SUMMER– 19 EXAMINATION

Subject Name: Engineering Mathematics **Model Answer** **Subject Code: 17216**

Important Instructions to Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.		**Solve any TEN of the following:**	**20**
	a)	Find x and y if $x(1-i)+y(2+i)+6=0$	**02**
	Ans	$x(1-i)+y(2+i)+6=0$	
		$\therefore x-xi+2y+iy+6=0$	
		$\therefore (x+2y+6)+i(-x+y)=0$	
		$\therefore x+2y+6=0$	1
		$-x+y=0$	
		$\Rightarrow y=x$	
		$\therefore x+2x+6=0$	
		$\therefore 3x=-6$	
		$\therefore x=-2$	½
		$y=-2$	½
		--	
	b)	If $Z=1+i\sqrt{3}$ show that $Z^2+4=2Z$	**02**
	Ans	$Z=1+i\sqrt{3}$	
		Z^2+4	
		$=(1+i\sqrt{3})^2+4$	

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1.	b)	$=1+2\sqrt{3}i-3+4$	1
		$=2+2\sqrt{3}i$	
		$=2\left(1+\sqrt{3}i\right)$	
		$=2Z$	1
	c)	If $f\left(x\right)=x^3-3x^2+5$, find $f\left(0\right)+f\left(2\right)$	**02**
	Ans	$f\left(x\right)=x^3-3x^2+5$	
		$\therefore f\left(0\right)=5$	½
		$f\left(2\right)=\left(2\right)^3-3\left(2\right)^2+5=1$	½
		$\therefore f\left(0\right)+f\left(2\right)=5+1$	
		$=6$	1
	d)	If $f\left(x\right)=\log\left(\tan x\right)$, find $f\left(\pi/4\right)$	**02**
	Ans	$f\left(x\right)=\log\left(\tan x\right)$	
		$\therefore f\left(\pi/4\right)=\log\left(\tan \pi/4\right)$	1
		$\therefore f\left(\pi/4\right)=\log\left(1\right)$	½
		$\therefore f\left(\pi/4\right)=0$	½
	e)	Evaluate: $\lim\limits_{x\to1}\left(\frac{1}{x-1}-\frac{1}{x^2-x}\right)$	**02**
	Ans	$\lim\limits_{x\to1}\left(\frac{1}{x-1}-\frac{1}{x^2-x}\right)$	
		$=\lim\limits_{x\to1}\left(\frac{1}{x-1}-\frac{1}{x\left(x-1\right)}\right)$	
		$=\lim\limits_{x\to1}\left(\frac{x-1}{x\left(x-1\right)}\right)$	1
		$=\lim\limits_{x\to1}\frac{1}{x}=1$	1

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Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	e)	**OR**	
		$\lim_{x\to1}\left(\frac{1}{x-1}-\frac{1}{x^2-x}\right)$	
		$=\lim_{x\to1}\left(\frac{1}{x-1}-\frac{1}{x(x-1)}\right)$	
		$=\lim_{x\to1}\frac{1}{x-1}\left(1-\frac{1}{x}\right)$	1
		$=\lim_{x\to1}\frac{1}{x-1}\left(\frac{x-1}{x}\right)$	
		$=\lim_{x\to1}\frac{1}{x}$	
		$=1$	1
	f)	Evaluate: $\lim_{x\to0}\frac{\sin 2x}{x}$	**02**
	Ans	$\lim_{x\to0}\frac{\sin 2x}{x}$	
		$=\lim_{x\to0}\frac{\sin 2x}{2x}\times2$	1
		$=1\times2$	
		$=2$	1
	g)	Evaluate: $\lim_{x\to0}\frac{e^{3x}-1}{2x}$	**02**
	Ans	$\lim_{x\to0}\frac{e^{3x}-1}{2x}$	
		$=\lim_{x\to0}\frac{e^{3x}-1}{3x}\times\frac{3}{2}$	1
		$=\log e\times\frac{3}{2}$	
		$=\frac{3}{2}$	1

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Q. No.	Sub Q. N.	Answers	Marking Scheme
1.	h)	If $y=a^{2x}\cos(3x)$, find $\frac{dy}{dx}$	**02**
	Ans	$y=a^{2x}\cos(3x)$	
		$\frac{dy}{dx}=a^{2x}(-\sin 3x)3+\cos 3x\ a^{2x}.\log a.2$	1+1
		$\frac{dy}{dx}=a^{2x}(-3\sin 3x+2\log a.\cos 3x)$	
	i)	If $y=\log\left[\tan(4-3x)\right]$, find $\frac{dy}{dx}$	**02**
	Ans	$y=\log\left[\tan(4-3x)\right]$	
		$\frac{dy}{dx}=\frac{1}{\tan(4-3x)}\sec^2(4-3x)(-3)$	2
		$\frac{dy}{dx}=\frac{-3\sec^2(4-3x)}{\tan(4-3x)}$	
	j)	If $\tan^{-1}(x^2+y^2)=\mathrm{a}^2$, find $\frac{dy}{dx}$	**02**
	Ans	$\tan^{-1}(x^2+y^2)=\mathrm{a}^2$	
		$\therefore x^2+y^2=\tan(\mathrm{a}^2)$	½
		$\therefore 2x+2y\frac{dy}{dx}=0$	1
		$\therefore \frac{dy}{dx}=-\frac{x}{y}$	½
	k)	Show that there exist a root of the equation $x^3-4x+1=0$ in the interval $(1,2)$.	**02**
	Ans	Let $f(x)=x^3-4x+1$	
		$f(1)=-2<0$	1
		$f(2)=1>0$	1
		$\therefore$ root is in $(1,2)$	
	l)	Find by Jacobi's method , the first iteration only, for the following equation $5x-y=9$, $x-5y+z=-4$, $y-5z=6$	**02**

