SUMMER -2017 EXAMINATION
Model Answer

Subject code: 17422 Subject : (Theory of Structures)

Important Instructions to examiners:

1) The answer should be examined by keywords and not as word-to- word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language error such as grammatical, spelling errors should not be given more importance. (Not applicable for subject English and communication skill).
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figure drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In the some cases, the assumed constants values may vary and there may be some difference in the candidates answer and model answer.
6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidates understanding

Question and sub Question	Model Answer/paper Solution with sketches	Marks
Q. 1 A	**Attempt any Six of the following**	**12 Marks**
a)	**State the condition of stability of Masonry dam**	
	Ans:	
	1. Stability against Overturning : $(P.h/3) \leq W\ (b\text{-}x)$	½ M
	2. Stability against no Tension : $e \leq \dfrac{b}{6}$	½ M
	3. Stability against Sliding : $P \leq F$	½ M
	4. Stability against crushing	½ M
b)	**Define the slope and deflection of beam**	1M
	Ans : Slope of a beam:-	
	Slope of a beam at a point is defined as rate of change of deflection with respect to longitudinal distance i.e.	
	slope dy/dx where, y = deflection The point of slope is " radian "	1M
	where Л radian = 180 degrees	
	Deflection of a beam :-	

The vertical displacement of a point on a beam after loading with respect to its original position before loading is called deflection It is denoted by 'y' unit is mm

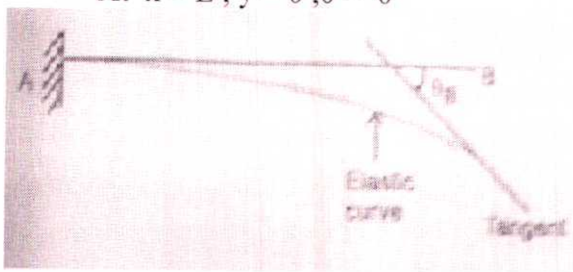
c) **What do you understand by boundry condition of a beam ?State the boundry condition for two different nature of beam supports,**

Ans : It is a general mathematical principle that the number of boundary condition necessary to determine a solution to a differential equation matches the order of the differential equation .

- Cantilever beam X is consider from free end

Boundary condition At $x = 0$, $y = y$ max ,$\theta = \theta_{max}$

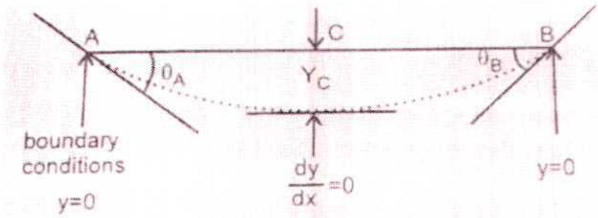
At $x = L$, $y = 0$,$\theta = 0$ 1M



- Simply supported beam The condition to locate point of maximum deflection is slope of tangent at that point is zero

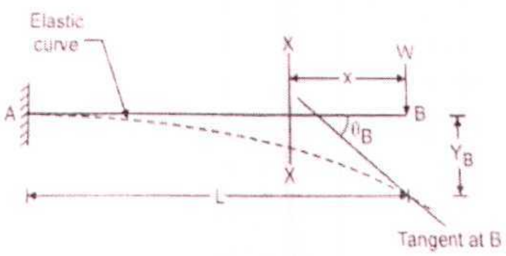
  Boundary condition At $x = 0$, $y = 0$,$\theta_A = \theta_{max}$ 1M

  $x = L$ $y = 0$,$\theta_A = \theta_{max}$



d) **State the values of maximum deflection for a cantilever beam of span L carrying a point load 'W' at a distance 'L' from the fixed end.**

Ans : The Values of maximum deflection for a cantilever beam of a span L carrying a point load 'W' at a distance 'L' from the fixed end 1M



$$Y_{max} = Y_B = -\frac{WL^3}{3EI}$$ 1M

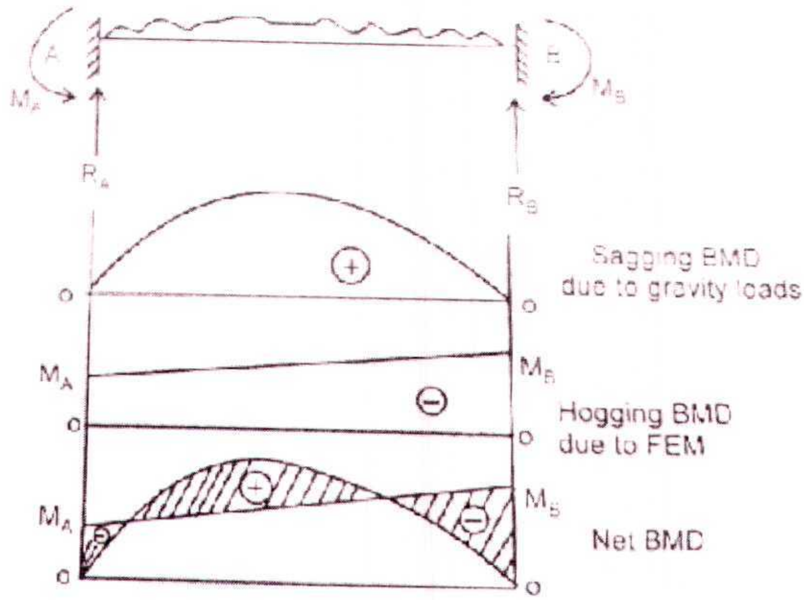
Where , W = Point load in KN

L = Span in m

Y = Deflection

e) State the Principle of Superposition

Ans: Principle of Superposition:-I states that " If number of forces are acting simultaneously on a body then their combined effect on the body is equal to the algebraic sum of the effects of the individual forces or moments considered separately 1M



1M

f) Define i) Distribution Factor ii) Portal Frames :

i) Distribution Factor :

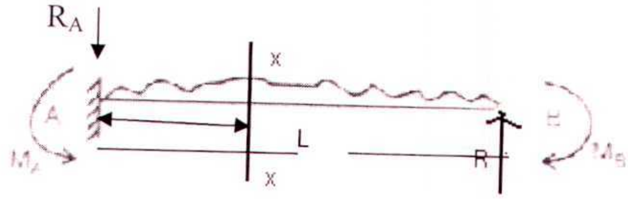
It is the ratio of relative stiffness of a member to the total stiffness of all the members meeting at a joint 1M

ii) Portal Frames :

Portal frame is a structure of least two columns and a connecting beam .It is a two dimensional structure assumed separate from a structure for analysis 1M

g) What is the carry over factor for a beam fixed at one end and simply supported at the other?

Ans:

		Consider any section x-x of x distance from A  $\frac{M_A}{M_B} = \frac{1}{2}$ Where, M_A = Moments at A in KNm M_B = Moments at A in KNm	1M 1M
	h)	**Define perfect and imperfect frame** 1) Perfect frame: A frame made up of just sufficient numbers so that it can remain in stable equilibrium, when loaded at joints. The condition of frame to be stable is : $n = 2j - 3$ Where, n = numbers of members j = number of joints 2) Imperfect frame : A frame made up of either more than or less than, just sufficient number of members to keep it in static equilibrium is called Imperfect frame . Where , $n < 2j - 3$-----------Deficient frame $n > 2j - 3$-----------Redundant frame	1M 1M
Q.1 B)		**Attempt any Two of the following**	**8 M**
	a)	**Explain the points to be considered to design 30 storey building with respect to wind pressure.** .Ans :While designing a 30 storey building the points to be considered are : 1. As the building is greater than 10 m height .The wind pressure will be considered 2. Location of building and topography of area which decide the coefficient of wind pressure. 3. The wind pressure is assumed to vary uniformly with increase in pressure with respect to height of building. 4. The wind pressure will cause large bending stresses and the self weight of the building will cause direct stresses and the material should be capable of bearing the total maximum stresses at base.	**4mark (any 4 for each 1)**

b)	**A short column of external 250 mm and internal diameter 200 mm carries an eccentric load .Find the eccentricity which the load can have without producing tension in the section of a column** Given :- D =250 mm d =200 mm Solution:-$\sigma_{min} = \sigma_d - \sigma_b$ $= P/A - \dfrac{Pe\,y}{I}$ $\dfrac{P}{\dfrac{\pi(D^2-d^2)}{4}} - \dfrac{P\ e}{\dfrac{\pi(D^4-d^4)}{64}} \times D/2$ -------------------- (1)	2 M
	For No Tension $\sigma_{min} = 0$ put in eq (1) $e = \dfrac{(D^2-d^2)}{8\,D}$ $= \dfrac{(250^2-200^2)}{8 \times 25}$ $e = 51.25$ mm	2 M
c)	**State four assumption made in the analysis of simple frame** Ans :Assumption made in analysis of frame frame are 1. The members are pin jointed at ends 2. The frame is perfect ,i.e. it satisfies n = 2j-3 condition 3. The load is acting at joints only. 4. Self weight of the member is neglected	**4mark (any 4 for each 1)**
Q. 2	**Attempt any Four of the following**	**16 M**
a)	**A chimney having diameter 5m and 50 m high . It is subjected to a horizontal wind pressure of 7KPa normal to the chimney ,Find the maximum bending stress in a chimney (use C = 0.70)** Ans :Given D =5 m H = 50m ρ = 7 Kpa =7 x 10^3 N/m^2 C = 0.7 Solution Maximum Bending stress , $\sigma_b = \dfrac{M}{I} \times y$ 1)Moment of 'P' about base = P (H/2)	1 M

