

Absolute Velocity :- Velocity of any point on a link with respect to another fixed point on the mechanism is known as Absolute Velocity. It is denoted as V_A or V_B or V_P etc.

2M Each

Relative Velocity :- Velocity of any point on a link with respect to another moving point on the mechanism is known as Relative Velocity. It is denoted as V_{AB} or V_{BC} or V_{PQ} etc.

d



2M Fig

Klein's velocity diagram

First of all, draw OM perpendicular to OP; such that it intersects the line PC produced at M.

2M
Explanation

The triangle OCM is known as **Klein's velocity diagram**. In this triangle OCM, OM may be regarded as a line perpendicular to PO, CM may be regarded as a line parallel to PC, and CO may be regarded as a line parallel to CO.

We have already discussed that the velocity diagram for given configuration is a triangle ocp as shown in Fig. If this triangle is revolved through 90° , it will be a triangle oc1p1, in which oc1 represents V_{CO} (i.e. velocity of C with respect to O or velocity of crank pin C) and is parallel to OC, op1 represents V_{PO} (i.e. velocity of P with respect to O or velocity of cross-head or piston P) and is perpendicular to OP, and c1p1 represents V_{PC} (i.e. velocity of P with respect to C) and is parallel to CP.

e

Roller follower is preferred over knife edge follower due to following reasons :-

2M

- i) Working is smooth due to rolling motion
- ii) There is no noise in operation
- iii) Life is more
- iv) Due to rolling action it results in more accuracy and reliability

2M

Two Applications: (Any two)

The Roller Follower is used in a wide range of applications such as cam mechanisms of automatic machines, dedicated machines as well as carrier systems, conveyors, bookbinding machines, tool changers of machining centers, pallet changers, automatic coating machines, sliding forks of automatic warehouses, I C Engine Valves & Fuel Injection Pump

f Problem on Belt:

Given : $d_1 = 450 \text{ mm} = 0.45 \text{ m}$ or $r_1 = 0.225 \text{ m}$; $d_2 = 200 \text{ mm} = 0.2 \text{ m}$ or $r_2 = 0.1 \text{ m}$; $x = 1.95 \text{ m}$; $N_1 = 200 \text{ r.p.m.}$; $T_1 = 1 \text{ kN} = 1000 \text{ N}$; $\mu = 0.25$

We know that speed of the belt,

$$v = \frac{\pi d_1 \cdot N_1}{60} = \frac{\pi \times 0.45 \times 200}{60} = 4.714 \text{ m/s}$$

Length of the belt

We know that length of the crossed belt,

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(0.225 + 0.1) + 2 \times 1.95 + \frac{(0.225 + 0.1)^2}{1.95} = 4.975 \text{ m Ans.}$$

Angle of contact between the belt and each pulley

Let θ = Angle of contact between the belt and each pulley.

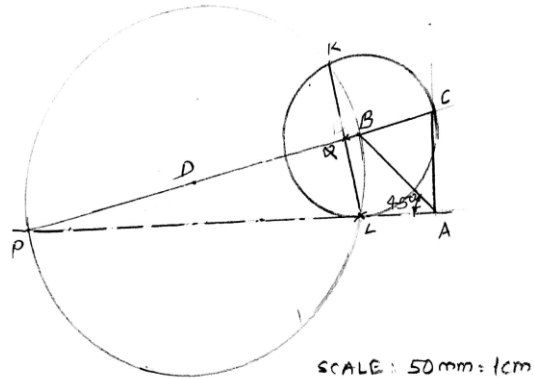
We know that for a crossed belt drive,

$$\sin \alpha = \frac{r_1 + r_2}{x} = \frac{0.225 + 0.1}{1.95} = 0.1667 \text{ or } \alpha = 9.6^\circ$$

$$\therefore \theta = 180^\circ + 2\alpha = 180^\circ + 2 \times 9.6^\circ = 199.2^\circ$$

$$= 199.2 \times \frac{\pi}{180} = 3.477 \text{ rad Ans.}$$

Klein's Construction



Given

$AB = 150 \text{ mm}$; $BP = 500 \text{ mm}$; $N = 500 \text{ rpm}$.

The Klein's velocity diagram ABC and

The Klein's Acceleration diagram BQLA

By measurement;

$AC = 2.7 \text{ cm}$ = By scale $2.7 \times 50 = 135 \text{ mm} \approx 0.135 \text{ m}$

$LA = 2.1 \text{ cm}$ = By scale $2.1 \times 50 = 105 \text{ mm} = 0.105 \text{ m}$

Velocity & Acceleration of Piston,

We know that the velocity of piston P,

$$V_p = \omega \times AC$$

$$= 52.33 \times 0.135$$

$$= 7.065 \text{ m/s.}$$

$$\therefore \omega = \frac{2\pi N}{60}$$

$$= \frac{2\pi \times 500}{60}$$

$$= 52.33 \text{ rad/sec.}$$

& acceleration of piston P,

$$a_p = \omega^2 \times LA$$

$$= (52.33)^2 \times 0.105$$

$$= 287.54 \text{ m}^2/\text{sec.}$$

1M

1M

2M

2M
Diagram

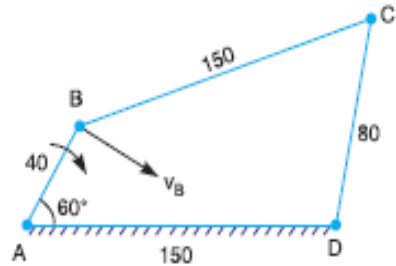
Ans. 2M

b **Four Bar Mechanism problem**

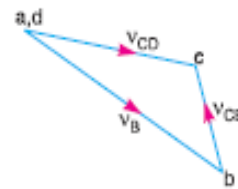
Given : $N_{BA} = 120$ r.p.m. or $\omega_{BA} = 2\pi \times 120/60 = 12.568$ rad/s

Since the length of crank $AB = 40$ mm = 0.04 m, therefore velocity of B with respect to A or velocity of B , (because A is a fixed point),

$$v_{BA} = v_B = \omega_{BA} \times AB = 12.568 \times 0.04 = 0.503 \text{ m/s}$$



Space diagram (All dimensions in mm).