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	l)		
	Ans	$x=\frac{9+y}{5}$	
		$y=\frac{-4-x-z}{-5}$	1
		$z=\frac{6-y}{-5}$	
		Initial approximations : $x_0=y_0=z_0=0$	
		$x=\frac{9}{5}=1.8$	
		$y=\frac{4}{5}=0.8$	1
		$z=\frac{6}{-5}=-1.2$	
		--	
2.		**Solve any FOUR of the following:**	**16**
	a)	Express $\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$ in Polar form.	**04**
	Ans	Let $z=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$	
		$x=-\frac{1}{2}, y=\frac{\sqrt{3}}{2}$	
		$r=\sqrt{x^2+y^2}=\sqrt{\left(-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$	
		$=\sqrt{\frac{1}{4}+\frac{3}{4}}$	
		$=1$	1
		$\because \theta=\pi-\tan^{-1}\left(\frac{y}{x}\right)$	
		$\therefore \theta=\pi-\tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right)$	

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2.	a)	$\therefore \theta = \pi - \tan^{-1}\left(\sqrt{3}\right) = 180 - 60 = 120^0$ or $\dfrac{2\pi}{3}$	2
		In polar form,	
		$z = r\left(\cos\theta + i\sin\theta\right)$	
		$= 1\left(\cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3}\right)$	1
		--	
	b)	Simplify using De-Moiver's theorem, $\dfrac{\left(\cos 3\theta + i\sin 3\theta\right)^4 \left(\cos 4\theta - i\sin 4\theta\right)^5}{\left(\cos 4\theta + i\sin 4\theta\right)^3 \left(\cos 5\theta + i\sin 5\theta\right)^4}$	**04**
	Ans	$\dfrac{\left(\cos 3\theta + i\sin 3\theta\right)^4 \left(\cos 4\theta - i\sin 4\theta\right)^5}{\left(\cos 4\theta + i\sin 4\theta\right)^3 \left(\cos 5\theta + i\sin 5\theta\right)^4}$	
		$= \dfrac{\left(\cos\theta + i\sin\theta\right)^{12} \left(\cos\theta + i\sin\theta\right)^{-20}}{\left(\cos\theta + i\sin\theta\right)^{12} \left(\cos\theta + i\sin\theta\right)^{20}}$	2
		$= \left(\cos\theta + i\sin\theta\right)^{12-12-20-20}$	
		$= \left(\cos\theta + i\sin\theta\right)^{-40}$	
		$= \cos 40\theta - i\sin 40\theta$	2
		--	
	c)	Using Euler's formula,prove that $\cos 2\theta = \cos^2\theta - \sin^2\theta$	**04**
	Ans	$\cos 2\theta = \left(\dfrac{e^{2i\theta} + e^{-2i\theta}}{2}\right) ------------ (1)$	
		$\cos^2\theta - \sin^2\theta = \left(\dfrac{e^{i\theta} + e^{-i\theta}}{2}\right)^2 - \left(\dfrac{e^{i\theta} - e^{-i\theta}}{2i}\right)^2$	1
		$= \dfrac{\left(e^{i\theta}\right)^2 + \left(e^{-i\theta}\right)^2 + 2e^{i\theta}e^{-i\theta}}{4} - \dfrac{\left(e^{i\theta}\right)^2 + \left(e^{-i\theta}\right)^2 - 2e^{i\theta}e^{-i\theta}}{4i^2}$	1
		$= \dfrac{\left(e^{i\theta}\right)^2 + \left(e^{-i\theta}\right)^2 + 2e^{i\theta}e^{-i\theta}}{4} + \dfrac{\left(e^{i\theta}\right)^2 + \left(e^{-i\theta}\right)^2 - 2e^{i\theta}e^{-i\theta}}{4}$	½
		$= \dfrac{e^{2i\theta} + e^{-2i\theta} + 2e^{i\theta}e^{-i\theta} + e^{2i\theta} + e^{-2i\theta} - 2e^{i\theta}e^{-i\theta}}{4}$	½
		$= \dfrac{2e^{2i\theta} + 2e^{-2i\theta}}{4}$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	c)	$=\dfrac{e^{2i\theta}+e^{-2i\theta}}{2}$ ------------ (2)	½
		From (1) and (2)	
		$\cos 2\theta=\cos^2\theta-\sin^2\theta$	½
	d)	Prove that : $(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n=2^{n+1}.\cos^n\left(\theta/2\right).\cos\left(\dfrac{n\theta}{2}\right)$	**04**