3) Total wind force $P = C \times \rho \times$ Projected area

4) $I = \dfrac{\pi (D^4)}{64}$ 1 M

$= \dfrac{0.7 \times 7000(5 \times 50)(50/2) \times 5/2}{\dfrac{\pi (5^4)}{64}}$ 1 M

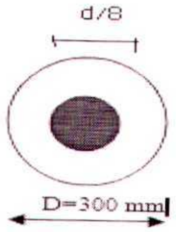
$\boxed{\sigma_{max} = 2.4950\ N/m^2}$ 1 M

b) **Calculate core of a section for following and draw neat sketches:**

i. Circular section 300mm diameter

ii Rectangular section is a 250 x 600 mm in size

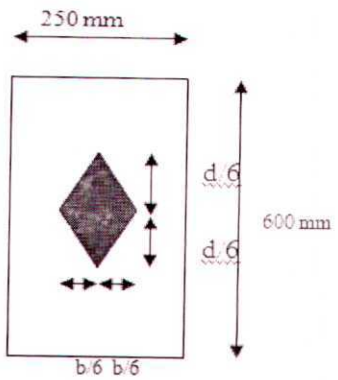
Ans i)Core of section foe a Circular section is a circle of diameter



1 M

$e_{max} = \dfrac{D}{8} = \dfrac{300}{8} = 37.5$ mm 1 M

ii)Core of section for a rectangular section is a 250 x 600 mm



1 M

i) $e_x = d/6 = 600/6 = 100$ mm

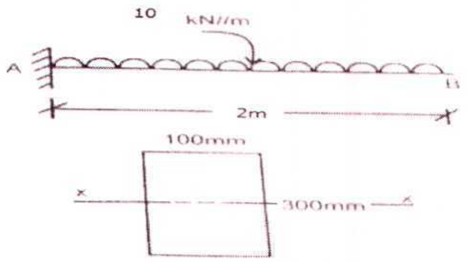
ii) $e_y = \dfrac{b}{6} = \dfrac{250}{6} = 41.6667$ mm 1 M

c)	**A square pillar 200 mm side is subjected to an eccentric load of 95 KN at an eccentricity of 50 mm in the plane bisecting thickness. Find the maximum and minimum intensities of stress at base section** Ans : Given b = 200 mm d = 200 mm P = 95 KN e = 50 Ans ; Solution : $\sigma 0$ = direct stress σb = Bending stress i) σmax = direct stress + Bending stress $= \sigma 0 + \sigma b$ $= P/A + \frac{M}{I} y$ $= \frac{95 \times 10^3}{200 \times 200} + \frac{95 \times 10^3 \times 50}{\frac{200 \times 200^3}{12}} \times 200/2$ $= 2.375 + 3.56$ $= 5.93 \text{ N/mm}^2$ ii) σmin = direct stress + Bending stress $= \sigma 0 - \sigma b$ $= P/A - \frac{M}{I} y$ $= \frac{95 \times 10^3}{200 \times 200} - \frac{95 \times 10^3 \times 50}{\frac{200 \times 200^3}{12}} \times 200/2$ $= 2.375 - 3.56$ (Comp) $= -1.185 \text{ N/mm}^2$ (Comp)	1M 1M 1M 1M
d)	**A cantilever of length 2m carries a UDL of 10 kn/m over a entire span .If the section is rectangle 100 mm wide and 300 mm deep . Find the slope at the free end. And deflection at free end Take E =100 KN/mm²** Given $E = 100 \text{ KN/mm}^2$	

$= 1 \times 10^{8}$ KN/m^2

$L = 2$ m

$w = 10$ kN/m



Solution :

1M

1) $I = \dfrac{100 \times 300^{3}}{12}$

$= 225 \times 10^{6}$ mm^4

2) Slope at B, 1M

$\dfrac{dy}{dx} = - \dfrac{W L^3}{6 EI} = \dfrac{10 \times 2^{3}}{6 EI}$

$= - \dfrac{10 \times 2^{3}}{6 \times 1 \times 10^{8} \times 2.25 \times 10^{-4}}$ 1M

$= - 5.94 \times 10^{-4}$

$= - 0.000592$ radians

3) Deflection at B

$= - \dfrac{W L^4}{8 EI} = \dfrac{10 \times 2^{4}}{8 \times 1 \times 10^{8} \times 2.25 \times 10^{-4}}$

$= - 8.88 \times 10^{-4}$m 1M

$= - 0.888$ mm

e) **A simply supported beam of 6m carries a UDL of 5 KN/m find the breadth and depth of beam , if maximum bending stress is not to exceed 8 N/mm^2 and maximum deflection 20 mm .Take E = 1x10^4 N/mm^2**

Given

$\sigma b = 8$ N/mm^2

$= 8000$ KN / m^2

$Y_{max} = 20$ mm

$E = 1 \times 10^4$ N/mm^2 1M

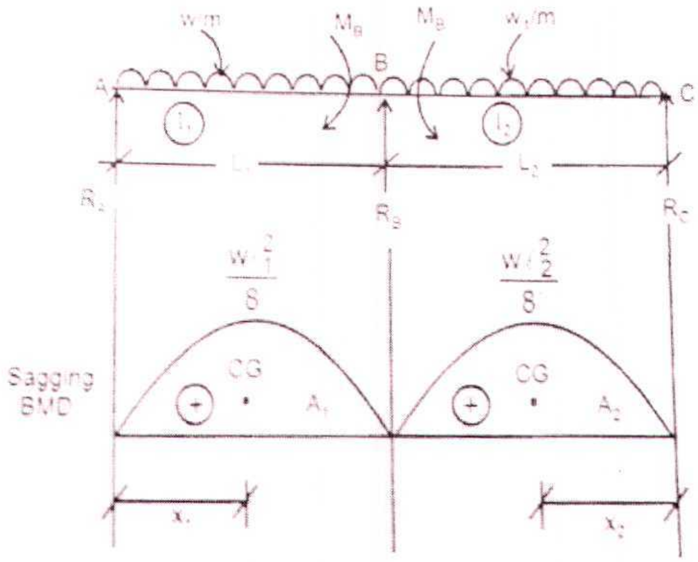
$= 1 \times 10^{7}$ KN/m^2

1) $BM_{max} = \dfrac{WL^2}{8} = \dfrac{5 \times 6^2}{8} = 22.5$ KN/m

$M = 22.5$ KN/m

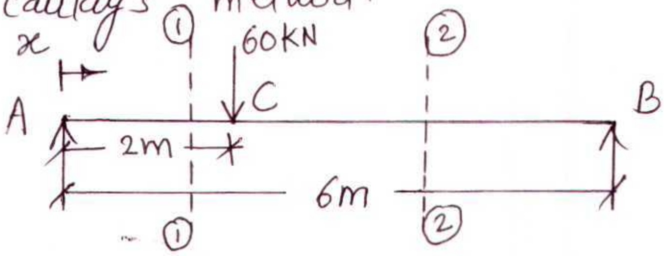
2) $I = \dfrac{bd^3}{12}$

	From Bending equation 3) $\sigma b = \frac{M\,y}{I}$ $bd^2 = 0.0168$ ------------------------------------(1) From deflection equation 4) $Y_{max} = \frac{-5\,W\,L^4}{384\,EI}$ $0.020 = \frac{5 \times 5 \times 6^4}{384 \times 1 \times 10^7 \times \frac{bd^3}{12}}$ $bd^3 = 5.06 \times 10^{-3}$---------------------------- (2) Put equation (1) in equation (2) $0.0168 = 5.06 \times 10^{-3}$ $d = 0.301$ m $\boxed{d = 301 \text{ mm}}$ Put in equation (1) $b = 0.185$ m $\boxed{b = 185\text{mm}}$	1M 1M 1M
	f)	**Write Clapeyron's moment theorem for beam with different M.I.giving meaning of each term along with the diagram of beam** Ans ; Clapeyron's moment theorem for beam with different M.I. Let ABC be a continuous beam, selected for analysis and loaded as shown below a1 and a2 are areas of sagging BMD Here M_A and $M_C = 0$ The support are simple supports Unknowns are R_A R, R_C and M_B To obtain M_B Clapeyrons theorem used as below $M_A \frac{L_1}{I_1} + 2M_B \left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_C \frac{L_2}{I_2}$ $= -6\left(\frac{a_1 x_1}{L_1 I_1} + \frac{a_2 x_2}{L_2 I_2}\right)$ Where, M_A = Support moment at A M_A = Support moment at A	1M

	M_B = Support moment at B M_C = Support moment at C L_1 = Length of the span AB L_2 = Length of the span BC $I1$ = Moment if Inertia for the span AB $I2$= Moment if Inertia for the span BC $a1$ = area of moment diagram for the span AB $x1$= Centroidal distance of moment diagram for the span AB $a2$= area of moment diagram for the span BC $x2$= Centroidal distance of moment diagram for the span BC The term above used above are as shown in Figure , M_A and $M_C = 0$ M_B can be obtained from the above equation. I1 and I2 are the moment of Inertia of beams respectively, If beams are of same size, then $I1 = I2= I$ hence the equation reduces to	1M
	$M_A\ L_1 + 2M_B\ (L_1 + L_2) + M_C\ L_2$ $= -6\ \left(\frac{a_1x_1}{I_1} + \frac{a_2x_2}{I_2} \right)$ Thus for 'n' number of unknown moments ,'n' such equation are required	1M
		1M

Question	Description	Marks
Q.3	Attempt any FOUR	16 Marks

3(a) A simply supported beam of span 6m, carries a point load of 60KN at 2m from left hand support. Calculate deflection under point load. Use Macaulay's method.



