SUMMER – 2013 EXAMINATION

MODEL ANSWER

Subject: BASIC MATHEMATICS

Subject Code: 17104

Important Instructions to examiners:

- The model answer shall be the complete solution for each and every question on the question paper.
- Numerical shall be completely solved in a step by step manner along with step marking.
- All alternative solutions shall be offered by the expert along with self-explanatory comments from the expert.
- In case of theoretical answers, the expert has to write the most acceptable answer and offer comments regarding marking scheme to the assessors.
- In should offer the most convincing figures / sketches / circuit diagrams / block diagrams / flow diagrams and offer comments for step marking to the assessors.
- In case of any missing data, the expert shall offer possible assumptions / options and the ensuing solutions along with comments to the assessors for effective assessment.
- In case of questions which are out of the scope of curricular requirement, the expert examiner shall solve the question and mention the marking scheme in the model answer. However, the experts are requested to submit their clear cut opinion about the scope of such question in the paper separately to the coordinator.
- Experts shall cross check the DTP of the final draft of the model answer prepared by them.

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1)		Attempt any ten of the following:		20
	(a)	Find k, if $\begin{vmatrix} 2 & -k & 7 \\ 3 & -4 & 13 \\ 8 & -11 & 33 \end{vmatrix}=0$		
	Ans.	$\begin{vmatrix} 2 & -k & 7 \\ 3 & -4 & 13 \\ 8 & -11 & 33 \end{vmatrix}=0$		
		$2(-4\times33+11\times13)+k(3\times33-13\times8)+7(-33+32)=0$	1	
		$22+k(-5)-7=0$		
		$15-5k=0$	½	
		$15=5k$		02
		$k=3$	½	
	(b)	Find A if , $2A+3\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}=\begin{bmatrix} 5 & 7 \\ 6 & 3 \end{bmatrix}$		
	Ans.	$2A+3\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}=\begin{bmatrix} 5 & 7 \\ 6 & 3 \end{bmatrix}$		
		$2A+\begin{bmatrix} 3 & 9 \\ 6 & 15 \end{bmatrix}=\begin{bmatrix} 5 & 7 \\ 6 & 3 \end{bmatrix}$	½	
		$2A=\begin{bmatrix} 5 & 7 \\ 6 & 3 \end{bmatrix}-\begin{bmatrix} 3 & 9 \\ 6 & 15 \end{bmatrix}$		
		$2A=\begin{bmatrix} 2 & -2 \\ 0 & -12 \end{bmatrix}$	1	02
		$A=\begin{bmatrix} 1 & -1 \\ 0 & -6 \end{bmatrix}$	½	
	c)	Prove that the matrix $\begin{bmatrix} 1 & 4 \\ 6 & 9 \end{bmatrix}$ is non-singular matrix.		
	Ans.	Consider, $\begin{vmatrix} 1 & 4 \\ 6 & 9 \end{vmatrix}$		
		$=9-24$		02
		$=-15\neq0$	2	
		$\therefore$ Given matrix is non-singular matrix.		
	d)	If $A=\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}, B=\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ and $C=\begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix}$ then show that $AB=AC$		
	Ans.	$AB=\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1.		$=\begin{bmatrix}12 & 16\\12 & 16\end{bmatrix}$	1	
		$AC=\begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}\begin{bmatrix}4 & 5\\2 & 3\end{bmatrix}$		
		$=\begin{bmatrix}12 & 16\\12 & 16\end{bmatrix}$	1	02
		$\therefore$ AB=AC		
	(e)	Resolve into partial fractions: $\frac{2x+3}{x^2-2x-3}$		
	Ans.	$\frac{2x+3}{x^2-2x-3}=\frac{2x+3}{(x-3)(x+1)}$		
		Let,		
		$\therefore \frac{2x+3}{(x-3)(x+1)}=\frac{A}{x-3}+\frac{B}{x+1}$	½	
		$2x+3=(x+1)A+(x-3)B$		
		Put $x=-1$ $9=4A$		
		$A=\frac{9}{4}$	½	
		Put $x=3$ $1=-4B$		
		$B=\frac{-1}{4}$	½	02
		$\frac{2x+3}{(x-3)(x+1)}=\frac{9/4}{x-3}+\frac{-1/4}{x+1}=\frac{1}{4}\left[\frac{9}{x-3}-\frac{1}{x+1}\right]$	½	
		OR		
		$\frac{2x+3}{x^2-2x-3}=\frac{2x+3}{(x-3)(x+1)}$		
		Let		
		$\therefore \frac{2x+3}{(x-3)(x+1)}=\frac{A}{x-3}+\frac{B}{x+1}$	½	
		$2x+3=(x+1)A+(x-3)B$ $2x+3=(A+B)x+A-3B$		
		By equating equal power coefficients $A+B=2$ $A-3B=3$ By solving above equations:		