Velocity diagram.

First of all, draw the space diagram to some suitable scale, as shown in Fig. Now the velocity diagram, as shown in Fig. , is drawn as discussed below :

1. Since the link AD is fixed, therefore points a and d are taken as one point in the velocity diagram. Draw vector ab perpendicular to BA , to some suitable scale, to represent the velocity of B with respect to A or simply velocity of B (i.e. v_{BA} or v_B) such that

$$\text{vector } ab = v_{BA} = v_B = 0.503 \text{ m/s}$$

2. Now from point b , draw vector bc perpendicular to CB to represent the velocity of C with respect to B (i.e. v_{CB}) and from point d , draw vector dc perpendicular to CD to represent the velocity of C with respect to D or simply velocity of C (i.e. v_{CD} or v_C). The vectors bc and dc intersect at c .

By measurement, we find that

$$v_{CD} = v_C = \text{vector } dc = 0.385 \text{ m/s}$$

We know that $CD = 80$ mm = 0.08 m

\therefore Angular velocity of link CD ,

$$\omega_{CD} = \frac{v_{CD}}{CD} = \frac{0.385}{0.08} = 4.8 \text{ rad/s (clockwise about D) Ans.}$$

Space Diag.
1M

Velocity
Diag. 2M

Ans. 1M

c **Advantages and Disadvantages of V-belt Drive Over Flat Belt Drive**

Following are the advantages and disadvantages of the V-belt drive over flat belt drive.

Advantages

1. The V-belt drive gives compactness due to the small distance between the centers of pulleys.
2. The drive is positive, because the slip between the belt and the pulley groove is negligible.
3. Since the V-belts are made endless and there is no joint trouble, therefore the drive is smooth.
4. It provides longer life, 3 to 5 years.

Any 4
points

1M each



5. It can be easily installed and removed.
6. The operation of the belt and pulley is quiet.
7. The belts have the ability to cushion the shock when machines are started.
8. The high velocity ratio (maximum 10) may be obtained.
9. The wedging action of the belt in the groove gives high value of limiting ratio of tensions.
Therefore the power transmitted by V-belts is more than flat belts for the same coefficient of friction,
arc of contact and allowable tension in the belts.
10. The V-belt may be operated in either direction with tight side of the belt at the top or bottom. The centre line may be horizontal, vertical or inclined.

Theories used in design of clutches and bearings:

i) Uniform pressure theory in clutches and bearings:

When the mating component in clutch, bearing are new, then the contact between surfaces may be good over the whole surface. It means that the pressure over the rubbing surfaces is uniform distributed.

This condition is not valid for old clutches, bearings because mating surfaces may have uneven friction.

The condition assumes that intensity of pressure is same.
 $P = W/A = \text{Constant}$; where, W = load, A = area

ii) Uniform wear theory in clutches and bearings:

When clutch, bearing become old after being used for a given period, then all parts of the rubbing surfaces will not move with the same velocity. The velocity of rubbing surface increases with the distance from the axis of the rotating element.

It means that wear may be different at different radii and rate of wear depends upon the intensity of pressure (P) and the velocity of rubbing surfaces (V). It is assumed that the rate of wear is proportional to the product of intensity of pressure and velocity of rubbing surfaces.

This condition assumes that rate of wear is uniform;

$P \cdot r = \text{Constant}$; where, P = intensity of pressure, r = radius of rotation

d

2M Each

e

Problem on Balancing

Let m = Balancing mass, and
 θ = The angle which the balancing mass makes with m_1 .

Since the magnitude of centrifugal forces are proportional to the product of each mass and its radius, therefore

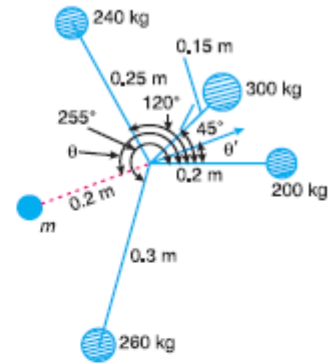
$$m_1 \cdot r_1 = 200 \times 0.2 = 40 \text{ kg-m}$$

$$m_2 \cdot r_2 = 300 \times 0.15 = 45 \text{ kg-m}$$

$$m_3 \cdot r_3 = 240 \times 0.25 = 60 \text{ kg-m}$$

$$m_4 \cdot r_4 = 260 \times 0.3 = 78 \text{ kg-m}$$

The problem may, now, be solved either analytically or graphically. But we shall solve the problem by both the methods one by one.



Space Diagram

The space diagram is shown in Fig.

