22232

3 Hours / 70 Marks Seat No. | | | | | | | | |

Instructions : (1) All Questions are *compulsory.*

(2) Illustrate your answers with neat sketches wherever necessary.

(3) Figures to the right indicate full marks.

(4) Assume suitable data, if necessary.

(5) Use of Non-programmable Electronic Pocket Calculator is permissible.

Marks

1. Attempt any FIVE of the following : **10**

(a) If $f(x, y) = x \sin y + y \sin x + xy$, find $\frac{\partial f}{\partial x}$.

(b) If $f(x, y) = x^4 \sin y + y^4 \cos x + x^3$

Find $\frac{\partial^2 f}{\partial x \partial y}$

(c) Find rank of matrix, $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$.

(d) Find the eigen values of the matrix

$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

(e) Find the value of p, if the vectors $\bar{a} = p\bar{i} + 5\bar{j} + \bar{k}$ & $\bar{b} = 2\bar{i} - \bar{j} - 5\bar{k}$ are perpendicular to each other.

(f) Find the projection of $\bar{a} = 2\bar{i} + \bar{j} + \bar{k}$ on $\bar{b} = \bar{i} + 3\bar{j} + \bar{k}$.

(g) Construct forward difference table for the following data :

x	0	1	2	3	4	5	6
$f(x)$	1	3	11	31	69	131	223

2. Attempt any THREE of the following : **12**

(a) Examine $f(x, y) = x^3 - y^2 - 3x$ for maximum and minimum values.

(b) Examine the following linear system of equation for consistency and solve it if consistent.

$$4x - 2y + 6z = 8$$
$$x + y - 3z = -1$$
$$15x - 3y + 9z = 21$$

(c) Reduce the following matrix to Normal form and hence find its rank.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}.$$

(d) Find the angle between the vectors

$$\bar{a} = 2\bar{i} + 2\bar{j} + \bar{k} \text{ and } \bar{b} = 3\bar{i} + 6\bar{j} + 2\bar{k}$$

3. Attempt any THREE of the following : **12**

(a) If $f(x, y) = x^3 + y^3 - 3axy$ then

show that $\dfrac{\partial^2 f}{\partial x \partial y} = \dfrac{\partial^2 f}{\partial y \partial x}$.

(b) Find inverse of following matrix by elementary transformation

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

(c) Determine the value of λ for which the system of linear equation

$3x + 2y + 4z = 3$, $x + y + z = \lambda$ and $5x + 4y + 6z = 15$ are consistent; also find corresponding solution.

(d) Find extreme value of $f(x, y) = xy$ subject to condition $x + y = 10$, using Lagrange method of undetermined multipliers.

4. Attempt any THREE of the following : **12**

(a) For which values of k the system of equation

$$x + 2y - z = 0$$
$$3x + (k + 7)\, y - 3z = 0$$
$$2x + 4y + (k - 3)\, z = 0$$

will possess non-trivial solution.

(b) Find a vector of magnitude $\sqrt{3}$ units and perpendicular to the vectors

$\bar{a} = 2\bar{i} + \bar{j} - 3\bar{k}$ and $\bar{b} = \bar{i} - 2\bar{j} + \bar{k}$

(c) The scalar product of the vector

$\bar{a} = \bar{i} + \bar{j} + \bar{k}$ with a unit vector along the sum of vectors

$\bar{b} = 2\bar{i} + 4\bar{j} - 5\bar{k}$ and $\bar{c} = \lambda\bar{i} + 2\bar{j} + 3\bar{k}$ is equal to one. Find value of λ and hence find the unit vector along $\bar{b} + \bar{c}$.

(d) Find the Eigen values and Eigen vectors of a matrix

$$A = \begin{bmatrix} 14 & -10 \\ 5 & -1 \end{bmatrix}$$

(e) If $\bar{a} = \bar{i} - 2\bar{j} + 3\bar{k}$, $\bar{b} = 2\bar{i} + \bar{j} - \bar{k}$ and $\bar{c} = \bar{j} + \bar{k}$

Find vector $\bar{a} \times (\bar{b} \times \bar{c})$

5. Attempt any TWO of the following : **12**

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 0$ using suitable interpolation for the following data :

x	0	1	2	3	4	5
y	4	8	15	7	6	2

(b) (i) Given

x	10	15	20
f(*x*)	14	18	28

Estimate f(12) using Newton's Forward difference interpolation formula.

(ii) Prove that

$$\Delta \log f(x) = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right].$$

(c) Solve the following linear programming problem graphically to find optimal solution

Maximize $Z = 2x + 5y$

Subject to $x + 2y \leq 16$

$5x + 3y \leq 45$

$x, y \geq 0$.

6. Attempt any TWO of the following : **12**

(a) Given the square of integers in the following data. Find the value of $(13)^2$, using extrapolation.

x	3	5	7	9
y	9	25	49	81

(b) (i) Evaluate $\int_2^7 \frac{1}{x}\,dx$ by using trapezoidal rule and by dividing the interval [2, 7] in to five equal sub-intervals.

(ii) Evaluate $\int_0^2 e^{-x}\,dx$ by using Simpson's one-third rule, by dividing the interval (0, 2) into four equal sub-intervals.

(c) Solve the following linear programming problem using simplex method to find optimal solution :

Maximize $Z = 12x_1 + 16x_2$

Subject to $10x_1 + 20x_2 \leq 120$

$8x_1 + 8x_2 \leq 80$

$x_1, x_2 \geq 0$
