16172

3 Hours / 100 Marks

Seat No.

Instructions –

- (1) All Questions are Compulsory.
- (2) Answer each next main Question on a new page.
- (3) Illustrate your answers with neat sketches wherever necessary.
- (4) Figures to the right indicate full marks.
- (5) Assume suitable data, if necessary.
- (6) Use of Non-programmable Electronic Pocket Calculator is permissible.
- (7) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Attempt any $\underline{\text{TEN}}$ of the following:

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- a) Evaluate $\int \frac{x}{(x+1)(x-2)} dx$
- b) Evaluate $\int x \cdot \sin x \, dx$
- c) Evaluate $\int \frac{1}{[1+\cos(2x)]} dx$
- d) Evaluate $\int_{4}^{9} \frac{1}{x^{3/2}} dx$
- e) Find order and degree of differential equation $\frac{d^2y}{dx^2} = \left[y + \frac{dy}{dx}\right]^{\frac{3}{2}}$
- f) Find area under the curve $y = \sin x$ from x = 0 to $x = 2\pi$. with x axis.

g) Evaluate :
$$\int_{1}^{5} \frac{x}{4+x^2} dx$$

h) Find 'k' if
$$\int_{-1}^{1} (3x^2 + 2x + k) dx = 0$$

- i) Find slope of tangent to the curve $x^2 + y^2 = 25$ at point (-3, 4).
- j) Form the differential equation if $x^2 + cy^2 = 4$
- k) Two fair dice are rolled. Determine the probability of getting two sixes.
- 1) A card is taken out at random from pack of well shuffled 52 cards. What is the probability that it is a heart?

2. Attempt any FOUR of the following:

16

a) Evaluate :
$$\int_{0}^{1} x^{3} \cdot \tan^{-1} x \, dx$$

b) Evaluate :
$$\int_{0}^{\pi/2} \frac{\tan x}{1 + \tan x} dx$$

c) Evaluate :
$$\int_{0}^{0} \frac{(2x+1)}{(x+1)(x+2)(x+3)} dx$$

d) Evaluate :
$$\int \frac{dx}{9\cos^2 x + 4\sin^2 x}$$

e) Evaluate :
$$\int \frac{dx}{x \log x \cdot \log(\log x)}$$

f) Find radius of curvatures of curve
$$\sqrt{x} + \sqrt{y} = \sqrt{a}$$
 at $\left(\frac{a}{4}, \frac{a}{4}\right)$

3. Attempt any FOUR of the following:

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a) Evaluate :
$$\int_{1}^{5} \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x} + \sqrt[3]{x+3}} dx$$

b) Evaluate :
$$\int \sec^3 x \ dx$$

c) Find area of circle
$$x^2 + y^2 = 4$$

d) Find maximum and minimum values of $x^3 - 9x^2 + 24x$

- e) Find equation of normal and tangent to the curve $x = \frac{1}{t}$, $y = t \frac{1}{t}$ when t = 2.
- f) Evaluate $\int \frac{e^x}{(e^x 1)(e^x + 1)} dx$

4. Attempt any FOUR of the following:

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- a) Determine a and b such that slope of curve $2y^3 = ax^2 + b$ at (1, -1) is same as the slope of x + y = 0.
- b) A beam is bent to form the curve $y = 2 \sin x \sin 2x$. Find the radius of curvatures of the beam at this point $x = \frac{\pi}{2}$.
- c) Evaluate $\int \sin^{-1} (3x 4x^3) dx$
- d) Evaluate $\int_{0}^{\pi/4} \log(1 + \tan x) dx$
- e) Solve $\frac{dy}{dx} = e^{x-y} \cdot x^2$
- f) Solve $x \frac{dy}{dx} y = x^2 \cdot \cos^2 x$.

5. Attempt any FOUR of the following:

16

- a) Solve $(2xy + y^2) dx + (x^2 + 2xy + \sin y) dy = 0$
- b) Solve $(x + y + 1)^2 \frac{dy}{dx} = 1$
- c) Solve $\sec^2 x \cdot \tan y \, dx + \sec^2 y \tan x \cdot dy = 0$ for $y = \frac{\pi}{4}$ when $x = \frac{\pi}{4}$
- d) Solve $xy dy = (x^2 + y^2) dx$
- e) The probability of getting an item defective is 0.005. What is the probability that exactly 3 items in a sample of 200 are defective? (give $e^{-1} = 0.3679$)
- f) A problem is given to the three students A, B and C whose chances of solving it are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ respectively. If they attempt to solve a problem independently, find probability that the problem is solved by at least one of them.

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[4]

Marks

6. Attempt any FOUR of the following:

16

- a) Solve $(1 + y^2) dx = (\tan^{-1} y x) dy$
- b) Find area bounded between curves $y = x^2 + 1$ and line y = 2x + 1
- c) A room has 3 electric lamps. From a collection of 15 electric bulbs of which only 10 are good, 3 bulbs are selected at random and put in the lamps. Find the probability. That the room is lighted by at least one of the bulbs.
- d) In a test of 2000 electric bulbs, it was found that the life of particular make was normally distributed with average life of 2040 Hr and standard deviation of 60 Hr. Estimate the no.s of bulbs likely to burn for:
 - (i) between 1920 Hr and 2160 Hr.
 - (ii) More than 2150 Hr.

Given area at z=2 is 0.4772 and area at z=1.83 is 0.4664

- e) If 20% of the bolts produced by a machine are defective, determine the probability that out of 4 bolts drawn:
 - (i) one is defective
 - (ii) at the most two are defective.
- f) If $P(A) = \frac{1}{2}$, $P(B') = \frac{2}{3}$ and $P(A \cup B) = \frac{2}{3}$ Find $P(A' \cap B')$ and $P(\frac{A}{B})$