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1.		$\therefore A=\frac{9}{4}$	½	
		$\therefore B=\frac{-1}{4}$	½	
		$\frac{2x+3}{(x-3)(x+1)}=\frac{9/4}{x-3}+\frac{-1/4}{x+1}$	½	02
	(f)	Prove that $2\sin^2\theta=1-\cos2\theta$		
	Ans.	$1-\cos2\theta=1-\cos(\theta+\theta)$	1	
		$=1-(\cos\theta.\cos\theta-\sin\theta.\sin\theta)$		
		$=1-(\cos^2\theta-\sin^2\theta)$		
		$=1-\cos^2\theta+\sin^2\theta$		
		$=\sin^2\theta+\sin^2\theta$	½	
		$=2\sin^2\theta$	½	02
	(g)	Define Allied Angles		
	Ans.	Allied Angle : Any two angles whose sum or difference is either zero or an integral multiple of $\frac{\pi}{2}$ are called as allied angle.	2	02
	(h)	If $2\cos60^0.\cos10^0=\cos A+\cos B$, then find A and B		
	Ans.	$\cos60^0.\cos10^0=\cos A+\cos B$		
		$\cos(60^0+10^0)+\cos(60^0-10^0)=\cos A+\cos B$	1	
		$\cos70^0+\cos50^0=\cos A+\cos B$		
		$\therefore A=70^0$ and $B=50^0$	1	02
		OR		
		$2\cos60^0.\cos10^0=\cos A+\cos B$		
		$2\cos60^0.\cos10^0=2\cos\frac{A+B}{2}.\cos\frac{A-B}{2}$	1	
		$\frac{A+B}{2}=60^0$ and $\frac{A-B}{2}=10^0$	½	
		$A+B=120^0$		
		$A-B=20^0$		
		$\therefore A=70^0$ and $B=50^0$	½	02
	(i)	Evaluate without using calculator $\frac{\tan85^0-\tan40^0}{1+\tan85^0.\tan40^0}$		

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1.	Ans.	$\frac{\tan 85^0-\tan 40^0}{1+\tan 85^0.\tan 40^0}$	1	
		$=\tan\left(85^0-40^0\right)$	½	
		$=\tan 45^0$		
		$=1$	½	02
	(j)	Prove that $\cos^{-1}x=\sec^{-1}\left(\frac{1}{x}\right)$		
	Ans.	Let $\cos^{-1}x=\theta$		
		$\therefore \cos\theta=x$	½	
		$\therefore \sec\theta=\frac{1}{x}$	1	
		$\theta=\sec^{-1}\left(\frac{1}{x}\right)$	½	02
		$\cos^{-1}x=\sec^{-1}\left(\frac{1}{x}\right)$		
	(k)	Find the perpendicular distance between the point (3,4) and the line $3x+4y=5.$		
	Ans.	Perpendicular distance=$\left\lvert\frac{\mathrm{ax}_1+by_1+c}{\sqrt{a^2+b^2}}\right\rvert$		
		$=\left\lvert\frac{3(3)+4(4)-5}{\sqrt{3^2+4^2}}\right\rvert$	1	
		$=\left\lvert\frac{20}{5}\right\rvert$		
		=4 units	1	02
	(l)	Find the range of the following distribution: 3,6,10,1,15,16,21,19,18.		
	Ans.	Range =Largest value-Smallest value		
		=21-1	1	
		=20	1	02
2.		Attempt any four of the following:		16
	(a)	Solve the following equations by using Cramers rule: $x-y-2z=1, 2x+3y+4z=4, 3x-2y-6z=5$		
	Ans.	Let $D=\begin{vmatrix}1 & -1 & -2\\ 2 & 3 & 4\\ 3 & -2 & -6\end{vmatrix}$		
		$=1(-18+8)+1(-12-12)-2(-4-9)$	1	
		$=-8$		