Resolving $m_1 \cdot r_1$, $m_2 \cdot r_2$, $m_3 \cdot r_3$ and $m_4 \cdot r_4$ horizontally,

$$\begin{aligned} \Sigma H &= m_1 \cdot r_1 \cos \theta_1 + m_2 \cdot r_2 \cos \theta_2 + m_3 \cdot r_3 \cos \theta_3 + m_4 \cdot r_4 \cos \theta_4 \\ &= 40 \cos 0^\circ + 45 \cos 45^\circ + 60 \cos 120^\circ + 78 \cos 255^\circ \\ &= 40 + 31.8 - 30 - 20.2 = 21.6 \text{ kg-m} \end{aligned}$$

Now resolving vertically,

$$\begin{aligned} \Sigma V &= m_1 \cdot r_1 \sin \theta_1 + m_2 \cdot r_2 \sin \theta_2 + m_3 \cdot r_3 \sin \theta_3 + m_4 \cdot r_4 \sin \theta_4 \\ &= 40 \sin 0^\circ + 45 \sin 45^\circ + 60 \sin 120^\circ + 78 \sin 255^\circ \\ &= 0 + 31.8 + 52 - 75.3 = 8.5 \text{ kg-m} \end{aligned}$$

$$\therefore \text{Resultant, } R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(21.6)^2 + (8.5)^2} = 23.2 \text{ kg-m}$$

We know that

$$m \cdot r = R = 23.2 \quad \text{or} \quad m = 23.2 / r = 23.2 / 0.3 = 77.33 \text{ kg}$$

and

$$\tan \theta' = \Sigma V / \Sigma H = 8.5 / 21.6 = 0.3935 \quad \text{or} \quad \theta' = 21.48^\circ$$

Since θ' is the angle of the resultant R from the horizontal mass of 200 kg, therefore the angle of the balancing mass from the horizontal mass of 200 kg,

$$\theta = 180^\circ + 21.48^\circ = 201.48^\circ$$

ΣH 1M

ΣV 1M

R 1M

θ 1M

f

- i) **Pitch curve.** It is the curve generated by the trace point as the follower moves relative to the cam. For a knife edge follower, the pitch curve and the cam profile are same whereas for a roller follower, they are separated by the radius of the roller.
- ii) **Pressure angle.** It is the angle between the direction of the follower motion and a normal to the pitch curve. This angle is very important in designing a cam profile. If the pressure angle is too large, a reciprocating follower will jam in its bearings.

2 M Each

4

a

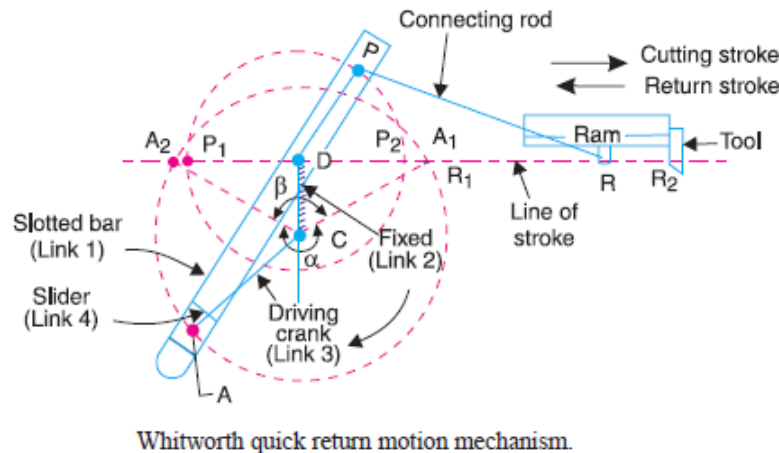
Following are the **parameters considered for selection of chain drive** for power transmission:

1. Type of application.
2. Shock load.
3. Source of power: motor type; rated power (kW); moment of inertia, ; rated torque at driving speed; starting torque; and stopping torque.
4. Drive sprocket rpm and shaft diameter.
5. Driven sprocket rpm and shaft diameter.
6. Center distance between sprockets.
7. Noise constraints.
8. Lubrication (possible or not).

Any 4 points
1M Each

b

Shaper Mechanism



2M Fig

Whitworth quick return motion mechanism. This mechanism is mostly used in shaping and slotting machines. In this mechanism, the link CD (link 2) forming the turning pair is fixed, as shown in Fig. The link 2 corresponds to a crank in a reciprocating steam engine. The driving crank CA (link 3) rotates at a uniform angular speed. The slider (link 4) attached to the crank pin at A slides along the slotted bar PA (link 1) which oscillates at a pivoted point D . The connecting rod PR carries the ram at R to which a cutting tool is fixed. The motion of the tool is constrained along the line RD produced, *i.e.* along a line passing through D and perpendicular to CD .

2M
Explanation

When the driving crank CA moves from the position CA_1 to CA_2 (or the link DP



from the position $DP1$ to $DP2$) through an angle α in the clockwise direction, the tool moves from the left hand end of its stroke to the right hand end through a distance $2 PD$. Now when the driving crank moves from the position $CA2$ to $CA1$ (or the link DP from $DP2$ to $DP1$) through an angle β in the clockwise direction, the tool moves back from right hand end of its stroke to the left hand end.

It is seen that the time taken during the left to right movement of the ram (*i.e.* during forward or cutting stroke) will be equal to the time taken by the driving crank to move from $CA1$ to $CA2$. Similarly, the time taken during the right to left movement of the ram (or during the idle or return stroke) will be equal to the time taken by the driving crank to move from $CA2$ to $CA1$.

Since the crank link CA rotates at uniform angular velocity therefore time taken during the cutting stroke (or forward stroke) is more than the time taken during the return stroke. In other words, the mean speed of the ram during cutting stroke is less than the mean speed during the return stroke.

c Difference between Flywheel and Governor (Any 4 points – 4 Marks)

Any 4 points
1M Each

FLYWHEEL	GOVERNOR
1.Function- To control the speed variations caused by fluctuations of engine turning moment during a cycle.	Function- To regulate the mean speed of engine within prescribed limit when there are variations of load.
2 .Mathematically it controls dN/ dT	2. Mathematically it controls dN
3. Flywheel acts as a reservoir; it stores energy due to its mass moment of inertia and releases energy when required during a cycle.	3. A governor regulates the speed by regulating the quantity of charge/working fluid of prime mover.
4.It regulates speed in one cycle only	4. It regulates speed over a period of time.
5.Flywheel has no control over supply of fluid/charge	5. Governor takes care of quantity of fluid
6. It is not an essential element of every prime mover. It is used when there are undesirable cyclic fluctuations.	6. It is an essential element of prime mover since varying demand of power is met by it.

d

Rope Brake dynamometer:

It is another form of absorption type dynamometer which is most commonly used for measuring the brake power of the engine. It consists of one, two or more ropes wound around the flywheel or rim of a pulley fixed rigidly to the shaft of an engine. The upper end of the ropes is attached to a spring balance while the lower end of the ropes is kept in position by applying a dead weight as shown in Fig.

