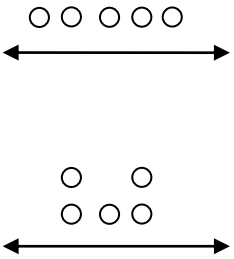
Template: Study Material

<table>
<tr><td colspan="3"><Basic Mathematics>: <22103>: <BMS>: <Statistics>: <UO_ 5.2>: <Study Material></td></tr>
<tr><td><Mrs.M.R.Abhang></td><td><8/7/2020></td><td><Mr.A.D.Wandhekar></td></tr>
<tr><td>Key words
Mean deviation,
Standard deviation</td><td>Learning Objective:
Calculate mean and standard deviation of discrete and grouped data related to the given simple engineering problem.</td><td rowspan="2">Diagram/ Picture
The more spread out a data distribution is, the greater its S.D.
 
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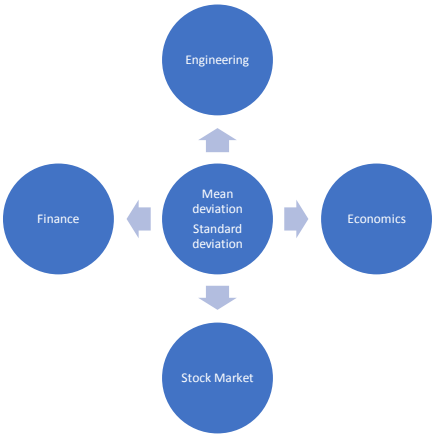
<tr><td>Key Questions

What is mean?
What is deviation?</td><td>Concept Map
 
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Explanation of Concept

Mean Deviation

Mean Deviation for raw data

i) Mean deviation about mean = $\frac{\Sigma|x_i - \bar{x}|}{N} = \frac{\Sigma |d_i|}{N}$

where $\bar{x}$ = mean of N observations.

1) Calculate the mean deviation about the mean of the following data.

3, 6, 5, 7, 10, 12, 15, 18

Solution :

Given data is raw data

$$\text{Mean} = \bar{x} = \frac{\Sigma x_i}{N} = \frac{3+6+5+7+10+12+15+18}{8}$$

$$= \frac{76}{8} = 9.5$$

x_i	$\lvert d_i \rvert = \lvert x_i - \bar{x} \rvert$
3	6.5
6	3.5
5	4.5
7	2.5
10	0.5
12	2.5
15	5.5
18	8.5
	$\Sigma \lvert d_i \rvert = 34$

$$\text{M.D.} = \frac{\Sigma |d_i|}{N} = \frac{34}{8}$$

$$\text{M.D.} = 4.25$$

Mean Deviation for Discrete (ungrouped) Frequency Distribution

$$\text{M.D. about mean} = \frac{\Sigma f_i|x_i - \bar{x}|}{\Sigma f_i} = \frac{\Sigma f_i|d_i|}{N}$$

where $N = \Sigma f_i$ and $|d_i| = |x_i - \bar{x}|$

1) Calculate the Mean deviation from Mean of the following data.

x_i	10	11	12	13	14
f_i	3	12	18	12	3

Solution: Mean deviation about mean :

x_i	f_i	$f_i x_i$	$\lvert d_i \rvert = \lvert x_i - \bar{x} \rvert$	$f_i \lvert d_i \rvert$
10	3	30	2	6
11	12	132	1	12
12	18	216	0	0
13	12	156	1	12
14	03	42	2	06
	$N = \Sigma f_i = 48$	$\Sigma f_i x_i = 576$		$\Sigma f_i \lvert d_i \rvert = 36$

$$\text{Mean} = \bar{x} = \frac{\Sigma f_i x_i}{N} = \frac{576}{48} = 12$$

Key Definitions/ Formulas

For Raw data:

M.D. $= \frac{\Sigma |d_i|}{N}$

S.D. $= \sqrt{\frac{\Sigma d_i^2}{N}}$

For ungrouped or grouped data:

M.D.$= \frac{\Sigma f_i |d_i|}{N}$

S.D.$= \sqrt{\frac{\Sigma f_i d_i^2}{N}}$

$$\text{M.D.} = \frac{\sum f_i | d_i |}{N} = \frac{36}{48} = 0.75$$

Mean deviation for grouped data :

$$\text{M.D.} = \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i} = \frac{\sum f_i | d_i |}{N}$$

where x_i = Mid - value or centre value

$\bar{x}$ = Mean

$N = \sum f_i$

Solved Examples:

1) Find the mean deviation from mean of the following distribution.

Marks	0-10	10-20	20-30	30-40	40-50
No. of students	5	8	15	16	6

Solution

C.I.	f_i	x_i	$f_i x_i$	$\|d_i\| = \|x_i - \bar{x}\|$	$f_i \| d_i \|$
0-10	5	5	25	22	110
10-20	8	15	120	12	96
20-30	15	25	375	2	30
30-40	16	35	560	8	128
40-50	6	45	270	18	108
	$N = \sum f_i =$ 50		$\Sigma f_i x_i = 1350$		$\Sigma f_i \| d_i \| =$ 472

$$\text{Mean} = \bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{1350}{50} = 27$$

$$\text{Mean Deviation about mean} = \frac{\sum f_i | d_i |}{N}$$

$$= \frac{472}{50}$$

$$= 9.44$$

Standard Deviation (S.D.)