2.		$D_x = \begin{vmatrix}1 & -1 & -2\\4 & 3 & 4\\5 & -2 & -6\end{vmatrix}$		
		$=1(-18+8)+1(-24-20)-2(-8-15)$		
		$=-8$	½	
		$D_y = \begin{vmatrix}1 & 1 & -2\\2 & 4 & 4\\3 & 5 & -6\end{vmatrix}$		
		$=1(-24-20)-1(-12-12)-2(10-12)$		
		$=-16$	½	
		$D_y = \begin{vmatrix}1 & -1 & 1\\2 & 3 & 4\\3 & -2 & 5\end{vmatrix}$		
		$=1(15+8)+1(10-12)+1(-4-9)$		
		$=8$	½	
		$x=\frac{D_x}{D}=\frac{-8}{-8}=1$	½	
		$y=\frac{D_y}{D}=\frac{-16}{-8}=2$	½	
		$z=\frac{D_z}{D}=\frac{8}{-8}=-1$	½	04
	(b)	Find x and y if $\begin{bmatrix}2 & -3\\1 & 5\end{bmatrix}\begin{bmatrix}x & -1 & 2\\1 & 0 & y\end{bmatrix}=\begin{bmatrix}2x-3 & -2 & 4-3y\\x+5 & -1 & 2+5y\end{bmatrix}$		
	Ans.	$\begin{bmatrix}2x-3 & -2 & 4-3y\\x+5 & -1 & 2+5y\end{bmatrix}=\begin{bmatrix}2x-3 & -2 & 4-3y\\x+5 & -1 & 2+5y\end{bmatrix}$	2	
		$\therefore 2x-3=2x-3 \quad , \quad 4-3y=4-3y$	2	
		and $x+5=x+5 \quad , \quad 2+5y=2+5y$		04
		In above all the cases finding the values of x and y are not possible. $\therefore$ it has no solution.		
	(c)	If $A=\begin{bmatrix}1 & 3 & 2\\-1 & 2 & 0\\4 & 0 & 3\end{bmatrix}, B=\begin{bmatrix}1 & 0 & 0\\1 & 2 & 0\\1 & 0 & 3\end{bmatrix}$ and $C=\begin{bmatrix}2 & 1 & 2\\2 & 2 & 1\\1 & 2 & 2\end{bmatrix}$ then find the matrix D such that 2A-3B-D=C		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
2.	Ans.	Given, $2A-3B-D=C$		
		$D=2A-3B-C$		
		$D=2\begin{bmatrix}1&3&2\\-1&2&0\\4&0&3\end{bmatrix}-3\begin{bmatrix}1&0&0\\1&2&0\\1&0&3\end{bmatrix}-\begin{bmatrix}2&1&2\\2&2&1\\1&2&2\end{bmatrix}$	1	
		$D=\begin{bmatrix}2&6&4\\-2&4&0\\8&0&6\end{bmatrix}-\begin{bmatrix}3&0&0\\3&6&0\\3&0&9\end{bmatrix}-\begin{bmatrix}2&1&2\\2&2&1\\1&2&2\end{bmatrix}$	1	
		$D=\begin{bmatrix}-3&5&2\\-7&-4&-1\\4&-2&-5\end{bmatrix}$	2	04
	(d)	If $A=\begin{bmatrix}2&3&-1\\1&0&4\end{bmatrix}$ and $B=\begin{bmatrix}-3&7\\-5&6\\-4&4\end{bmatrix}$, then show that $\left(\text{AB}\right)'=B'A'$		
	Ans.	Consider $AB=\begin{bmatrix}2&3&-1\\1&0&4\end{bmatrix}\begin{bmatrix}-3&7\\-5&6\\-4&4\end{bmatrix}$		
		$AB=\begin{bmatrix}-6-15+4&14+18-4\\-3-16&7+16\end{bmatrix}$	1	
		$AB=\begin{bmatrix}-17&28\\-19&23\end{bmatrix}$	½	
		$\left(AB\right)'=\begin{bmatrix}-17&-19\\28&23\end{bmatrix}$	½	
		$B'A'=\begin{bmatrix}-3&-5&-4\\7&6&4\end{bmatrix}\begin{bmatrix}2&1\\3&0\\-1&4\end{bmatrix}$	1	
		$B'A'=\begin{bmatrix}-17&-19\\28&23\end{bmatrix}$	1	04
		$\therefore\left(AB\right)'=B'A'$		
	(e)	Resolve into partial fractions: $\dfrac{x}{x^2+x-2}$		
	Ans.	$\dfrac{x}{x^2+x-2}=\dfrac{2x+3}{\left(x+2\right)\left(x-1\right)}$		
		Let,		
		$\therefore\dfrac{x}{\left(x+2\right)\left(x-1\right)}=\dfrac{A}{x+2}+\dfrac{B}{x-1}$	1	