In order to prevent the slipping of the rope over the flywheel, wooden blocks are placed at intervals around the circumference of the flywheel.

In the operation of the brake, the engine is made to run at a constant speed. The frictional torque, due to the rope, must be equal to the torque being transmitted by the engine.

- Let W = Dead load in newtons,
 S = Spring balance reading in newtons,
 D = Diameter of the wheel in metres,
 d = diameter of rope in metres, and
 N = Speed of the engine shaft in r.p.m.

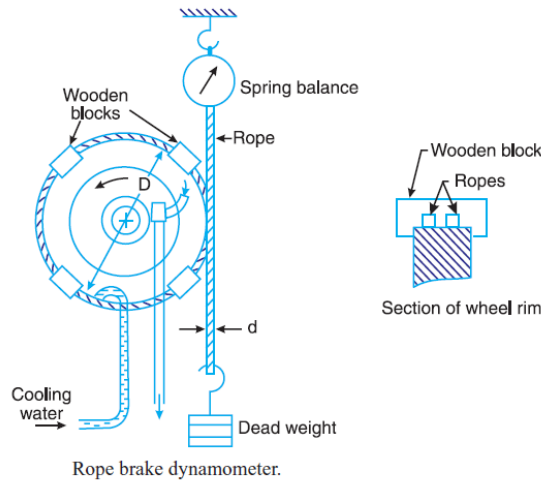
\therefore Net load on the brake
 $= (W - S) N$

We know that distance moved in one revolution

$$= \pi(D + d) \text{ m}$$

\therefore Work done per revolution
 $= (W - S) \pi(D + d) \text{ N-m}$

and work done per minute
 $= (W - S) \pi(D + d) N \text{ N-m}$



\therefore Brake power of the engine,

$$\text{B.P.} = \frac{\text{Work done per min}}{60} = \frac{(W - S) \pi(D + d)N}{60} \text{ watts}$$

If the diameter of the rope (d) is neglected, then brake power of the engine,

$$\text{B.P.} = \frac{(W - S) \pi D N}{60} \text{ watts}$$

2M Fig

2M
Explanation



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e

Given

$P = 50 \text{ kW}$; $N = 1750 \text{ rpm}$; $\mu = 0.12$; $p = 0.15 \text{ N/mm}^2$
 $r_1 = 120 \text{ mm}$; $r_2 = 90 \text{ mm}$; find no. of plates to transmit
the required torque.

Since the intensity of pressure is max. at the inner radius r_2 ,
 $\therefore p \cdot r_2 = C$ or $C = 0.15 \times 90 = 13.5 \text{ N/mm}$

We know that axial force required to engage the clutch,

$$W = 2\pi C (r_1 - r_2) = 2\pi \times 13.5 (120 - 90) = 2543 \text{ N}$$

and mean radius of friction surfaces,

$$R = \frac{r_1 + r_2}{2} = \frac{120 + 90}{2} = 105 \text{ mm} \approx 0.105 \text{ m}$$

We know that torque transmitted,

$$T = n \cdot \mu \cdot W \cdot R =$$

$$\therefore 272.98 = n \times 0.12 \times 2543 \times 0.105 = 8.52$$

$$\therefore n = 9$$

Power transmitted, $P = T \cdot \omega$

$$\therefore T = \frac{P}{\omega} = \frac{50 \times 10^3}{2\pi \times 1750/60}$$
$$= 272.98 \text{ N-m}$$

1M

1M

1M

1M

f

Problem on Balancing

Space
Diagram 1M

Given

$$m_1 = 12 \text{ kg}; m_2 = 10 \text{ kg}; m_3 = 18 \text{ kg}; m_4 = 15 \text{ kg}$$

$$r_1 = 40 \text{ mm}; r_2 = 50 \text{ mm}; r_3 = 60 \text{ mm}; r_4 = 30 \text{ mm}$$

$$\theta_1 = 0^\circ; \theta_2 = 60^\circ; \theta_3 = 135^\circ$$

$$\theta_4 = 270^\circ; r = 100 \text{ mm}$$

Let m = Balancing mass

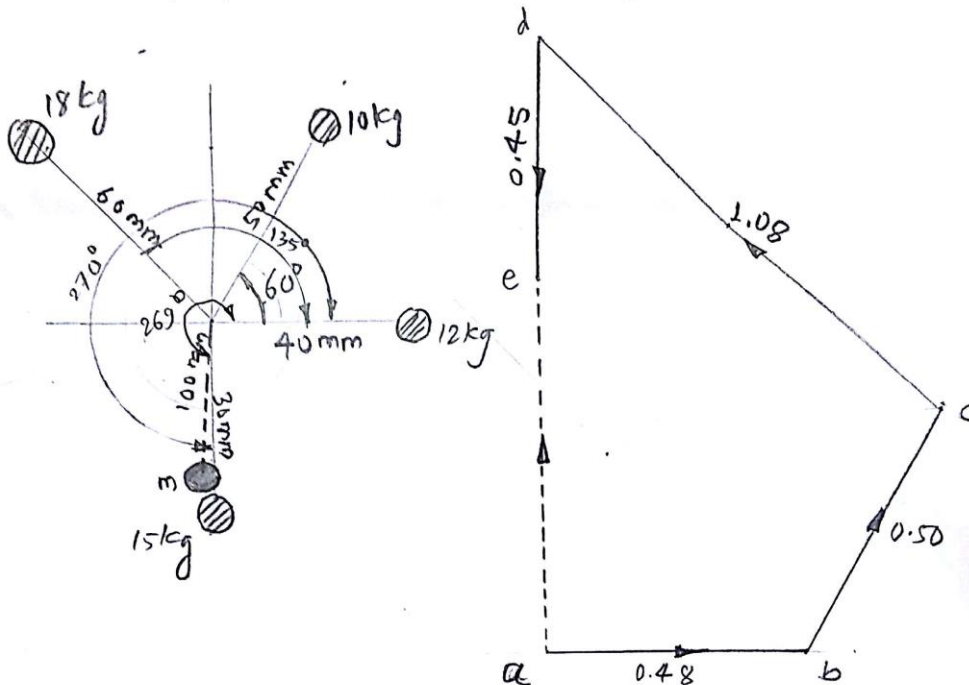
θ = The angle which the balancing mass makes with m_1