	Ans	Consider $(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n$	
		$=\left(2\cos^2\dfrac{\theta}{2}+i2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}\right)^n+\left(2\cos^2\dfrac{\theta}{2}-i2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}\right)^n$	1
		$=\left[2\cos\dfrac{\theta}{2}\left(\cos\dfrac{\theta}{2}+i\sin\dfrac{\theta}{2}\right)\right]^n+\left[2\cos\dfrac{\theta}{2}\left(\cos\dfrac{\theta}{2}-i\sin\dfrac{\theta}{2}\right)\right]^n$	
		$=2^n\cos^n\dfrac{\theta}{2}\left(\cos\dfrac{\theta}{2}+i\sin\dfrac{\theta}{2}\right)^n+2^n\cos^n\dfrac{\theta}{2}\left(\cos\dfrac{\theta}{2}-i\sin\dfrac{\theta}{2}\right)^n$	1
		$=2^n\cos^n\dfrac{\theta}{2}\left(\cos\dfrac{n\theta}{2}+i\sin\dfrac{n\theta}{2}\right)+2^n\cos^n\dfrac{\theta}{2}\left(\cos\dfrac{n\theta}{2}-i\sin\dfrac{n\theta}{2}\right)$ --------Using De Moiver's theorem	1
		$=2^n\cos^n\dfrac{\theta}{2}\left(\cos\dfrac{n\theta}{2}+i\sin\dfrac{n\theta}{2}+\cos\dfrac{n\theta}{2}-i\sin\dfrac{n\theta}{2}\right)$	
		$2^n\cos^n\dfrac{\theta}{2}\left(2\cos\dfrac{n\theta}{2}\right)$	
		$=2^{n+1}.\cos^n\left(\theta/2\right).\cos\left(\dfrac{n\theta}{2}\right)$	1
	e)	If $f(x)=\dfrac{2x+5}{3x-4}$ and $t=\dfrac{5+4x}{3x-2}$ show that $f(t)=x$	**04**
	Ans	$f(x)=\dfrac{2x+5}{3x-4}$	
		$\therefore f(t)=\dfrac{2t+5}{3t-4}$	1
		$\therefore f(t)=\dfrac{2\left(\dfrac{5+4x}{3x-2}\right)+5}{3\left(\dfrac{5+4x}{3x-2}\right)-4}$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	e)	$\therefore f(t)=\dfrac{\dfrac{2(5+4x)+5(3x-2)}{3x-2}}{\dfrac{3(5+4x)-4(3x-2)}{3x-2}}$	
		$\therefore f(t)=\dfrac{10+8x+15x-10}{15+12x-12x+8}$	1
		$\therefore f(t)=\dfrac{23x}{23}$	
		$\therefore f(t)=x$	1
	f)	If $f(x)=\log\left(\dfrac{1+x}{1-x}\right)$ then prove that $f\left(\dfrac{2x}{1+x^2}\right)=2.f(x)$	**04**
	Ans	Consider $f(x)=\log\left(\dfrac{1+x}{1-x}\right)$	
		$\therefore f\left(\dfrac{2x}{1+x^2}\right)=\log\left(\dfrac{1+\left(\dfrac{2x}{1+x^2}\right)}{1-\left(\dfrac{2x}{1+x^2}\right)}\right)$	1
		$\therefore f\left(\dfrac{2x}{1+x^2}\right)=\log\left(\dfrac{\dfrac{1+x^2+2x}{1+x^2}}{\dfrac{1+x^2-2x}{1+x^2}}\right)$	½
		$\therefore f\left(\dfrac{2x}{1+x^2}\right)=\log\left(\dfrac{(1+x)^2}{(1-x)^2}\right)$	1
		$\therefore f\left(\dfrac{2x}{1+x^2}\right)=\log\left(\dfrac{1+x}{1-x}\right)^2$	½
		$\therefore f\left(\dfrac{2x}{1+x^2}\right)=2.\log\left(\dfrac{1+x}{1-x}\right)$	½
		$\therefore f\left(\dfrac{2x}{1+x^2}\right)=2.f(x)$	½
		OR	
		Consider $2.f(x)=2.\log\left(\dfrac{1+x}{1-x}\right)$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	f)	$\therefore\ 2.f(x)=\log\left(\frac{1+x}{1-x}\right)^2$	1
		$\therefore\ 2.f(x)=\log\left(\frac{(1+x)^2}{(1-x)^2}\right)$	
		$\therefore\ 2.f(x)=\log\left(\frac{1+x^2+2x}{1+x^2-2x}\right)$ -----------------------(1)	1
		Consider $f\left(\frac{2x}{1+x^2}\right)=\log\left(\frac{1+\left(\frac{2x}{1+x^2}\right)}{1-\left(\frac{2x}{1+x^2}\right)}\right)$	
		$\therefore\ f\left(\frac{2x}{1+x^2}\right)=\log\left(\frac{\frac{1+x^2+2x}{1+x^2}}{\frac{1+x^2-2x}{1+x^2}}\right)$	1
		$\therefore\ f\left(\frac{2x}{1+x^2}\right)=\log\left(\frac{1+x^2+2x}{1+x^2-2x}\right)$ -----------------------(2)	
		From (1) and (2)	
		$f\left(\frac{2x}{1+x^2}\right)=2f(x)$	1