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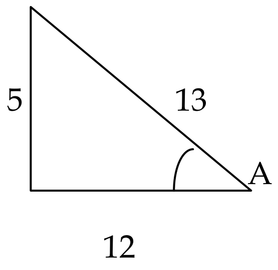
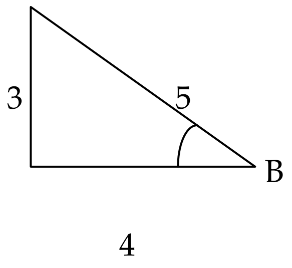
2.		$x=(x-1)A+(x+2)B$		
		Put $x=-2$		
		$-2=A(-3)$		
		$A=\frac{2}{3}$	1	
		Put $x=1$		
		$1=3B$		
		$B=\frac{1}{3}$	1	
		$\frac{x}{(x+2)(x-1)}=\frac{2/3}{x-3}+\frac{1/3}{x+1}$	1	04
	(f)	Resolve into partial fractions: $\frac{9}{(x-1)(x+2)^2}$		
	Ans.	$\frac{9}{(x-1)(x+2)^2}=\frac{A}{x-1}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$	½	
		$9=(x+2)^2A+(x+2)(x-1)B+(x-1)C$	½	
		Put $x=1$		
		$9=9A$		
		$\therefore$ A=1	½	
		Put$x=-2$		
		$9=-3C$		
		$\therefore C=-3$	½	
		Put $x=0$		
		$9=4A-2B-C$		
		$9=4(1)-2B+3$		
		$9=7-2B$		
		$2B=-2$		
		$\therefore B=-1$	1	
		$\frac{9}{(x-1)(x+2)^2}=\frac{1}{x-1}+\frac{-1}{x+2}+\frac{-3}{(x+2)^2}$	1	04
3.		Attempt any four of the following:		16
	(a)	Using matrix inversion method, solve the following equations:		
		$x+y+z=3, x+2y+3z=4, x+4y+9z=6$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3.	Ans.	Let $A=\begin{bmatrix}1&1&1\\1&2&3\\1&4&9\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix}$ and $B=\begin{bmatrix}3\\4\\6\end{bmatrix}$		
		Consider, $\|A\|=\begin{vmatrix}1&1&1\\1&2&3\\1&4&9\end{vmatrix}$		
		$=1(18-12)-1(9-3)+1(4-2)$		
		$=6-6+2$		
		$=2\neq 0$	½	
		$\therefore A^{-1}$exists		
		Matrix of minors= $\begin{bmatrix}\begin{vmatrix}2&3\\4&9\end{vmatrix}&\begin{vmatrix}1&3\\1&9\end{vmatrix}&\begin{vmatrix}1&2\\1&4\end{vmatrix}\\\begin{vmatrix}1&1\\4&9\end{vmatrix}&\begin{vmatrix}1&1\\1&9\end{vmatrix}&\begin{vmatrix}1&1\\1&4\end{vmatrix}\\\begin{vmatrix}1&1\\2&3\end{vmatrix}&\begin{vmatrix}1&1\\1&3\end{vmatrix}&\begin{vmatrix}1&1\\1&2\end{vmatrix}\end{bmatrix}$		
		$=\begin{bmatrix}6&6&2\\5&8&3\\1&2&1\end{bmatrix}$		
		matrix of cofactors $=\begin{bmatrix}6&-6&2\\-5&8&-3\\1&-2&1\end{bmatrix}$	1	
		Adj.$A=\begin{bmatrix}6&-5&1\\-6&8&-2\\2&-3&1\end{bmatrix}$	½	
		$A^{-1}=\dfrac{1}{\|A\|}.adj.A$		
		$=\dfrac{1}{2}\begin{bmatrix}6&-5&1\\-6&8&-2\\2&-3&1\end{bmatrix}$	½	
		$X=A^{-1}B$		
		$=\dfrac{1}{2}\begin{bmatrix}6&-5&1\\-6&8&-2\\2&-3&1\end{bmatrix}\begin{bmatrix}3\\4\\6\end{bmatrix}$	½	