$$m_1 r_1 = 12 \times 0.040 = 0.48 \text{ kg-m}$$

$$m_2 r_2 = 10 \times 0.050 = 0.50 \text{ kg-m}$$

$$m_3 r_3 = 18 \times 0.060 = 1.08 \text{ kg-m}$$

$$m_4 r_4 = 15 \times 0.030 = 0.45 \text{ kg-m}$$



Vector
Diagram
1M

m 1M

θ 1M

By measurement $ae = 0.69 \text{ kg-m}$

The balancing force is equal to the resultant force, but opposite in direction as shown in fig. Since the balancing force is proportional to $m \cdot r$, therefore $m \times 0.1 = \text{vector } ea = 0.69 \text{ kgm}$.

$$\therefore m = 0.69 / 0.1 = \underline{6.9 \text{ kg}}$$

Angle of inclination of the balancing mass (m) from the horizontal mass of 12 kg, $\theta = 269^\circ$

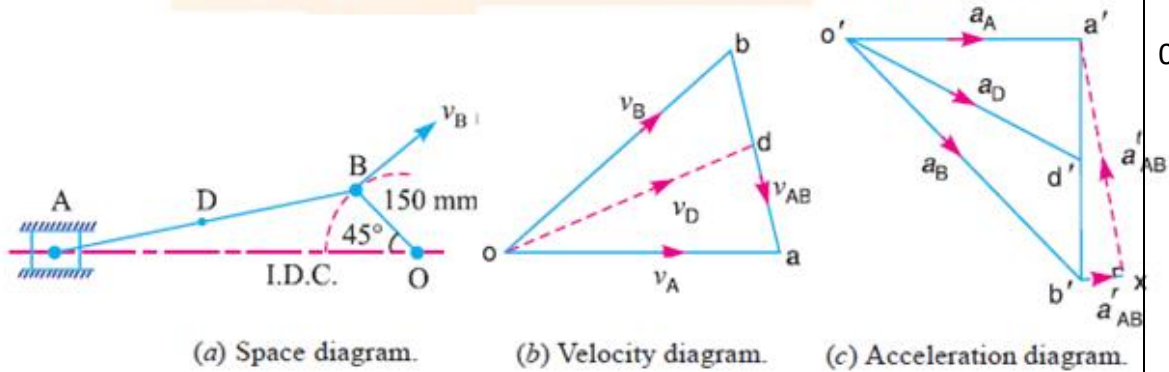
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a

Given : $N_{RO} = 300$ r.p.m. or $\omega_{RO} = 2\pi \times 300/60 = 31.42$ rad/s; $OB = 150$ mm = 0.15 m
 $BA = 500$ mm = 0.5 m

We know that linear velocity of B with respect to O or velocity of B ,

$$v_{BO} = v_B = \omega_{BO} \times OB = 31.42 \times 0.15 = 4.713 \text{ m/s} \dots (\text{Perpendicular to } BO)$$



(a) Space diagram.

(b) Velocity diagram.

(c) Acceleration diagram.

1. Linear velocity of the midpoint of the connecting rod

First of all draw the space diagram, to some suitable scale; as shown in Fig. 8.4 (a). Now the velocity diagram, as shown in Fig. 8.4 (b), is drawn as discussed below:

1. Draw vector ob perpendicular to BO , to some suitable scale, to represent the velocity of B with respect to O or simply velocity of B i.e. v_{BO} or v_B , such that

$$\text{vector } ob = v_{BO} = v_B = 4.713 \text{ m/s}$$

2. From point b , draw vector ba perpendicular to BA to represent the velocity of A with respect to B i.e. v_{AB} , and from point o draw vector oa parallel to the motion of A (which is along AO) to represent the velocity of A i.e. v_A . The vectors ba and oa intersect at a .

By measurement, we find that velocity of A with respect to B ,

$$v_{AB} = \text{vector } ba = 4.1 \text{ m/s}$$

and Velocity of A , $v_A = \text{vector } oa = 4.92 \text{ m/s}$

3. In order to find the velocity of the midpoint D of the connecting rod AB , divide the vector ba at d in the same ratio as D divides AB , in the space diagram. In other words,

$$bd / ba = BD / BA$$

Note: Since D is the midpoint of AB , therefore d is also midpoint of vector ba .