S.D. for raw data :

$$\text{S.D.} = \sigma = \sqrt{\frac{\sum \left(x_i - \bar{x}\right)^2}{N}} = \sqrt{\frac{\sum d_i^2}{N}}$$

$\bar{x}$ = Mean

$d_i = |x_i - \bar{x}|$

N = Total number of observations

1)Calculate S.D. of the following data :

25, 50, 30, 70, 42, 36, 48, 34, 60

Solution : Given data is raw data

$$\bar{x} = \frac{\sum x_i}{N} = \frac{25+50+30+70+42+36+48+34+60}{9} = \frac{395}{9}$$

$\bar{x}$ = **43.888**

x_i	$d_i = x_i - \bar{x}$	d_i^2
25	–18.88	356.45
50	6.12	37.45
30	–13.88	192.65
70	26.12	682.25
42	– 1.88	3.53
36	– 7.88	62.09
48	4.12	16.97
34	– 9.88	97.614
60	16.12	259.85
		$\Sigma d_i^2 = 1708.85$

Standard Deviation $= \sqrt{\frac{d_i^2}{N}}$

$= \sqrt{\frac{1708.85}{9}}$

$= \sqrt{189.872}$

S.D. = 0.538

Standard deviation for ungrouped data :

$$S.D. = \sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N}}$$

where $N = \Sigma f_i$

$\bar{x}$ = Mean $= \frac{\Sigma f_i x_i}{N}$

$$d_i = x_i - \bar{x}$$

Solved Examples:

1) Find the standard deviation of the following frequency distribution.

x_i	6	7	8	9	10	11	12
f_i	3	6	9	13	8	5	4

Solution :

x_i	f_i	$f_i x_i$	$d_i = x_i - \bar{x}$	d_i^2	$f_i d_i^2$
6	3	18	–3	9	27
7	6	42	–2	4	24
8	9	72	–1	1	9
9	13	117	0	0	0
10	8	80	1	1	8
11	5	55	2	4	20
12	4	48	3	9	36
	$N = \Sigma f_i =$ 48	$\Sigma f_i x_i =$ 432			$\Sigma f_i d_i^2 = 124$

Mean $= \bar{x} = \frac{\Sigma f_i x_i}{N}$

$= \frac{432}{48} = 9$

S.D. $= \sqrt{\frac{\Sigma f_i d_i^2}{N}}$

$= \sqrt{\frac{124}{48}}$

$= \sqrt{2.583}$

Solved word Problem

Calculate M.D.
1, 2, 3, 4, 5

Sol.

$\bar{x} = \frac{1+2+3+4+5}{5}$

=3

x_i	$\lvert d_i \rvert$
1	**2**
2	**1**
3	**0**
4	**1**
5	**2**
	$\sum \lvert d_i \rvert$ =6

M.D.= $\frac{\Sigma \lvert d_i \rvert}{N}$

$= \frac{6}{5}$

=1.2

=**1.607**

S.D. for grouped data

$$S.D. = \sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N}}$$

where, $N = \Sigma f_i$

$d_i = | x_i - \bar{x} |$

$\bar{x}$ =Mean = $\frac{\Sigma f_i x_i}{N}$

x_i = Mid - Value

Solved Example:

Calculate Mean and Standard deviation for the following data.

Class	20-29	30-39	40-49	50-59	60-69	70-79
Frequency	10	15	30	20	15	10

Solution : Here class interval are not continuous making them continuous as :

Class	**Cont. Class**	$\mathbf{f_i}$	$\mathbf{x_i}$	$\mathbf{f_i x_i}$	$\mathbf{d_i = \lvert x_i - \bar{x} \rvert}$	$\mathbf{d_i^2}$	$\mathbf{f_i d_i^2}$
20-29	19.5-29.5	10	24.5	245	24.5	600.25	6002.5
30-39	29.5-39.5	15	34.5	517.5	14.5	210.25	3153.75
40-49	39.5-49.5	30	44.5	1335	4.5	20.25	607.5
50-59	49.5-59.5	20	54.5	1090	5.5	30.25	605
60-69	59.5-69.5	15	64.5	967.5	15.5	240.25	3603.75
70-79	69.5-79.5	10	74.5	745	25.5	650.25	6502.5
		$N = \Sigma f_i =$ 100		$\Sigma f_i x_i =$ 4900			$\Sigma f_i d_i^2 =$ 20475

$$\text{Mean} = \bar{x} = \frac{\Sigma f_i x_i}{N}$$

$$= \frac{4900}{100}$$

$$= 49$$

Standard Deviation =σ

$$= \sqrt{\frac{\Sigma f_i d_1^2}{N}}$$

$$= \sqrt{\frac{20475}{100}}$$

$$= 14.309$$

	Application of Concept/ Examples in real life Standard deviation is a widely used measure of variability. S.D. measures the spread of a data distribution. It is used in various fields such as economics, finance, engineering etc.	Link to YouTube/ OER/ video **khanacademy.org**
Key Take away from this UO : Mean deviation Standard deviation		