3.		**Solve any FOUR of the following:**	**16**
	a)	If $f(x)=x^2-4x+11,$ solve the equation $f(x)=f(3x-1)$	**04**
	Ans	$f(3x-1)=(3x-1)^2-4(3x-1)+11$	
		$=9x^2-6x+1-12x+4+11=9x^2-18x+16$	2
		Given $f(x)=f(3x-1)$	
		$\therefore x^2-4x+11=9x^2-18x+16$	
		$\therefore 8x^2-14x+5=0$	1
		$\therefore x=\frac{5}{4}\ ,\ x=\frac{1}{2}$	1

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3.	b)	If $f(t)=50\sin(100\pi t+0.04)$, then show that $f\left(\frac{2}{100}+t\right)=f(t)$	**04**
	Ans	$f\left(\frac{2}{100}+t\right)=50\sin\left(100\pi\left(\frac{2}{100}+t\right)+0.04\right)$	1
		$=50\sin(2\pi+100\pi t+0.04)$	1
		$=50\sin(100\pi t+0.04)$	1
		$=f(t)$	1
		--	
	c)	Evaluate: $\lim_{x\to 4}\frac{x^4-64x}{\sqrt{x^2+9}-5}$	**04**
	Ans	$\lim_{x\to 4}\frac{x^4-64x}{\sqrt{x^2+9}-5}$	
		$=\lim_{x\to 4}\frac{x^4-64x}{\sqrt{x^2+9}-5}\times\frac{\sqrt{x^2+9}+5}{\sqrt{x^2+9}+5}$	1
		$=\lim_{x\to 4}\frac{(x^4-64x)\left(\sqrt{x^2+9}+5\right)}{x^2+9-25}$	
		$=\lim_{x\to 4}\frac{x(x^3-64)\left(\sqrt{x^2+9}+5\right)}{x^2-16}$	
		$=\lim_{x\to 4}\frac{x(x^3-4^3)\left(\sqrt{x^2+9}+5\right)}{x^2-4^2}$	
		$=\lim_{x\to 4}\frac{x(x-4)(x^2+4x+16)\left(\sqrt{x^2+9}+5\right)}{(x-4)(x+4)}$	
		$=\lim_{x\to 4}\frac{x(x^2+4x+16)\left(\sqrt{x^2+9}+5\right)}{(x+4)}$	1
		$=\frac{4(4^2+4(4)+16)\left(\sqrt{4^2+9}+5\right)}{(4+4)}$	1
		$=240$	1
		--	
	d)	Evaluate: $\lim_{x\to\infty}\left(\sqrt{x^2+x}-x\right)$	**04**

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3.	d) Ans	$\lim_{x\to\infty}\left(\sqrt{x^2+x}-x\right)$	
		$=\lim_{x\to\infty}\left(\sqrt{x^2+x}-x\right)\times\frac{\left(\sqrt{x^2+x}+x\right)}{\left(\sqrt{x^2+x}+x\right)}$	1
		$=\lim_{x\to\infty}\frac{x^2+x-x^2}{\sqrt{x^2+x}+x}$	
		$=\lim_{x\to\infty}\frac{x}{\sqrt{x^2+x}+x}$	
		$=\lim_{x\to\infty}\frac{x}{x\left(\sqrt{1+\frac{1}{x}}+1\right)}$	
		$=\lim_{x\to\infty}\frac{1}{\left(\sqrt{1+\frac{1}{x}}+1\right)}$	1
		$=\frac{1}{\left(\sqrt{1+0}+1\right)}$	1
		$=\frac{1}{2}$	1
		--	
	e) Ans	Evaluate: $\lim_{x\to 0}\frac{5^x+5^{-x}-2}{x^2}$	**04**
		$\lim_{x\to 0}\frac{5^x+5^{-x}-2}{x^2}$	
		$=\lim_{x\to 0}\frac{5^x+\frac{1}{5^x}-2}{x^2}$	½
		$=\lim_{x\to 0}\frac{\left(5^x\right)^2+1-2\left(5^x\right)}{x^2\left(5^x\right)}$	½
		$=\lim_{x\to 0}\frac{\left(5^x-1\right)^2}{x^2}\times\lim_{x\to 0}\left(\frac{1}{5^x}\right)$	1