4. Join od . Now the vector od represents the velocity of the midpoint D of the connecting rod i.e. v_D .

By measurement, we find that

$$v_D = \text{vector } od = 4.9 \text{ m/s Ans.}$$

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Acceleration of the midpoint of the connecting rod

We know that the radial component of the acceleration of B with respect to O or the acceleration of B ,

$$a_{BO}^r = a_B = \frac{v_{BO}^2}{OB} = \frac{(4.713)^2}{0.15} = 148.1 \text{ m/s}^2$$

and the radial component of the acceleration of A with respect to B ,

$$a_{AB}^r = \frac{v_{AB}^2}{BA} = 33.6 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. 8.4 (c) is drawn as discussed below:

1. Draw vector $o'b'$ parallel to BO , to some suitable scale, to represent the radial component of the acceleration of B with respect to O or simply acceleration of B i.e. a_{BO}^r or a_B , such that

$$\text{vector } o'b' = a_{BO}^r = a_B = 148.1 \text{ m/s}^2$$

2. The acceleration of A with respect to B has the following two components:

- (a) The radial component of the acceleration of A with respect to B i.e. a_{AB}^r , and
- (b) The tangential component of the acceleration of A with respect to B i.e. a_{AB}^t . These two components are mutually perpendicular.

Therefore from point b' , draw vector $b'x$ parallel to AB to represent $a_{AB}^r = 33.6 \text{ m/s}^2$ and from point x draw vector xa' perpendicular to vector $b'x$ whose magnitude is yet unknown.

3. Now from o' , draw vector $o'a'$ parallel to the path of motion of A (which is along AO) to represent the acceleration of A i.e. a_A . The vectors xa' and $o'a'$ intersect at a' . Join $a'b'$.

4. In order to find the acceleration of the midpoint D of the connecting rod AB , divide the vector $a'b'$ at d' in the same ratio as D divides AB . In other words

$$b'd' / b'a' = BD / BA$$

5. Join $o'd'$. The vector $o'd'$ represents the acceleration of midpoint D of the connecting rod i.e. a_D .

By measurement, we find that

$$a_D = \text{vector } o'd' = 140 \text{ m/s}^2 \text{ Ans.}$$

2. Angular velocity of the connecting rod

We know that angular velocity of the connecting rod AB ,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{4.1}{0.5} = 8.2 \text{ rad/s}^2$$

Angular acceleration of the connecting rod

From the acceleration diagram, we find that

$$a_{AB}^t = 176 \text{ m/s}^2$$

We know that angular acceleration of the connecting rod AB ,

$$\alpha_{AB} = \frac{a_{AB}^t}{BA} = 352 \text{ rad/s}^2$$

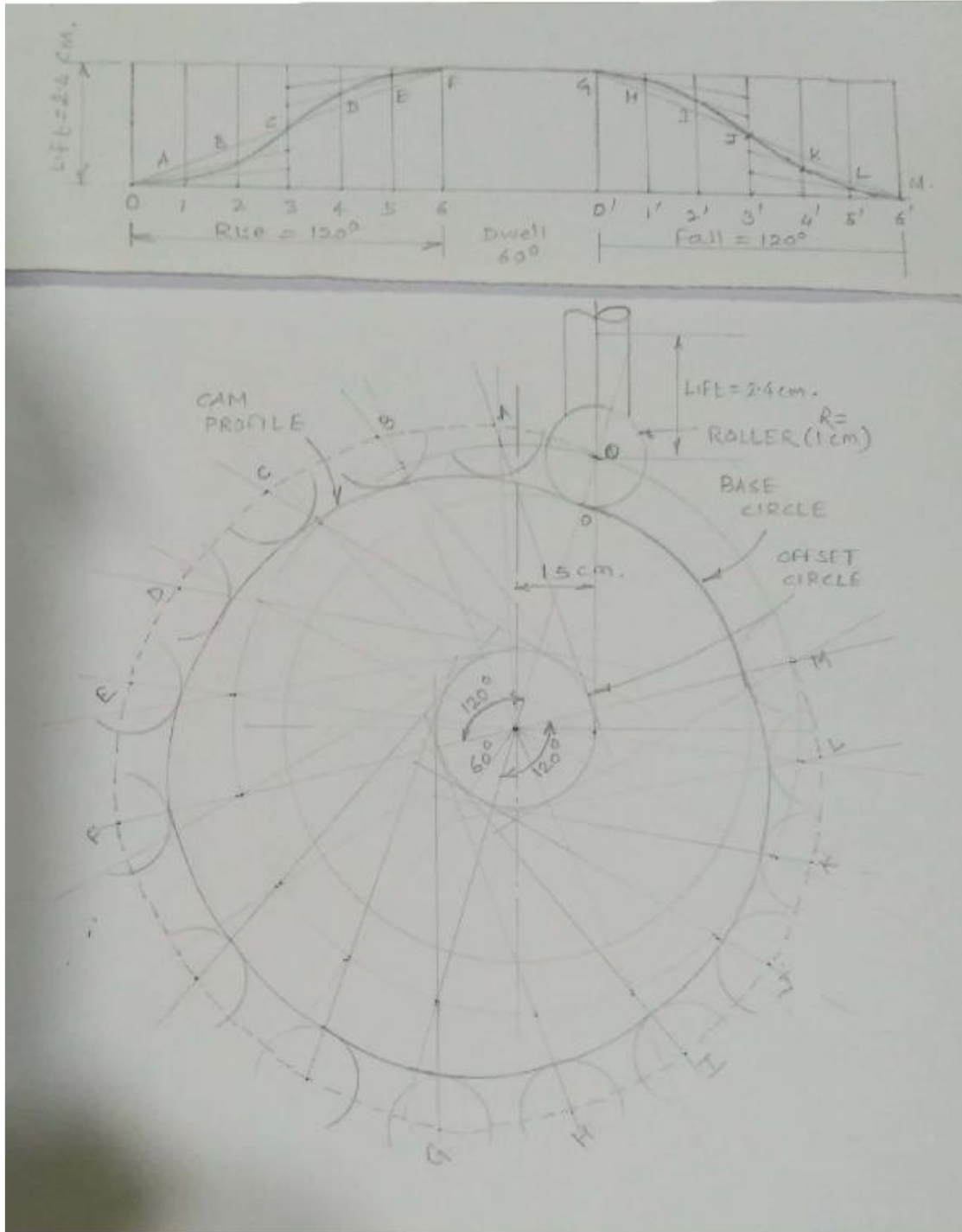
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b



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C

Solution. Given : $N_1 = 200$ r.p.m. ; $N_2 = 300$ r.p.m. ; $P = 6 \text{ kW} = 6 \times 10^3 \text{ W}$; $b = 100 \text{ mm}$;
 $t = 10 \text{ mm}$; $x = 4 \text{ m}$; $d_2 = 0.5 \text{ m}$; $\mu = 0.3$

Let $\sigma =$ Stress in the belt.