1) To Find support reactions

$$\Sigma M_A = 0 \; (\curvearrowright +)$$

$$-R_B \times 6 + 60 \times 2 = 0$$

$$\boxed{R_B = 20KN}$$ 1 Mark

$$R_A = 60 - 20 = 40KN$$

2) To Find slope & deflection

$$EI \frac{d^2y}{dx^2} = M \quad \text{— Differential equation.}$$

$$EI \frac{d^2y}{dx^2} = 40 \times x \;\Big|_{①}\; - 60(x-2) \;\Big|_{②} \quad \text{— Moment eq}^n$$

① $x = 2m$ ② $x = 6m$

Integrating w.r. to x

$$EI \frac{dy}{dx} = \frac{40x^2}{2} + C_1 \;\Big|_{①}\; - \frac{60(x-2)^2}{2} \;\Big|_{②} \quad \text{— slope equation (I)}$$

Again Integrating w.r. to x

$$EI \cdot y = \frac{20x^3}{3} + C_1 x + C_2 \;\Big|_{①}\; - \frac{30(x-2)^3}{3} \;\Big|_{②} \quad \text{— Deflection eq}^n \text{ (II)}$$ 1 Mark

3) To calculate Constants of Integration

Boundary Conditions.

At $x = 0$, $y = 0$ Putting in Deflection eq^n (I)

$$EI(0) = \frac{20(0)^3}{3} + C_1(0) + C_2 \;\Big|\; - \frac{30}{3}(0-2)^3$$

$$\boxed{C_2 = 0}$$

At $x = 6m$, $y = 0$ Putting in Deflection eq^n

$$EI(0) = \frac{20(6)^3}{3} + C_1(06) + C_2 \;\Big|\; -\frac{30}{3}(6-2)^3$$

$$0 = 800 + 6C_1$$

$$\boxed{C_1 = -133.33}$$ 1 Mark

Putting values of C_1 and C_2 in slope and deflection equation & rewriting eq^n

$$EI \cdot \frac{dy}{dx} = 20x^2 - 133.33 \;\Big|_{1}\; -30(x-2)^2 \;\Big|_{2}$$ Final slope eq^n

$$EI \cdot y = \frac{20}{3}x^3 - 133.33x + 0 \;\Big|_{1}\; -\frac{30}{3}(x-2)^3$$ final defl^n eq^n

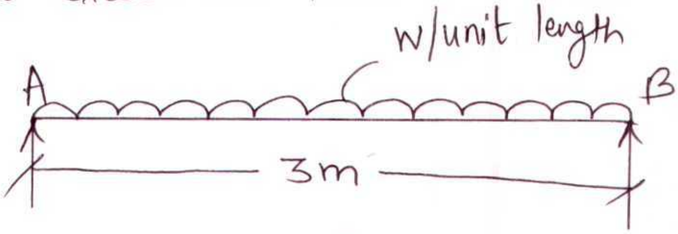
4) Deflection under point load

At $x = 2m$, $y = y_c$ Putting in Final deflection eq^n

$$EI \cdot y_c = \frac{20}{3}(2)^3 - 133.33(2) - 0$$

$$\boxed{y_c = -\frac{213.326}{EI}}$$ — Deflection below point load 1 Mark

Q.3(b) A simply supported beam having 3m span carries a UDL 'W' per unit length. If slope at the end is not to exceed 1.5°. Find the maximum deflection.

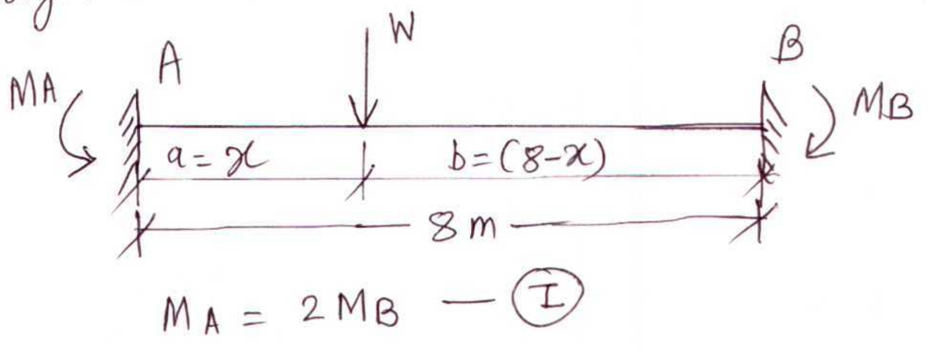


$$\text{slope } (\theta) = 1.5^\circ$$

$$\theta = 1.5 \times \frac{\pi}{180} \text{ radians.}$$

$$\theta = 0.02618 \text{ radians.}$$ $\frac{1}{2}$ Mark

Question	Description	Marks.
	slope of the beam is calculated as	
	$\theta = \frac{WL^3}{24EI}$	$\frac{1}{2}$ Mark
	$0.02618 = \frac{W \times 3^3}{24 EI}$	
	$\therefore \boxed{\frac{W}{EI} = 0.02327}$	1 Mark
	Maximum Deflection of beam	
	$y_{max} = \frac{5WL^4}{384 EI}$	
	$y_{max} = \frac{5 \times W (3)^4}{384 EI}$	1 Mark
	Putting value of $\frac{W}{EI}$ in deflection equation	
	$y_{max} = \frac{5 \times 0.02327 \times (3)^4}{384}$	
	$y_{max} = 0.02454\ m.$	
	$\therefore \boxed{y_{max} = 24.54\, mm}$	1 Mark.
Q.3(c)	State two advantages and two disadvantages of fixed beam over simply supported beam.	
	<u>Advantages of Fixed Beam.</u>	
	1) The slopes at the end support in case of fixed beam are zero but in case of simply supported beam slopes are maximum.	Any Two for two marks.
	2) The maximum bending moment and deflection are less as compared to simply supported beam for same span and loading.	
	3) Fixed beam is more strong, stable and stiff than simply supported beam.	

Question	Description	Marks
	4) The cross section required for fixed beam is smaller and steel required is less due to lesser B.M. and hence it is economical as compared to simply supported beam.	
	Disadvantages of fixed beam –	
	1) If any one of the support sinks to a small extent, it induces additional moment at each end.	Any TWO for TWO Marks.
	2) Since both the end of beam are fixed, temprature stresses are developed due to variation in temperature	
	3) Complete fixity cannot be achieved.	
Q.3(d)	A fixed beam of span 8m carries a point load 'W' at a distance 'x' from the left hand support. If the moment at the left end is twice that at the right end evaluate 'x'	
		
	$M_A = 2M_B$ — (I)	1 Mark
	To find fixed end moments.	
	$M_A = -\dfrac{Wab^2}{L^2} = \dfrac{-W(x)(8-x)^2}{8^2}$	$\frac{1}{2}$ Mark
	$M_B = -\dfrac{Wa^2b}{L^2} = -\dfrac{W(x^2)(8-x)}{8^2}$	$\frac{1}{2}$ Mark
	Putting values of MA & MB in equation I	
	$\therefore \; - M_A = 2M_B$	
	$\dfrac{W(x)(8-x)^2}{8^2} = 2\left[\dfrac{W(x^2)(8-x)}{8^2}\right]$	1 Mark

Question	Description	Marks

$$(8-x) = 2x$$

$$3x = 8$$

$$x = \frac{8}{3}$$

$$\boxed{x = 2.67\, m}$$

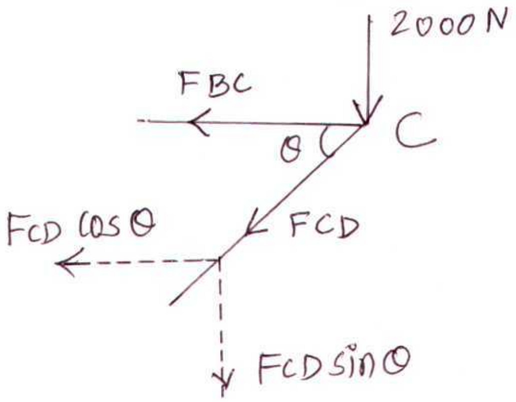
1 Mark

Q.3 (e) Using method of joints, find nature and magnitude of forces in member AB and member DE of truss.