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3.	e)	$=\left(\lim_{x\to0}\frac{5^x-1}{x}\right)^2\times\left(\frac{1}{5^0}\right)$	1
		$=(\log 5)^2$	1
		--	
	f)	Evaluate: $\lim_{x\to0}\left(\frac{2x+1}{1-2x}\right)^{1/x}$	**04**
	Ans	$\lim_{x\to0}\left(\frac{2x+1}{1-2x}\right)^{1/x}$	
		$=\lim_{x\to0}\frac{(2x+1)^{1/x}}{(1-2x)^{1/x}}$	
		$=\frac{\left[\lim_{x\to0}(1+2x)^{1/2x}\right]^2}{\left[\lim_{x\to0}(1-2x)^{1/-2x}\right]^{-2}}$	2
		$=\frac{e^2}{e^{-2}}$	1
		$=e^4$	1
		--	
4.		**Solve any <u>FOUR</u> of the following:**	**16**
	a)	Using first principle find derivative of $f(x)=\log x$	**04**
	Ans	$f(x)=\log x$	
		Using first principle	
		$\frac{dy}{dx}=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$	
		$\therefore\frac{dy}{dx}=\lim_{h\to0}\frac{\log(x+h)-\log x}{h}$	**1**
		$=\lim_{h\to0}\frac{\log\left(\frac{x+h}{x}\right)}{h}$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	a)	$\therefore \frac{dy}{dx} = \lim_{h\to 0} \log\left(1+\frac{h}{x}\right)^{\frac{x}{h}\times\frac{1}{x}}$	1
		$= \log e^{\frac{1}{x}}$	1
		$= \frac{1}{x}$	½
		--	
	b)	If u and v are differentiable functions of x and $y=\frac{u}{v}$, then prove that $\frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$	**04**
	Ans	Let $\delta u, \delta v, \delta y$ are small increments in u, v, y respectively corresponding to increment δx in x.	
		$\therefore y+\delta y=\frac{u+\delta u}{v+\delta v}$	1
		$\delta y=\frac{u+\delta u}{v+\delta v}-y$	
		$\delta y=\frac{u+\delta u}{v+\delta v}-\frac{u}{v}$	
		$\delta y=\frac{uv+v\delta u-u(v+\delta v)}{v(v+\delta v)}$	
		$\delta y=\frac{v\delta u-u\delta v}{v^2+v\delta v}$	
		$\frac{\delta y}{\delta x}=\frac{\frac{v\delta u-u\delta v}{\delta x}}{v^2+v\delta v}$	1
		$\lim_{\delta x\to 0}\frac{\delta y}{\delta x}=\lim_{\delta x\to 0}\frac{\frac{v\delta u-u\delta v}{\delta x}}{v^2+v\delta v}$	
		$\lim_{\delta x\to 0}\frac{\delta y}{\delta x}=\frac{v\lim_{\delta x\to 0}\frac{\delta u}{\delta x}-u\lim_{\delta x\to 0}\frac{\delta v}{\delta x}}{v^2}$	1

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4.	b)	$\therefore \frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$ $\quad (\because \text{as } \delta x \to 0, \delta v \to 0)$	1
		--	
	c)	If $e^y = y^x$ prove that $\frac{dy}{dx}=\frac{(\log y)^2}{\log y-1}$	**04**
	Ans	$e^y = y^x$ taking log on both sides,	
		$\therefore \log e^y = \log y^x$	½
		$\therefore y\log e = x\log y$ $\therefore y = x\log y$	
		$\therefore x=\frac{y}{\log y}$	½
		diff.w.r.t.y	
		$\therefore \frac{dx}{dy}=\frac{\log y(1)-y\frac{1}{y}}{(\log y)^2}$	2
		$\therefore \frac{dx}{dy}=\frac{\log y-1}{(\log y)^2}$	
		$\therefore \frac{dy}{dx}=\frac{(\log y)^2}{\log y-1}$	1
		OR	
		$e^y = y^x$ taking log on both sides,	
		$\therefore \log e^y = \log y^x$	½
		$\therefore y\log e = x\log y$	
		$\therefore y = x\log y$ ------(i)	½
		diff.w.r.t.x	
		$\therefore \frac{dy}{dx}=x\frac{1}{y}\frac{dy}{dx}+\log y(1)$	1

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Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	c)	$\therefore \left(1-\frac{x}{y}\right)\frac{dy}{dx}=\log y$	
		$\therefore \frac{dy}{dx}=\frac{\log y}{\left(1-\frac{x}{y}\right)}$	1
		$\therefore \frac{dy}{dx}=\frac{\log y}{\left(1-\frac{1}{\log y}\right)}$ -----$\left(\because eq^n.(i)\right)$	
		$\therefore \frac{dy}{dx}=\frac{(\log y)^2}{\log y-1}$	1
		--	
	d)	If $x^3+y^3=3axy$, find $\frac{dy}{dx}$ at the point $\left(\frac{3a}{2},\frac{3a}{2}\right)$	**04**
	Ans	$x^3+y^3=3axy$	
		$\therefore 3x^2+3y^2\frac{dy}{dx}=3a\left(x\frac{dy}{dx}+y\right)$	2
		$\therefore 3x^2+3y^2\frac{dy}{dx}=3ax\frac{dy}{dx}+3ay$	
		$\therefore 3y^2\frac{dy}{dx}-3ax\frac{dy}{dx}=3ay-3x^2$	
		$\therefore \left(3y^2-3ax\right)\frac{dy}{dx}=3ay-3x^2$	
		$\therefore \frac{dy}{dx}=\frac{3ay-3x^2}{3y^2-3ax}=\frac{ay-x^2}{y^2-ax}$	1
		at $\left(\frac{3a}{2},\frac{3a}{2}\right)$	
		$\therefore \frac{dy}{dx}=\frac{a\cdot\frac{3a}{2}-\left(\frac{3a}{2}\right)^2}{\left(\frac{3a}{2}\right)^2-a\cdot\frac{3a}{2}}$	
		$=-1$	1
		--	