1. Stress in the belt for an open belt drive

First of all, let us find out the diameter of larger pulley (d_1). We know that

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \text{ or } d_1 = \frac{N_2 \cdot d_2}{N_1} = \frac{300 \times 0.5}{200} = 0.75 \text{ m}$$

and velocity of the belt, $v = \frac{\pi d_2 \cdot N_2}{60} = \frac{\pi \times 0.5 \times 300}{60} = 7.855 \text{ m/s}$

Now let us find the angle of contact on the smaller pulley. We know that, for an open belt drive,

$$\sin \alpha = \frac{r_1 - r_2}{x} = \frac{d_1 - d_2}{2x} = \frac{0.75 - 0.5}{2 \times 4} = 0.03125 \text{ or } \alpha = 1.8^\circ$$

$$\therefore \text{Angle of contact } \theta = 180^\circ - 2\alpha = 180 - 2 \times 1.8 = 176.4^\circ \\ = 176.4 \times \pi / 180 = 3.08 \text{ rad}$$

Let $T_1 =$ Tension in the tight side of the belt, and
 $T_2 =$ Tension in the slack side of the belt.

We know that

$$2.3 \log \left(\frac{T_1}{T_2} \right) = \mu \cdot \theta = 0.3 \times 3.08 = 0.924$$

$$\therefore \log \left(\frac{T_1}{T_2} \right) = \frac{0.924}{2.3} = 0.4017 \text{ or } \frac{T_1}{T_2} = 2.52 \quad \dots(i)$$

We also know that power transmitted (P),

$$6 \times 10^3 = (T_1 - T_2) v = (T_1 - T_2) 7.855$$

$$\therefore T_1 - T_2 = 6 \times 10^3 / 7.855 = 764 \text{ N} \quad \dots(ii)$$

From equations (i) and (ii),

$$T_1 = 1267 \text{ N, and } T_2 = 503 \text{ N}$$

We know that maximum tension in the belt (T_1),

$$1267 = \sigma \cdot b \cdot t = \sigma \times 100 \times 10 = 1000 \sigma$$

$$\therefore \sigma = 1267 / 1000 = 1.267 \text{ N/mm}^2 = 1.267 \text{ MPa} \quad \text{Ans.}$$

02

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Stress in the belt for a cross belt drive

We know that for a cross belt drive,

$$\sin \alpha = \frac{r_1 + r_2}{x} = \frac{d_1 + d_2}{2x} = \frac{0.75 + 0.5}{2 \times 4} = 0.1562 \text{ or } \alpha = 9^\circ$$

$$\begin{aligned} \therefore \text{Angle of contact, } \theta &= 180^\circ + 2\alpha = 180 + 2 \times 9 = 198^\circ \\ &= 198 \times \pi / 180 = 3.456 \text{ rad} \end{aligned}$$

We know that

$$2.3 \log \left(\frac{T_1}{T_2} \right) = \mu \cdot \theta = 0.3 \times 3.456 = 1.0368$$

$$\log \left(\frac{T_1}{T_2} \right) = \frac{1.0368}{2.3} = 0.4508 \text{ or } \frac{T_1}{T_2} = 2.82 \quad \dots(iii)$$

From equations (ii) and (iii),

$$T_1 = 1184 \text{ N and } T_2 = 420 \text{ N}$$

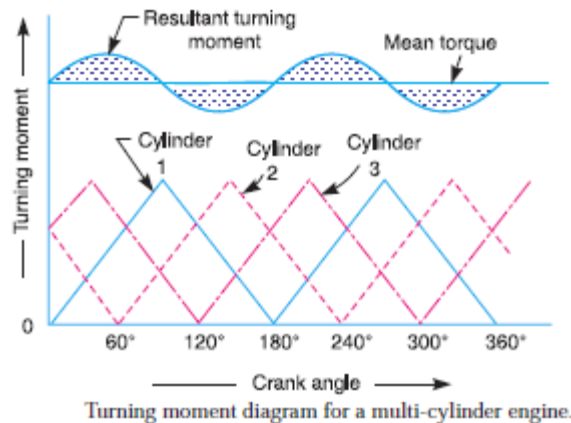
We know that maximum tension in the belt (T_1),

$$1184 = \sigma \cdot b \cdot t = \sigma \times 100 \times 10 = 1000 \sigma$$

$$\therefore \sigma = 1184 / 1000 = 1.184 \text{ N/mm}^2 = 1.184 \text{ MPa Ans.}$$

i) Working of Flywheel with the help of Turning moment diagram:

A flywheel used in machines serves as a reservoir, which stores energy during the period when the supply of energy is more than the requirement, and releases it during the period when the requirement of energy is more than the supply.