$$\text{Slope } (\theta) = \tan^{-1}\left(\frac{4}{6}\right)$$

$$\boxed{\theta = 33.69^\circ}$$

1) Consider joint C



$$\Sigma F_x = 0$$

$$-F_{BC} - F_{CD}\cos\theta = 0 \quad \text{—(I)}$$

$$\Sigma F_y = 0$$

$$-2000 - F_{CD}\sin\theta = 0$$

$$F_{CD} = \frac{-2000}{\sin 33.69}$$

$$\left[F_{CD} = -3605.55\, N\right] \text{—(Compression)}$$

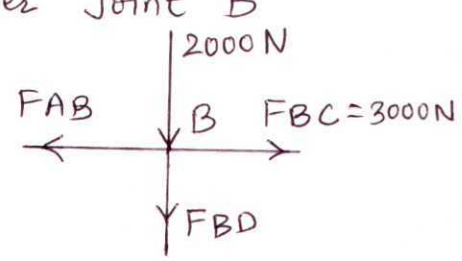
Putting value of F_{CD} in eq^n (I)

$$-F_{BC} + 3605.55 \cos 33.69 = 0$$

$$\left[F_{BC} = 3000\, N\right] \text{—(Tension)}$$

1 Mark

2) Consider Joint B



$$\Sigma F_x = 0$$

$$+3000 - F_{AB} = 0$$

$$\left[F_{AB} = 3000\, N\right] \text{—(Tension)}$$

$$\Sigma F_y = 0$$

$$-F_{BD} - 2000 = 0$$

$$\left[F_{BD} = -2000\, N\right] \text{— (Compression)}$$

1 Mark

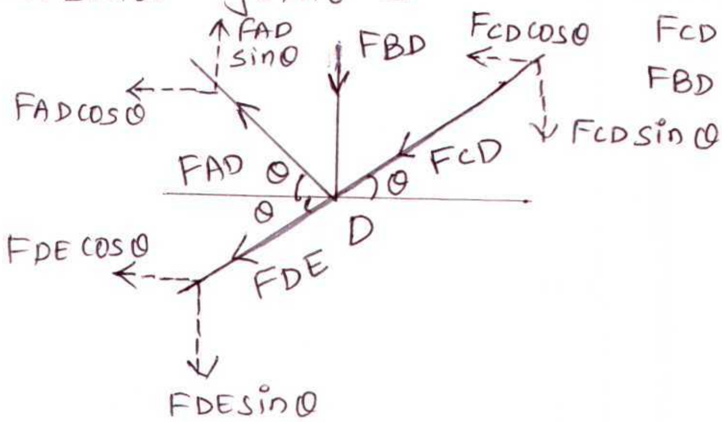
Question	Description	Marks

Consider joint D

$\theta = 33.69^\circ$

$F_{CD} = 3605.55\ N.(C)$

$F_{BD} = 2000\ N\ (T)$



$$\Sigma F_x = 0$$

$$-F_{AD}\cos\theta - F_{CD}\cos\theta - F_{DE}\cos\theta = 0$$

$$-F_{AD} - F_{CD} - F_{DE} = 0$$

$$F_{AD} + F_{DE} = -3605.55 \quad \text{—} \quad (I)$$

1 Mark

$$\Sigma F_y = 0$$

$$F_{BD} - 3605.55\sin\theta - F_{DE}\sin\theta + F_{AD}\sin\theta = 0$$

$$-2000 - 3605.55\sin 33.69 - F_{DE}\sin 33.69 + F_{AD}\sin 33.69$$

$$-2000 - 2000 - F_{DE}\sin 33.69 + F_{AD}\sin 33.69 = 0$$

$$\therefore\ F_{AD} - F_{DE} = 7211.11 \quad \text{——} \quad (II)$$

Addition of eq^n (I) & (II)

$$\begin{aligned} & F_{AD} + F_{DE} = -3605.55 \\ + \quad & F_{AD} - F_{DE} = 7211.11 \\ \hline & 2F_{AD} = +3605.55 \end{aligned}$$

$$\boxed{F_{AD} = 1802.78\,N} - (\text{Tension})$$

Putting value of F_{AD} in eq^n (I)

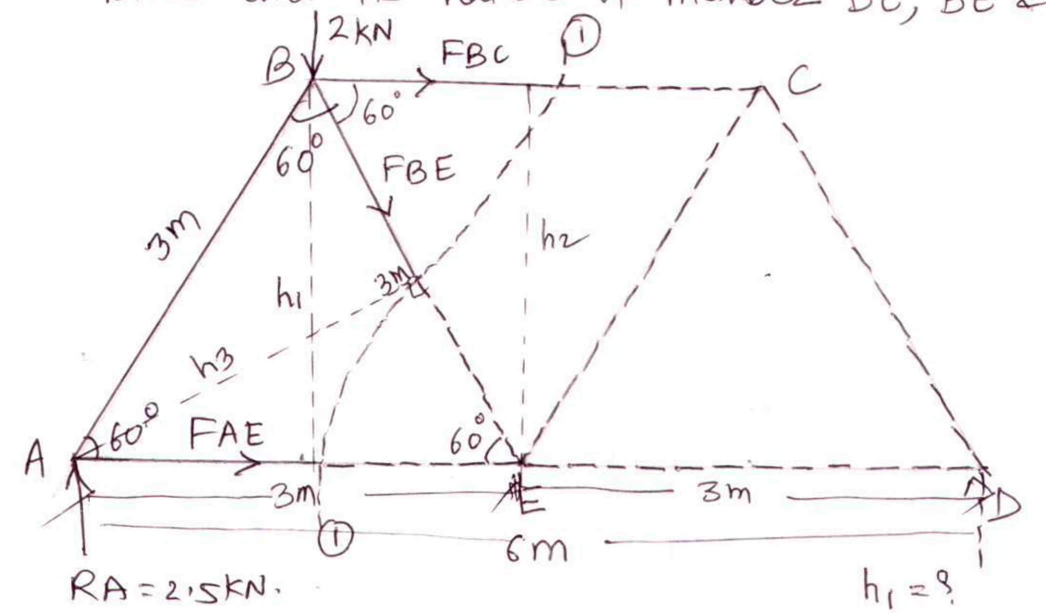
$$1802.78 + F_{DE} = -3605.55$$

$$\boxed{F_{DE} = -5408.33\ N}\ \text{Compression.}$$

1 Mark

MEMBER	FORCE	NATURE
AB	3000 N	Tension.
DE	5408.33 N	Compression.

Question	Description	Marks
Q.3(f)	Using Method of section calculate the magnitude of forces and its nature of member BC, BE & AE	



1) To Calculate support Reactions.

$$R_A + R_B = 2 + 4 = 06$$

Taking moment at A

$$\Sigma M_A = 0$$

$$-R_D \times 6 + 4 \times 4.5 + 2 \times 1.5 = 0$$

$$\boxed{R_D = 3.5 KN}$$

$$\therefore R_A = 06 - 3.5 =$$

$$\boxed{R_A = 2.5 kN}$$

1 Mark

Consider section ①–① passing through member BC, BE & AE assuming all forces tensile.

Taking moment At B considering left side of section.

$$\Sigma M_B = 0$$

$$2.5 \times 1.5 - F_{AE} \times h_1 = 0$$

$$3.75 - 2.598 F_{AE} = 0$$

$$\sin 60^\circ = \frac{h_1}{3}$$

$$\therefore \boxed{F_{AE} = +1.443 KN} \text{ (Tension)} \quad \boxed{h_1 = 2.598 m}$$

1 Mark

$$\Sigma M_E = 0$$

$$\sin 60 = \frac{h_2}{3}$$

$$2.5 \times 3 - 2 \times 1.5 + F_{BC} \times h_2 = 0 \quad \boxed{h_2 = 2.598 m}$$

Question	Description	Marks

$$7.5 - 3 + 2.598\,F_{BC} = 0$$

$$\left[F_{BC} = -1.732\text{ kN}\right] \text{ — Compression.}$$ 1 Mark

$$\Sigma M_A = 0$$

$$2 \times 1.5 - 1.5 \times 3 + F_{BE} \times h_3 = 0 \qquad \sin 60 = \frac{h_3}{3}$$

$$\left[h_3 = 2.598\text{m}\right]$$

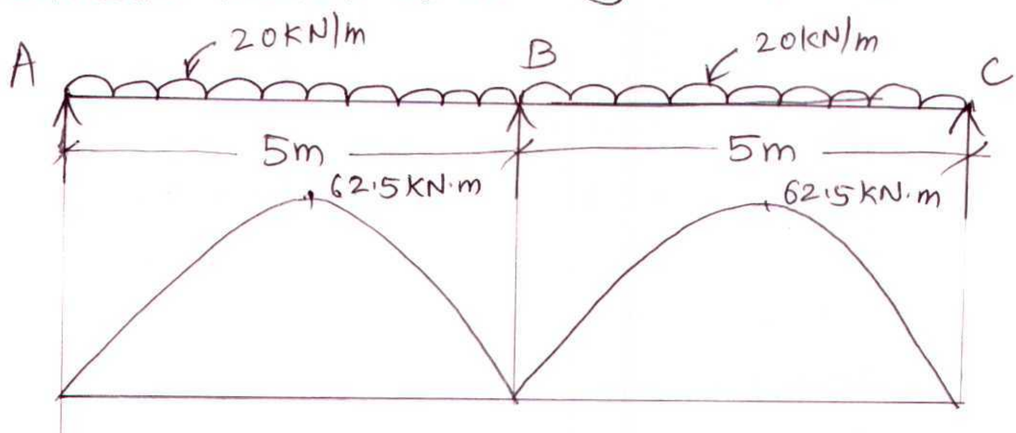
$$-0.75 + 2.598\,F_{BE} = 0$$

$$\left[F_{BE} = 0.577\text{ kN}\right] \text{ — Tension.}$$ 1 Mark

Member	Force	Nature
AE	1.443 kN	Tension
BC	1.732 kN	Compression
BE	0.577 kN.	Tension.