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Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	e)	If $x = 3\sin\theta - 2\sin^3\theta,\ y = 3\cos\theta - 2\cos^3\theta$ find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$	**04**
	Ans	$x = 3\sin\theta - 2\sin^3\theta$	
		$\therefore \frac{dx}{d\theta} = 3\cos\theta - 6\sin^2\theta\cdot\cos\theta$	
		$= 3\cos\theta\left(1 - 2\sin^2\theta\right)$	1
		$y = 3\cos\theta - 2\cos^3\theta$	
		$\frac{dy}{d\theta} = -3\sin\theta + 6\cos^2\theta\cdot\sin\theta$	
		$= -3\sin\theta\left(1 - 2\cos^2\theta\right)$	1
		$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-3\sin\theta\left(1-2\cos^2\theta\right)}{3\cos\theta\left(1-2\sin^2\theta\right)}$	
		$= \frac{-3\sin\theta\left(-\cos 2\theta\right)}{3\cos\theta\left(\cos 2\theta\right)}$	
		$= \tan\theta$	1
		$\therefore$ at $\theta = \frac{\pi}{4}$	
		$\therefore \frac{dy}{dx} = \tan\frac{\pi}{4}$	
		$= 1$	1
		Note: Many times in such problems, students first find separately the values of $\frac{dx}{d\theta}$ *and* $\frac{dy}{d\theta}$ at given θ. And then they find the required value of $\frac{dy}{dx}$. Such method is applicable in the problems where $\frac{dx}{d\theta}$ is non-zero. In this problem both $\frac{dx}{d\theta}$ *and* $\frac{dy}{d\theta}$ are **zero** at given value of θ.	
		Consequently, the simplified form of $\frac{dy}{dx}$ is **MUST** in this case. Without simplification if it is done (which is in fact applicable in other problems), then we get	
		$\frac{dy}{dx} = \frac{-3\sin\theta + 6\cos^2\theta\cdot\sin\theta}{3\cos\theta - 6\sin^2\theta\cdot\cos\theta}$	
		$\therefore$ at $\theta = \frac{\pi}{4}$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	e)	$\therefore \frac{dy}{dx}=\frac{-3\sin\frac{\pi}{4}+6\cos^2\frac{\pi}{4}\cdot\sin\frac{\pi}{4}}{3\cos\frac{\pi}{4}-6\sin^2\frac{\pi}{4}\cdot\cos\frac{\pi}{4}}=\frac{0}{0}$	1
		--	
	f)	Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ w.r.t. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$	**04**
	Ans	Let $u=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$	
		Put $x=\tan\theta \quad \therefore \theta=\tan^{-1}x$	½
		$\therefore u=\tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$	
		$=\tan^{-1}(\tan 2\theta)$	
		$=2\theta$	
		$=2\tan^{-1}x$	
		$\therefore \frac{du}{dx}=2\frac{1}{1+x^2}$	1
		Let $v=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$	
		Put $x=\tan\theta \quad \therefore \theta=\tan^{-1}x$	½
		$\therefore v=\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$	
		$=\sin^{-1}(\sin 2\theta)$	
		$=2\theta$	
		$=2\tan^{-1}x$	
		$\therefore \frac{dv}{dx}=2\frac{1}{1+x^2}$	1
		$\therefore \frac{du}{dv}=\frac{\frac{du}{dx}}{\frac{dv}{dx}}$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	f)	$\therefore \frac{du}{dv} = \frac{\frac{2}{1+x^2}}{\frac{2}{1+x^2}}$	
		$= 1$	1
5.		**Solve any <u>FOUR</u> of the following:**	**16**
	a)	Evaluate $\lim_{x\to 0} \frac{e^{\tan 2x}-1}{\sin 3x}$	**04**
	Ans	$\lim_{x\to 0} \frac{e^{\tan 2x}-1}{\sin 3x}$	
		$= \lim_{x\to 0} \frac{\frac{e^{\tan 2x}-1}{\tan 2x} \cdot \frac{\tan 2x}{2x} \cdot 2x}{\frac{\sin 3x}{3x} \cdot 3x} = \frac{\lim_{x\to 0}\left(\frac{e^{\tan 2x}-1}{\tan 2x}\right) \cdot \lim_{x\to 0}\left(\frac{\tan 2x}{2x}\right) \cdot 2}{\lim_{x\to 0}\left(\frac{\sin 3x}{3x}\right) \cdot 3}$	2
		$= \frac{(\log e)(1)(2)}{(1)(3)}$	1
		$= \frac{2}{3}$	1
	b)	Evaluate $\lim_{x\to 0} \frac{\log(5+x)-\log(5-x)}{x}$	**04**
	Ans	$\lim_{x\to 0} \frac{\log(5+x)-\log(5-x)}{x}$	
		$= \lim_{x\to 0} \frac{1}{x} \log\left(\frac{5+x}{5-x}\right)$	½
		$= \lim_{x\to 0} \log\left(\frac{1+\frac{x}{5}}{1-\frac{x}{5}}\right)^{\frac{1}{x}}$	½
		$= \lim_{x\to 0} \log \frac{\left(1+\frac{x}{5}\right)^{\frac{1}{x}}}{\left(1-\frac{x}{5}\right)^{\frac{1}{x}}}$	