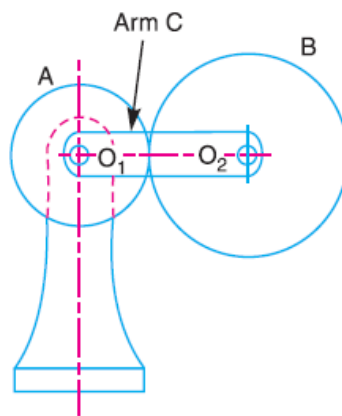


The fluctuation of energy may be determined by the turning moment diagram for one complete cycle of operation. Consider the turning moment diagram for a single cylinder double acting steam engine as shown in Fig. We see that the mean resisting torque line AF cuts the turning moment diagram at points B, C, D and E . When the crank moves from a to p , the work done by the engine is equal to the area aBp , whereas the energy required is represented by the area $aABp$. In other words, the engine has done less work (equal to the area aAB) than the requirement. This amount of energy is taken from the flywheel and hence the speed of the flywheel decreases. Now the crank moves from p to q , the work done by the engine is equal to the area $pBbCq$, whereas the requirement of energy

is represented by the area $pBCq$. Therefore, the engine has done more work than the requirement. This excess work (equal to the area BbC) is stored in the flywheel and hence the speed of the flywheel increases while the crank moves from p to q . Similarly, when the crank moves from q to r , more work is taken from the engine than is developed. This loss of work is represented by the area CcD . To supply this loss, the flywheel gives up some of its energy and thus the speed decreases while the crank moves from q to r . As the crank moves from r to s , excess energy is again developed given by the area DdE and the speed again increases. As the piston moves from s to e , again there is a loss of work and the speed decreases. The variations of energy above and below the mean resisting torque line are called **fluctuations of energy**. The areas BbC , CcD , DdE , etc. represent fluctuations of energy.

ii) Epicyclic gear train:

A simple epicyclic gear train is shown in Fig. where a gear A and the arm C have a common axis at O_1 about which they can rotate. The gear B meshes with gear A and has its axis on the arm at O_2 , about which the gear B can rotate. If the arm is fixed, the gear train is simple and gear A can drive gear B or *vice-versa*, but if gear A is fixed and the arm is rotated about the axis of gear A (i.e. O_1), then the gear B is forced to rotate upon and around gear A . Such a motion is called **epicyclic** and the gear trains arranged in such a manner that one or more of their members move upon and around another member are known as **epicyclic gear trains**



Epicyclic gear train.

Data: $P = 25 \text{ kW}$, $N = 900 \text{ rpm}$, $P = 85 \text{ kN/m}^2$, $d_1 = 360 \text{ mm}$, $r_1 = 180 \text{ mm}$, $\mu = 0.25$

$$d_2 = ?, f = ?$$

$$\omega = 2\pi N/60 = 2 \times 3.142 \times 900/60 = 94.24 \text{ rad/sec}$$

$$P = T \times \omega$$

$$25 \times 1000 = T \times 94.24 \quad T = 265.28 \text{ N-m}$$

$$\text{Intensity of pressure } p \times r_2 = C \quad \text{i.e.} \quad 85 \times 10^3 \times r_2 = C$$

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	$C = 85 \times 10^3 \times r_2 \text{ N/m}$	
	$W = 2 \times \pi \times C (r_1 - r_2)$	
	$W = 2 \times 3.142 \times 85 \times 10^3 \times r_2 \times (0.180 - r_2) \text{ N}$	01
	$W = 533.8 \times 10^3 \times r_2 (0.180 - r_2)$	
	$T = n \times \mu \times W \times (r_1 + r_2)/2$	
	$T = 2 \times 0.25 \times 533.8 \times 10^3 \times r_2 \times (0.180 - r_2) \times (r_2 + 0.180)/2$	
	$T = 133.45 \times 10^3 \times r_2 \times (0.0324 - r_2^2)$	02
	$265.28 = 133.45 \times 10^3 \times r_2 \times (0.0324 - r_2^2) \quad r_2 = 0.146 \text{ m} = 146 \text{ mm} \quad \text{so, } d_2 = 292 \text{ mm}$	
	$r_2^3 - 0.0324 \times r_2 + 0.002 = 0$	
	$r_2 = 0.1 \text{ m} = 100 \text{ mm}$	02
	Axial Thrust $W = W = 2 \times \pi \times C (r_1 - r_2)$	
	$= 2 \times 3.142 \times 85 \times 10^3 \times r_2 (180 - 100)/1000$	02
	$= 4273.12 \text{ N}$	
c	Data: $m = 300 \text{ Kg}$, $k = 30 \text{ cm} = 0.3 \text{ m}$, $N = 300 \text{ rpm}$, $\mu = 0.25$,	
	$d = 1 \text{ m}$, $r = 0.5 \text{ m}$, $P = 100 \text{ N}$,	
	$w = \omega = 2 \pi N/60 = 2 \times 3.142 \times 300/60 = 31.42 \text{ rad/sec}$,	02
	$\theta = 210 \times \pi / 180 = 3.66 \text{ radians}$	
	$T_1 / T_2 = e^{\mu \theta} = 2.1$ ----- (i)	
	Taking moments about fulcrum " O "	
	$T_1 \times 10 = 100 \times 40$ ----- (ii)	02
	From equation (i) and (ii) ,	
	$T_1 = 400 \text{ N}$ $T_2 = 190.5 \text{ N}$	
	Torque $T = (T_1 - T_2) \times r$	02
	$= (400 - 190.5) \times 0.5 = 104.75 \text{ N-m}$ ----- Ans	
	Let N = no. of turns required /revolutions	



$$\text{K. E. of the drum} = \frac{1}{2} \times I \times \omega^2$$

$$= \frac{1}{2} m \times k^2 \times \omega^2 = \frac{1}{2} \times 300 \times (0.3)^2 \times (31.42)^2 = 13327 \text{ N-m}$$

This energy is used to overcome the work done due to torque

$$\text{Therefore } 13327 = T \times 2 \pi N \quad \text{No. of turns } N = 20.26 \quad \text{----- Ans}$$

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