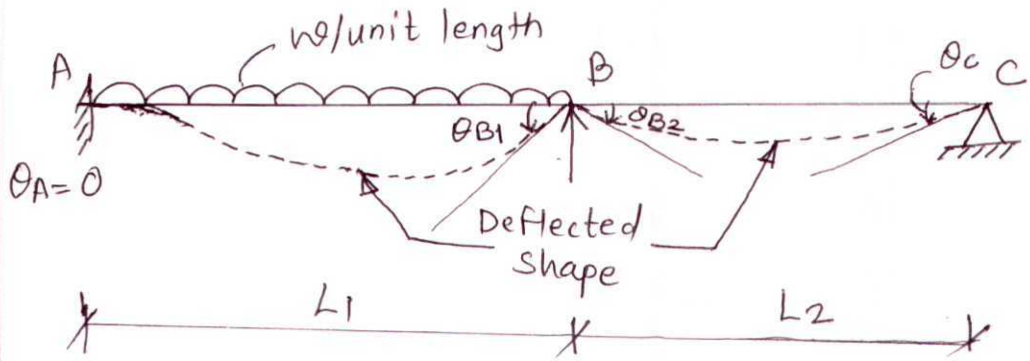
Q. 4 Attempt Any FOUR — 16 Marks

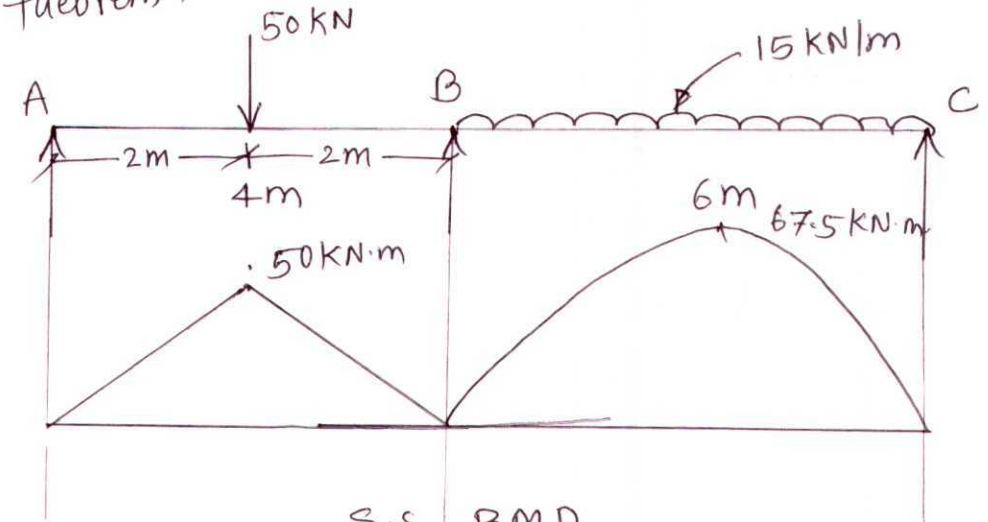
4(a) A continuous beam ABC is supported on three supports at same level AB = BC = 5m Both spans carry UDL of intensity 20 kN/m over entire span. Calculate moment at B using theorem of three moments.



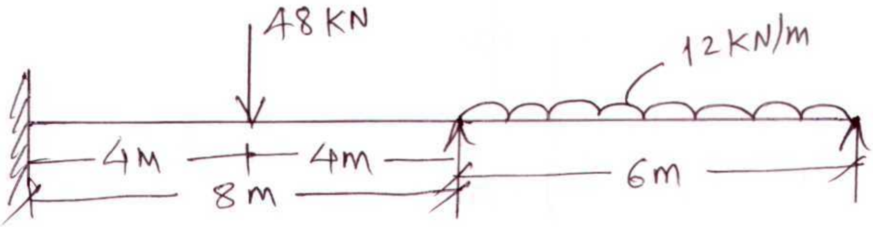
1) To Calculate S.S. Bending moments for Span AB & BC

$$M_{max} = \frac{WL^2}{8} = \frac{20 \times 5^2}{8} = 62.5\text{ kN·m}$$ 1 Mark

Question	Description	Marks.
	Applying Three moment theorem for span AB & BC	
	$\cancel{M_A}^{0}(L_1) + 2M_B(L_1+L_2) + \cancel{M_C}^{0}(L_2) = -\left[\frac{6A_1\bar{x}_1}{L_1} + \frac{6A_2\bar{x}_2}{L_2}\right]$	1 Mark
	$M_A = M_C = 0$ for simple supports.	
	$A_1 = \frac{2}{3}bh = \frac{2}{3} \times 5 \times 62.5 = 208.33\ kN \cdot m^2$	
	$\therefore A_2 = 208.33\ kN \cdot m^2$	
	$\bar{x}_1 = \bar{x}_2 = \frac{5}{2} = 2.5\ m.$	
	$\therefore 2M_B(5+5) = -\left[\frac{6 \times 208.33 \times 2.5}{5} + \frac{6 \times 208.33 \times 2.5}{5}\right]$	1 Mark
	$20M_B = -1249.98$	
	$\boxed{M_B = 62.5\ kN \cdot m}$ – Support moment.	1 Mark
Q.4(b)	Draw deflected shape of two span continuous beam ABC, fixed at 'A' and hinged at 'C' Continuous over B. The beam AB is subjected to UDL over entire span.	
		
	for sketch of beam —	2 marks
	for deflected shape and slopes —	2 Marks

Question	Description	Marks
Q.4 (c)	A beam ABC is simply supported at A, B & C. Span AB and BC are of lengths 4m and 6m respectively. AB carries a Central point load of 50 KN and BC carries a UDL of 15 KN/m over the entire span. Calculate support moment at B using three moment theorem.	
	 S.S. BMD	
	1) Calculate S.S. Bending moments. for span AB $M_{max} = \frac{WL}{4} = \frac{50 \times 4}{4} = 50\ KN.m$ for span BC $M_{max} = \frac{WL^2}{8} = \frac{15 \times 6^2}{8} = 67.5\ KN.m$	1 Mark
	2) Applying Three moment theorem for span AB & BC $M_A \times L_1 + 2M_B(L_1+L_2) + M_C(L_2) = -\left[\frac{6A_1\bar{X}_1}{L_1} + \frac{6A_2\bar{X}_2}{L_2}\right]$	1 Mark
	$M_A = M_C = 0$ - Simple supports. $A_1 = \frac{1}{2} \times 4 \times 50 = 100\ KN.m^2$ $A_2 = \frac{2}{3} bh = \frac{2}{3} \times 6 \times 67.5 = 270\ KN.m^2$ $\bar{X}_1 = \frac{4}{2} = 2m \quad \bar{X}_2 = \frac{6}{2} = 3m$	1 Mark

Question	Description	Marks.
	$0 \times (L_1) + 2M_B(6+4) + \cancel{M}\, 0\,(6) = -\left[\frac{6\times100\times2}{4} + \frac{6\times270\times3}{6}\right]$	
	$20M_B = [300 + 810]$	
	$\boxed{M_B = 55.5\ kN{\cdot}m}$ – Support moment.	1 Mark.
Q.4d)	Define Carryover factor and stiffness factor with respect to moment distribution method.	
	<u>Carryover Factor</u> :- The ratio of moment produced at a joint to the moment applied at the other joint, without displacing it. is called as Carryover factor.	2 Marks
	<u>Stiffness Factor</u> :- It is the moment required to rotate the end, while acting on it, through a unit angle without translation of the far end. OR The moment required at the simply supported end of a beam so as to produce unit rotation at that end without translation of the either end is called stiffness factor	2 Marks.
Q.4(e)	A Continuous beam of uniform flexural rigidity is fixed at A and supported over B & C such that AB = 8m, BC = 6m. A UDL of 12 kN/m acts on AB and a point load of 48 kN acts at the centre of BC. Calculate distribution factor using MDM.	



Joint	Member	stiffness Factor	Total stiffness	Distribution Factor	
B	BA	$K_{BA} = \frac{4EI}{L} = \frac{4EI}{8}$ $K_{BA} = 0.5EI$	$\Sigma K_B = K_{BA} + K_{BC}$	$DF_{BA} = \frac{K_{BA}}{\Sigma K_B}$ $DF_{BA} = \frac{0.5EI}{1.0EI}$ $DF_{BA} = 0.5$	2 Mark
	BC	$K_{BC} = \frac{3EI}{L} = \frac{3EI}{6}$ $K_{BC} = 0.5EI$	$\Sigma K_B = 0.5EI + 0.5EI$ $\Sigma K_B = 1.0EI$	$DF_{BC} = \frac{K_{BC}}{\Sigma K_B}$ $DF_{BC} = \frac{0.5EI}{1.0EI}$ $= 0.5$	2 Mark

$\therefore$ $DF_{BA} = 0.50$

$DF_{BC} = 0.50$

Q.4 (f) Explain the procedure of Moment Distribution Method.