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Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	b)	$= \lim_{x\to 0} \log \dfrac{\left(1+\frac{x}{5}\right)^{\frac{5}{x}\times\frac{1}{5}}}{\left(1-\frac{x}{5}\right)^{\frac{-5}{x}\times\frac{1}{-5}}} = \log\left\{\dfrac{\left(\lim_{x\to 0}\left(1+\frac{x}{5}\right)^{\frac{5}{x}}\right)^{\frac{1}{5}}}{\left(\lim_{x\to 0}\left(1-\frac{x}{5}\right)^{\frac{-5}{x}}\right)^{\frac{-1}{5}}}\right\}$	1
		$= \log\left(\dfrac{e^{\frac{1}{5}}}{e^{\frac{1}{-5}}}\right) = \log e^{\frac{1}{5}+\frac{1}{5}} = \log e^{\frac{2}{5}}$	1
		$= \dfrac{2}{5}\log e$ $= \dfrac{2}{5}$	1
	c)	Find a real root of the equation $x^3 - 2x - 5 = 0$ using the bisection method in the interval $(2, 3)$ $($carry out three iterations$)$.	**04**
	Ans	Let $f(x) = x^3 - 2x - 5$ $\therefore f(2) = -1$ $f(3) = 16$	
		$\therefore$ the root is in (2, 3).	1
		$\therefore x_1 = \dfrac{2+3}{2} = 2.5$ $\therefore f(2.5) = 5.625$ $\therefore$ the root is in (2, 2.5).	1
		$\therefore x_2 = \dfrac{2+2.5}{2} = 2.25$ $\therefore f(2.25) = 1.891$ $\therefore$ the root is in (2, 2.25).	1
		$\therefore x_3 = \dfrac{2+2.25}{2} = 2.125$ OR	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	c)	Let $f(x) = x^3 - 2x - 5$	
		$\therefore f(2) = -1$	
		$f(3) = 16$	
		$\therefore$ root is in $(2,3)$	1

Iteration	a	b	$x = \frac{a+b}{2}$	$f(x)$
I	2	3	2.5	5.625
II	2	2.5	2.25	1.891
III	2	2.25	2.125	----

Marks: 1, 1, 1

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d) Using false position method, find the root of the equation $x^2 + x - 3 = 0$ in the interval $(1,2)$ by performing three iterations. **04**

Ans

Let $f(x) = x^2 + x - 3$

$f(1) = -1 < 0$

$f(2) = 3 > 0$

$\therefore$ the root is in $(1,2)$ — 1

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$= \frac{1(3) - 2(-1)}{3 - (-1)} = 1.25$$ — 1

$f(x_1) = -0.188 < 0$

$\therefore$ the root is in $(1.25, 2)$

$$x_2 = \frac{1.25(3) - 2(-0.188)}{3 + 0.188} = 1.294$$ — 1

$f(x_2) = -0.032 < 0$

$\therefore$ the root is in $(1.294, 2)$

$$x_3 = \frac{1.294(3) - 2(-0.032)}{3 + 0.032} = 1.301$$ — 1

<u>OR</u>

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Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	d)	OR Let $f(x)=x^2+x-3$ $f(1)=-1<0$ $f(2)=3>0$ $\therefore$ the root is in $(1,2)$	1

a	b	$f(a)$	$f(b)$	$x=\frac{af(b)-bf(a)}{f(b)-f(a)}$	$f(x)$	
1	2	-1	3	1.25	-0.188	1
1.25	2	-0.188	3	1.294	-0.032	1
1.294	2	-0.032	3	1.301	---	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
	e)	Solve $x^3-x-1=0$ by Newton-Raphson method (up to three iterations.)	**04**
	Ans	Let $f(x)=x^3-x-1$ $f(1)=-1<0$ $f(2)=5>0$ $f'(x)=3x^2-1$ Initial root $x_0=1$	1
		$\therefore f'(1)=2$ $x_1=x_0-\frac{f(x_0)}{f'(x_0)}=1-\frac{f(1)}{f'(1)}=1.5$	1
		$x_2=1.5-\frac{f(1.5)}{f'(1.5)}=1.348$	1
		$x_3=1.348-\frac{f(1.348)}{f'(1.348)}=1.325$	1
		OR Let $f(x)=x^3-x-1$ $f(1)=-1<0$	

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Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	e)	$f(2)=5>0$	
		$f'(x)=3x^2-1$	
		Initial root $x_0=1$	1
		$x_i=x-\frac{f(x)}{f'(x)}=x-\frac{x^3-x-1}{3x^2-1}$	
		$=\frac{2x^3+1}{3x^2-1}$	
		$x_1=1.5$	1
		$x_2=1.348$	1
		$x_3=1.325$	1
		--	
	f)	Find the root of $e^{-x}-x=0$ by bisection method (up to three iterations.)	**04**
	Ans	Let $f(x)=e^{-x}-x$	
		$\therefore f(0)=e^{-0}-0=1$	
		$f(1)=e^{-1}-1=-0.632$	
		$\therefore$ root is in $(0,1)$	1
		$\therefore x_1=\frac{0+1}{2}=0.5$	1
		$\therefore f(0.5)=0.107$	
		$\therefore$ root is in $(0.5,1)$	
		$\therefore x_2=\frac{0.5+1}{2}=0.75$	1
		$\therefore f(0.75)=-0.278$	
		$\therefore$ root is in $(0.5,0.75)$	
		$\therefore x_3=\frac{0.5+0.75}{2}=0.625$	1
		OR	
		Let $f(x)=e^{-x}-x$	
		$\therefore f(0)=e^{-0}-0=1$	
		$f(1)=e^{-1}-1=-0.632$	
		$\therefore$ root is in $(0,1)$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	f)		