Procedure -

1) Assume all the supports are to be fixed and find fixed end moments (FEM) acting at the joints by using Hardy Cross sign convention. ½ Mark

2) Calculate the stiffness factors K and distribution factors (Df) for all members by considering the intermediate support as a joint. ½ Mark

3) Prepare moment distribution table consisting of Distribution Factors, fixed end moments. 1 Mark

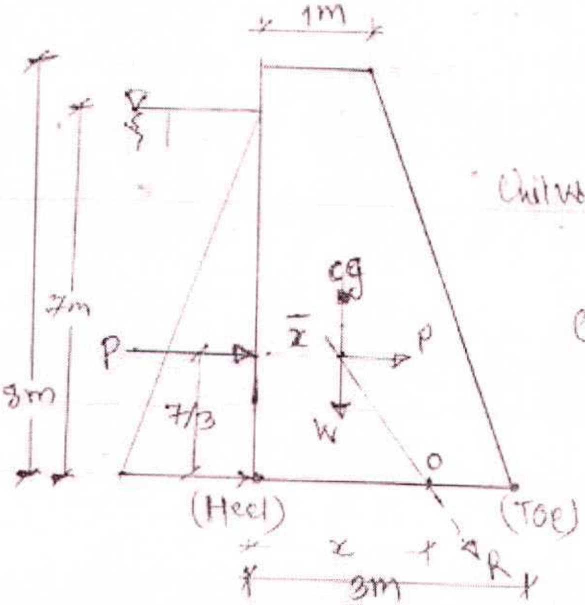
4) Release the joints by applying an equal & opposite moment in case of simply support. take carryover to the opposite joints for fixed end & continuous end. 1 Mark

5) The method of balancing at the joints & carryover to opposite joints should be done till it gives the required degree of accuracy of final moments. 1 Mark.

The final moments are obtained by the algebraic sum of end moments of all members mentioned in table.

Q.5 Attempt Any TWO

Q.5a) Given



Unit wt. of water = $10 kN/m^3$

Unit wt. of masonary = $22 kN/m^3$

Considering length of dam = 1m

Soln →

1) Self wt. of Dam :-

$$W = 22\left[\frac{1.5+3}{2} \times 8\right]1 = \underline{396 \text{ kN}}$$ (1M)

Acting at CG of section,

$$\bar{x} = \frac{(1.5)^2 + (1.5 \times 3) + (3)^2}{3[1.5+3]} = \underline{1.17 \text{ m}}$$ (1M)

2) Force due to water

$$P = (10 \times 7)\frac{7}{2} = \underline{245 \text{ kN}}$$ acts $\frac{7}{3}$ from base of dam. (1M)

3) Point where resultant cuts the base,

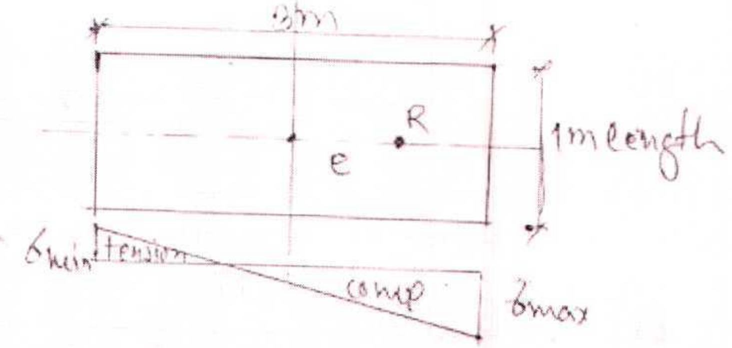
Taking moment @ 'O'

$$\Sigma M_{@O} = 0 = +(245 \times 7/3) - (396(x - 1.17))$$ (1M)

$\therefore$ $\underline{x = 2.61 m}$

$\therefore$ Eccentricity $e = x - b/2 = 2.61 - \frac{3}{2} = \underline{1.11 \text{ m}}$ (1M)

Dam section at base



(1M)

4)

stresses develops at base

At toe side

$$\sigma_{max} = \frac{396}{3}\left[1 + \frac{6(1.11)}{3}\right]$$

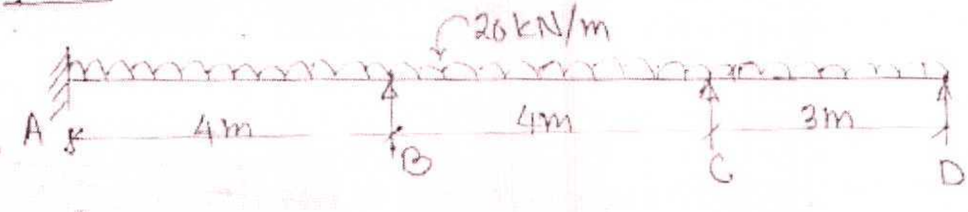
$\therefore \sigma_{max} = \underline{\underline{+425.40}}\ kN/m^2$ (comp) (1M)

& At heel side

$$\sigma_{min} = \frac{396}{3}\left[1 - \frac{6(1.11)}{3}\right]$$

$\therefore \sigma_{min} = \underline{\underline{-161.40}}\ kN/m^2$ (Tension) (1M)

Q5b) Given



Soln →

1) Calculation of Distribution Factor (DF)

Joint	Member	Relative Stiffness	Total Stiff.	D.F.	
B	BA	$\frac{4EI}{4} = EI$	2EI	0.5	(½M)
	BC	$\frac{4EI}{4} = EI$		0.5	(½M)
C	CB	$\frac{4EI}{4} = EI$	2EI	0.5	(½M)
	CD	$\frac{3EI}{3} = EI$		0.5	(½M)

2) Calculation of fixed end Moments (FEM)

$M_{AB} = \frac{-20 \times (4)^2}{12} = -26.67$ kN·m (anticlockwise) (½M)

$M_{BA} = +\frac{20 \times (4)^2}{12} = +26.67$ kN·m (clockwise)

$M_{BC} = \frac{-20 \times (4)^2}{12} = -26.67$ kN·m (anticlockwise) (½M)

$M_{CB} = \frac{+20 \times (4)^2}{12} = +26.67$ kN·m (clockwise)

$M_{CD} = \frac{-20 \times (3)^2}{12} = -15$ kN·m (anticlockwise) (½M)

$M_{DC} = \frac{+20 \times (3)^2}{12} = +15$ kN·m (clockwise)

3) Calculation of sagging BM

$M^{+}_{AB} = M^{+}_{BC} = \frac{20 \times (4)^2}{8} = 40$ kN·m (½M)

& $M^{+}_{CD} = \frac{20(3)^2}{8} = 22.5$ kN·m (½M)

4) Calculation of support moment by MDM.

Joint	A	B		C		D
D.F.	0	0.5	0.5	0.5	0.5	0
FEM	-26.67	+26.67	-26.67	+26.67	+15	+15
Release D					-7.5	-15
Initial Mom.	-26.67	+26.67	-26.67	+26.67	-22.5	0
Balance				-2.085	-2.085	
C.O.			-1.043			
Balance		+0.521	+0.521			
C.O.	0.261			+0.261		
Balance				-0.130	-0.13	
CO			-0.065			
Balance		+0.033	+0.033			
Final Moment	-26.41	+27.22	-27.22	+24.72	-24.72	0

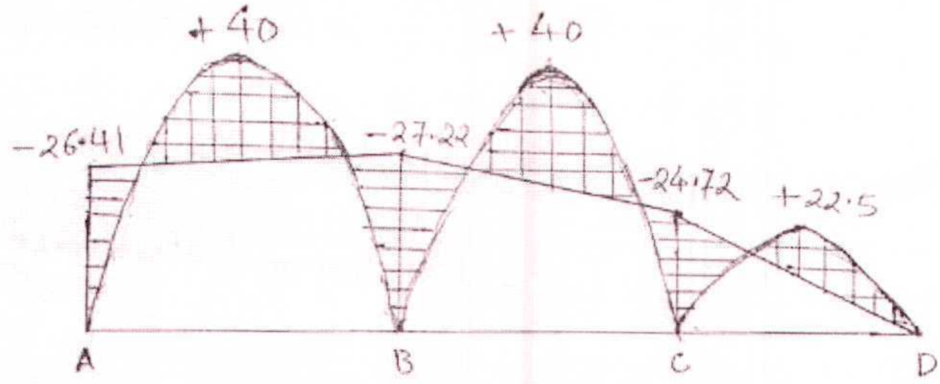
(½M)

(½M)

(½M)

(1M)

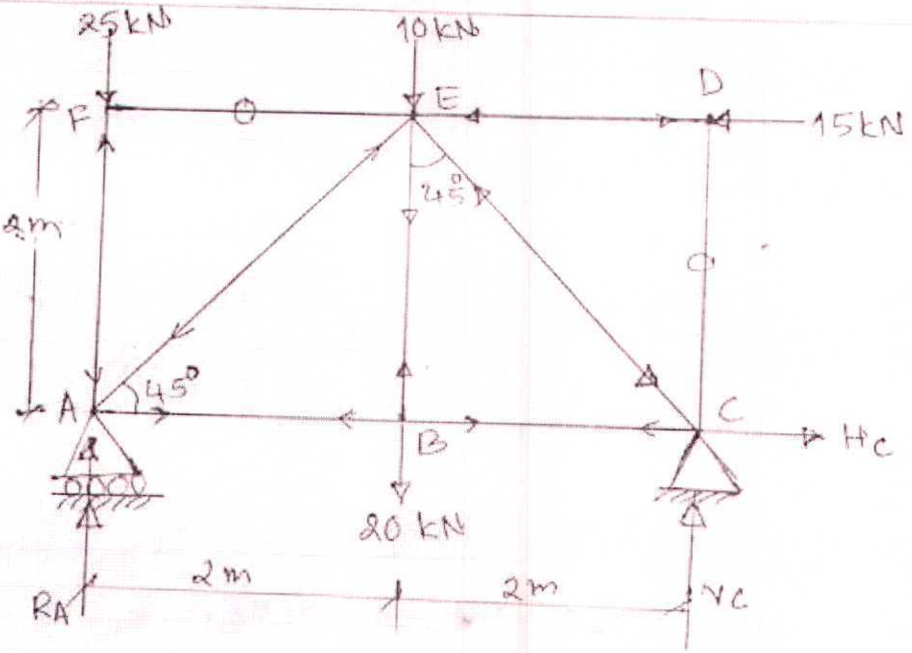
5)



(1M)

BMD (kN·m) [+ Sagging, − Hogging]

Q.5C) Given



(½M)

1) Using Equations of equilibrium, support reactions of frame, -

Taking moment from Pt. A

$\Sigma M@A = 0 = 25\times0 + (10+20)2 - (15\times2) - (V_C\times4)$

$\therefore \quad \underline{V_C = 7.5\text{kN}}$ (½M)

$\Sigma F_x = 0 = +H_C - 15 = 0$

$\therefore \quad \underline{H_C = +15\text{ kN}}$ (½M)

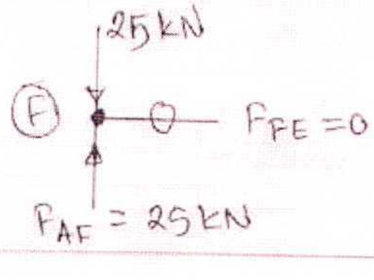
& $\Sigma F_y = 0 = R_A + V_C - 25 - 10 - 20 = 0$

$\therefore \quad \underline{R_A = 47.5\text{ kN}}$ (½M)

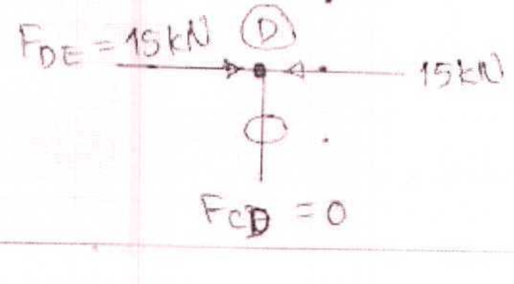
2) Using method of Joint forces in Frame

As all joints are in equilibrium using conditions of equilibrium at every joint

I) At jt F

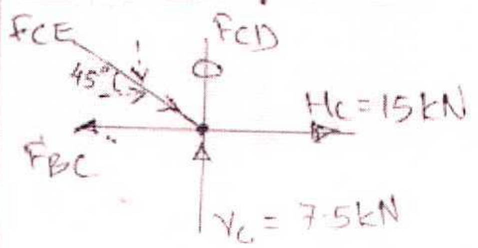


II) At jt D



(1M)

iii) At jt C



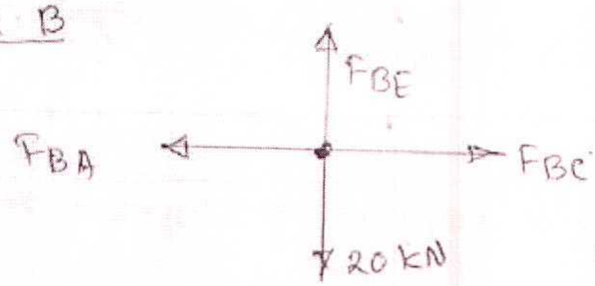
$F_{CE} \sin 45 = 7.5$

$\therefore$ $\underline{F_{CE} = 10.61 \text{ kN}}$ (1M)

& $F_{BC} = 15 + F_{CE} \cos 45$

$\therefore$ $\underline{F_{BC} = 22.5 \text{ kN}}$

iv) At jt B

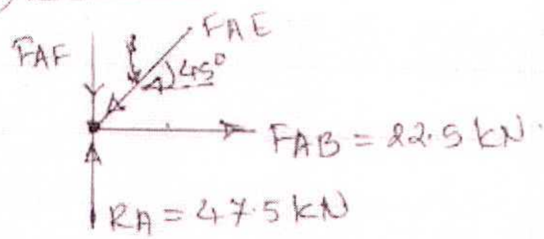


$\underline{F_{BE} = 20 \text{ kN}}$

$F_{BC} = F_{BA}$ (1M)

$\therefore$ $\underline{F_{BA} = 22.5 \text{ kN}}$

v) At jt A



$F_{AE} \cos 45 = 22.5$

$\therefore$ $\underline{F_{AE} = 31.82 \text{ kN}}$ (1M)

Member	Force (kN)	Nature
AB	22.5	Tension
BC	22.5	Tension
CD	0	—
DE	15	compression
FE	0	—
AF	25	Compression
AE	31.82	Compression
BE	20	Tension
CE	10.61	Compression

(2M)

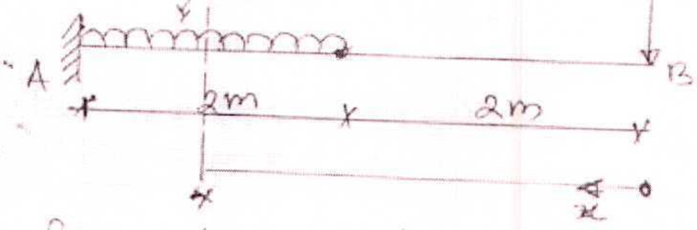
Q.No.	Question and Model Answers	Marks

Q.6 Attempt Any TWO

Q.6 a) Given

$EI = 2 \times 10^{?}\ kN \cdot mm^2$

$EI = 20\ kN \cdot m^2$



Soln → Take free end as origin and consider a section xx at a distance 'x' from free end as shown in fig.

$$EI\left(\frac{d^2y}{dx^2}\right) = M_x$$

$$= -40 \times x \;\Big|\; -20\frac{(x-2)^2}{2} \qquad (1M)$$

Integrating wrt x,

$$EI\left(\frac{dy}{dx}\right) = -\frac{40x^2}{2} + C_1 \;\Big|\; -\frac{20}{2}\cdot\frac{(x-2)^3}{3}$$

$$\therefore EI\left(\frac{dy}{dx}\right) = -20x^2 + C_1 \;\Big|\; -3.33(x-2)^3 \;\ldots\ldots (1) \qquad (\tfrac{1}{2}M)$$

Integrating w.r.t x,

$$\therefore EI(y) = -20\frac{x^3}{3} + C_1x + C_2 \;\Big|\; -3.33\frac{(x-2)^4}{4}$$

$$\therefore EI(y) = -6.67x^3 + C_1x + C_2 \;\Big|\; -0.833(x-2)^4 \;\ldots (2) \qquad (\tfrac{1}{2}M)$$

Where C_1 & C_2 are integration constants.

Using Boundary Conditions find C_1 & C_2, -

At fixed end $\frac{dy}{dx} = 0$ at $x = 4m$.

$$\therefore EI(0) = -20(4)^2 + C_1 \;\Big|\; -3.33(4-2)^3 \qquad (\tfrac{1}{2}M)$$

$$\therefore \underline{C_1 = +346.64} \qquad (\tfrac{1}{2}M)$$

& $y = 0$ at $x = 4m$

$$\therefore EI(0) = -6.67(4)^3 + (346.64 \times 4) + C_2 \;\Big|\; -0.833(4-2)^4 \qquad (\tfrac{1}{2}M)$$

$$\therefore \underline{C_2 = -946.56} \qquad (\tfrac{1}{2}M)$$

Max. slope & deflection occures at free end.

At $x = 0m$ using eqⁿ (1) valid upto 1st part (1m)

$$EI\left(\frac{dy}{dx}\right) = -20(0)^2 + 346.64$$

$\therefore$ Max. slope $\theta_B = \frac{346.64}{20} = 17.332$ rad. (1m)

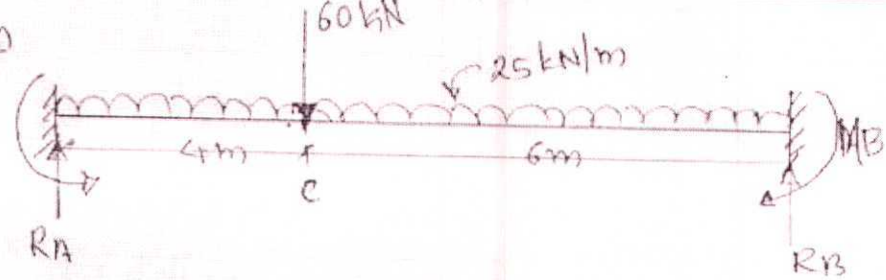
Using eqⁿ (2) valid upto 1st part at $x = 0m$, - (1m)

$$EI(y) = -6.67(0)^3 + (346.64 \times 0) + (-946.56)$$

$$\therefore \quad y_B = \frac{-946.56}{20}$$

Max. deflection $y_B = -47.328$ m (1m)

Q.6 b) Given



Solⁿ →

1) From first principle, fixed end moment (Hogging) is at bellow,

$$M_A = -\frac{25(10)^2}{12} - \frac{60(4)(6)^2}{(10)^2} = -294.73 \text{ kN·m} \quad (1m)$$

$$M_B = -\frac{25(10)^2}{12} - \frac{60(4)^2(6)}{(10)^2} = -265.93 \text{ kN·m} \quad (1M)$$

2) Calculation of Reactions using conditions of Eqm,

$$\Sigma M @ A = 0 = -294.73 + 265.93 + (60 \times 4) + \left(25 \times 10 \times \frac{10}{2}\right) - (R_B \times 10)$$

Q.No.	Question and Model Answers	Marks

$\therefore \quad R_B = 146.12\ kN$ (1M)

& $\Sigma F_y = 0 = R_A + R_B - 60 - (25 \times 10) = 0$ (1M)

$\therefore \quad R_A = 163.88\ kN$

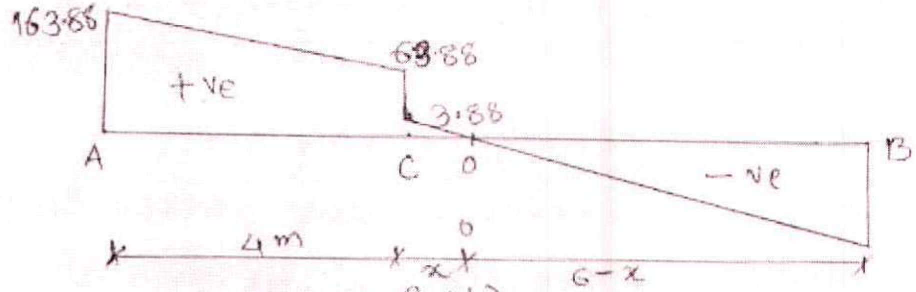
3) S.F. calculations.