Iteration	a	b	$x=\frac{a+b}{2}$	$f(x)$	Marking Scheme
I	1	0	0.5	0.107	1
II	1	0.5	0.75	-0.278	1
III	0.75	0.5	0.625	----	1

---

6. **Solve any FOUR of the following:** **16**

a) If $y=2\cos(\log x)+3\sin(\log x)$, prove that $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$ **04**

Ans $y=2\cos(\log x)+3\sin(\log x)$

$\therefore \frac{dy}{dx}=-2\sin(\log x)\cdot\frac{1}{x}+3\cos(\log x)\cdot\frac{1}{x}$ 1

$\therefore x\frac{dy}{dx}=-2\sin(\log x)+3\cos(\log x)$ ½

$\therefore x\frac{d^2y}{dx^2}+\frac{dy}{dx}=-2\cos(\log x)\cdot\frac{1}{x}-3\sin(\log x)\cdot\frac{1}{x}$ 1

$\therefore x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}=-2\cos(\log x)-3\sin(\log x)$ ½

$\therefore x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}=-\left[2\cos(\log x)+3\sin(\log x)\right]$ ½

$=-y$

$\therefore x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$ ½

---

b) Differentiate $\tan^{-1}\left(\frac{x}{1+12x^2}\right)$ w.r.t. x **04**

Ans Let

$y=\tan^{-1}\left(\frac{x}{1+12x^2}\right)$

$=\tan^{-1}\left(\frac{4x-3x}{1+4x\cdot 3x}\right)$ 1

$=\tan^{-1}4x-\tan^{-1}3x$ 1

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Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	b)	$\therefore \frac{dy}{dx}=\frac{1}{1+(4x)^2}\cdot 4-\frac{1}{1+(3x)^2}\cdot 3$ $=\frac{4}{1+16x^2}-\frac{3}{1+9x^2}$	1+1
	c)	Solve the following equations by Gauss elimination method: $x+y+z=6$, $2x-3y+3z=5$, $3x+2y-z=4$	**04**
	Ans	$x+y+z=6$ $2x-3y+3z=5$ $3x+2y-z=4$	
		$3x+3y+3z=18$ $2x-3y+3z=5$ and $-$ ______ $x+6y=13$ $2x-3y+3z=5$ $9x+6y-3z=12$ $+$ ______ $11x+3y=17$	1
		$x+6y=13$ $22x+6y=34$ $-$ ______ $-21x=-21$	
		$\therefore x=1$	1
		$y=2$	1
		$z=3$	1
	d)	Solve the following equtions by Jacobi's method (take three iterations.) $20x+y-2z=17$, $2x-3y+20z=25$, $3x+20y-z=18$	**04**
	Ans	$20x+y-2z=17$,$3x+20y-z=18$, $2x-3y+20z=25$ $x=\frac{1}{20}(17-y+2z)$ $y=\frac{1}{20}(18-3x+z)$	1

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Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	d)	$z=\frac{1}{20}(25-2x+3y)$	
		Starting with $x_0=y_0=z_0=0$	
		$x_1=0.85$ $y_1=0.9$ $z_1=1.25$	1
		$x_2=0.93$ $y_2=0.835$ $z_2=1.3$	1
		$x_3=0.938$ $y_3=0.826$ $z_3=1.282$	1
		--	
	e)	Solve the equations by Gauss Seidal method (up to three iterations.) $8x+2y+3z=30$, $x-9y+2z=1$, $2x+3y+6z=31$	**04**
	Ans	$8x+2y+3z=30$, $x-9y+2z=1$, $2x+3y+6z=31$	
		$\therefore x=\frac{1}{8}(30-2y-3z)$ $y=\frac{1}{-9}(1-x-2z)$ $z=\frac{1}{6}(31-2x-3y)$	1
		Starting with $x_0=y_0=z_0=0$	
		$x_1=3.75$ $y_1=0.306$ $z_1=3.764$	1
		$x_2=2.262$ $y_2=0.977$ $z_2=3.924$	1

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Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	e)	$x_3 = 2.034$ $y_3 = 0.987$ $z_3 = 3.996$	1
		--	
	f)	With the following system of equations: $3x+2y=4.5,\ 2x+3y-z=5,\ -y+2z=0.52$ Set up the Gauss-Seidal iterations scheme for solution.Iterate two times, using initial approximations$x_0=0.4, y_0=1.6, z_0=0.4$	**04**
	Ans	$3x+2y=4.5,\ 2x+3y-z=5,\ -y+2z=0.52$ $\therefore x=\frac{1}{3}(4.5-2y)$ $y=\frac{1}{3}(5-2x+z)$ $z=\frac{1}{2}(0.52+y)$	1
		Starting with $x_0=0.4, y_0=1.6, z_0=0.4$ $x_1=0.433$ $y_1=1.511$ $z_1=1.016$	1½
		$x_2=0.493$ $y_2=1.677$ $z_2=1.099$	1½
		--	
		Important Note *In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.*	