SF @ A = +163.88 kN.

SF @ C without 60 kN = 163.88 − (25×4) = 63.88 kN

SF @ C with 60 kN = 63.88 − 60 = 03.88 kN.

SF @ B = −146.12 kN.



SFD (kN) (1M)

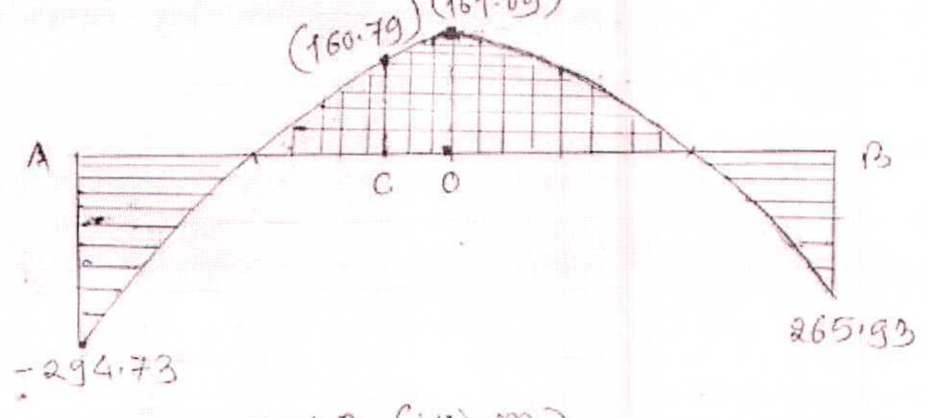
Position of pt 'O' from 'C' $\quad \frac{x}{3.88} = \frac{(6-x)}{146.12} \quad \therefore x = 0.155m$

4) BM. calculations

Net BM @ C $= -294.73 + (163.88 \times 4) - \left(25 \times 4 \times \frac{4}{2}\right) = 160.79$ kN·m (1M)

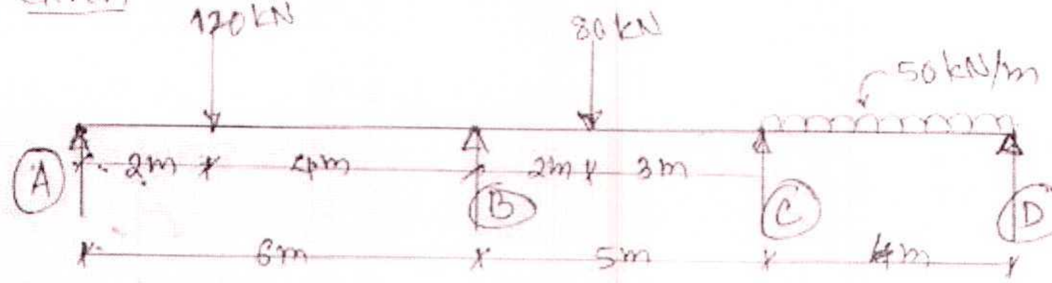
Net BM @ O $= -294.73 + (163.88 \times 4.155) - \left(25 \times \frac{(4.155)^2}{2}\right)$ (1M)

$= 161.09$ kN·m



BMD (KN·m) (1M)

Q.6c) Given

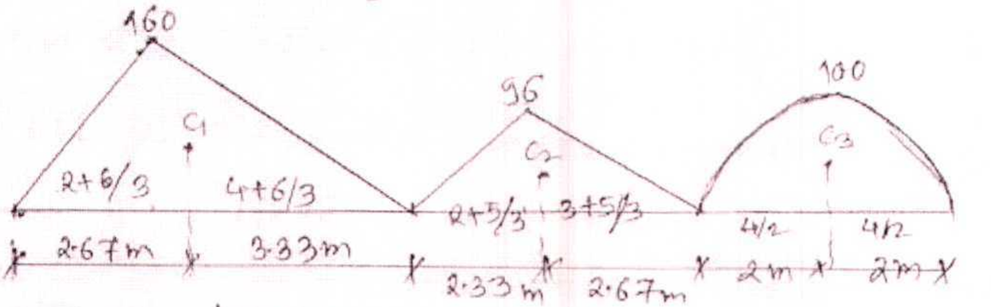


Soln → 1) Calculation for sagging moment (+ve)

for span AB $= \frac{120 \times 2 \times 4}{6} = 160$ kN·m (½ M)

for span BC $= \frac{80 \times 2 \times 3}{5} = 96$ kN·m (½ M)

for span CD $= \frac{50 \times 4^2}{8} = 100$ kN·m (½ M)



(½ M)

$a_1 = \frac{1}{2} \times 6 \times 160$ $\quad a_2 = \frac{1}{2} \times 5 \times 96$ $\quad a_3 = \frac{2}{3} \times 4 \times 100$ (½ M)

$a_1 = 480$ $\quad a_2 = 240$ $\quad a_3 = 266.67$ (½ M)

Sagging moment area diagram.

2) Using Three moment Theorem for Pair span.

$\boxed{\text{PAIR A-B-C}}$

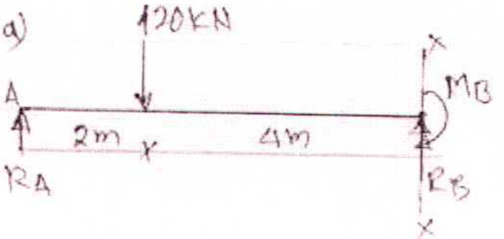
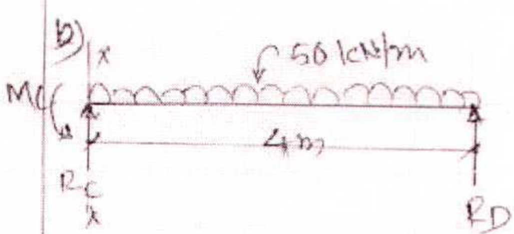
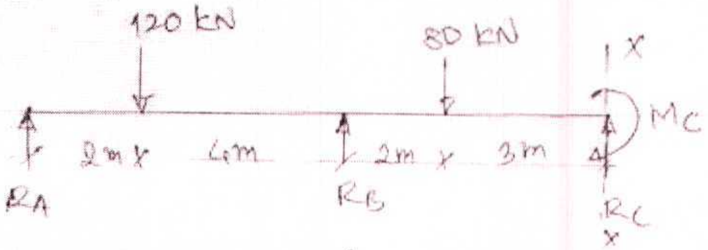
$$(M_A \times 6) + 2M_B(6+5) + (M_C \times 5) = -6\left[\left(\frac{480 \times 2.67}{6}\right) + \left(\frac{240 \times 2.67}{5}\right)\right]$$ (½ M)

As support A is s/s $\because M_A = 0$

$$22 M_B + 5 M_C = -2050.56 \quad \cdots\cdots (1)$$ (½ M)

$\boxed{\text{PAIR B-C-D}}$

$$(M_B \times 5) + 2M_C(5+4) + (M_D \times 4) = -6\left[\left(\frac{240 \times 2.33}{5}\right) + \left(\frac{266.67 \times 2}{4}\right)\right]$$ (½ M)

Q.No.	Question and Model Answers	Marks
	As support 'D' is s/s hence $M_D = 0$,	(½ M)
	$\therefore \quad 5M_B + 18M_C = -1471.05 \quad \cdots \quad (2)$	
	Solving eq^n (1) & (2)	
	$M_B = \underline{\underline{-79.67}}$ kN·m — -ve sign indicates	(½ M)
	& $M_C = \underline{\underline{-59.58}}$ kN·m Hogging BM.	(½ M)
	3) Using method of parts for finding support reactions.	
	a)  BM at section xx. $(R_A \times 6) - (120 \times 4) = -79.67$ $\underline{R_A = 66.72 \text{ kN}}$	(½ M)
	b)  BM at section xx $(R_D \times 4) - \frac{50 \times (4)^2}{2} = -59.58$ $\underline{R_D = 85.11 \text{ kN}}$	(½ M)
	c) 	
	BM @ section xx, $(R_A \times 11) + (R_B \times 5) - (120 \times 9) - (80 \times 3) = -59.58$	
	$\therefore \quad \underline{R_B = 105.3 \text{ kN}}$	(½ M)
	d) $\Sigma F_y = 0 = R_A + R_B + R_C + R_D - 120 - 80 - (50 \times 4)$	
	$\therefore \quad \underline{R_D = 142.87 \text{ kN}}$	(½ M)