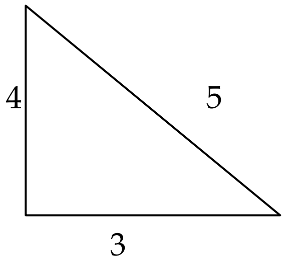
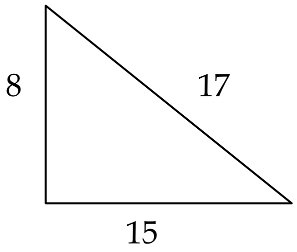
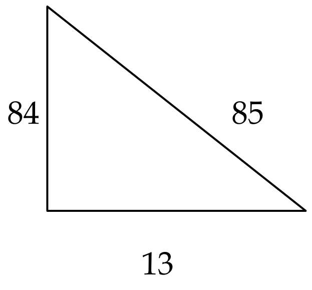
Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3.		$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ *OR*	1	04
		Matrix of cofactors can be evaluated by following method: $Matrix\ of\ cofactors = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$ $By\ c_{ij} = (-1)^{i+j}M_{ij}$ $c_{11} = (-1)^{1+1}\begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix} = 6$ $c_{12} = (-1)^{1+2}\begin{vmatrix} 1 & 3 \\ 1 & 9 \end{vmatrix} = -6$ $c_{13} = (-1)^{1+3}\begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} = 2$ $c_{21} = (-1)^{2+1}\begin{vmatrix} 1 & 1 \\ 4 & 9 \end{vmatrix} = -5$ $c_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 1 \\ 1 & 9 \end{vmatrix} = 8$ $c_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix} = -3$ $c_{31} = (-1)^{3+1}\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 1$ $c_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = -2$ $c_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1$ $\therefore Matrix\ of\ cofactors = \begin{bmatrix} 6 & -6 & 2 \\ -5 & 8 & -3 \\ 1 & -2 & 1 \end{bmatrix}$	1	
	(b) Ans.	Resolve into partial fractions: $\frac{x^3+2}{x^2-1}$ $x^2-1\overset{x}{\overline{)x^3+2}}$ x^3-x $-\ \ +$ $x+2$ $\therefore \frac{x^3+2}{x^2-1} = x + \frac{x+2}{x^2-1}$	1	
		Consider, $\frac{x+2}{x^2-1} = \frac{x+2}{(x+1)(x-1)}$ $\therefore \frac{x+2}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$	½	
		$x+2 = (x-1)A + (x+1)B$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3.		Put $x=-1$		
		$1=-2A$		
		$\therefore A=-\frac{1}{2}$	1	
		Put $x=1$		
		3=2B		
		$\therefore B=\frac{3}{2}$	1	
		$\therefore \frac{x+2}{(x+1)(x-1)}=\frac{-1/2}{x+1}+\frac{3/2}{x-1}$		
		$\therefore \frac{x^3+2}{x^2-1}=x+\frac{-1/2}{x+1}+\frac{3/2}{x-1}$	½	04
	(c)	Resolve into partial fractions: $\frac{e^x}{e^{2x}+4e^x+3}$		
	Ans.	put $e^x=t$		
		$\frac{t}{t^2+4t+3}=\frac{t}{(t+3)(t+1)}$	½	
		$\therefore \frac{t}{(t+3)(t+1)}=\frac{A}{t+3}+\frac{B}{t+1}$	½	
		$\therefore t=(t+1)A+(t+3)B$		
		put $t=-3$		
		$-3=-2A$		
		$\therefore A=\frac{3}{2}$	1	
		put $t=-1$		
		$-1=2B$		
		$\therefore B=-\frac{1}{2}$	1	
		$\therefore \frac{t}{(t+3)(t+1)}=\frac{3/2}{t+3}+\frac{-1/2}{t+1}$	½	
		$\therefore \frac{e^x}{(e^x+3)(e^x+1)}=\frac{3/2}{e^x+3}+\frac{-1/2}{e^x+1}$	½	04

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3.	(d)	In any ΔABC,prove that $\sin\frac{A+B-C}{2}+\sin\frac{B+C-A}{2}+\sin\frac{C+A-B}{2}=\cos A+\cos B+\cos C$		
	Ans.	$A+B+C=180$ $A+B=180-C$ $A+B-C=180-2C$ $\frac{A+B-C}{2}=90-C$	1	
		$\sin\left(\frac{A+B-C}{2}\right)=\sin\left(90-C\right)$ $\sin\left(\frac{A+B-C}{2}\right)=\cos C$	1	
		Similarly, $\sin\left(\frac{B+C-A}{2}\right)=\cos A$	½	
		$\sin\left(\frac{C+A-B}{2}\right)=\cos B$	½	
		$\therefore\sin\left(\frac{A+B-C}{2}\right)+\sin\left(\frac{B+C-A}{2}\right)+\sin\left(\frac{C+A-B}{2}\right)$ = cosA+cosB+cosC	1	04
	(e)	If A and B are both obtuse angles and $\sin A=\frac{5}{13}$ and $\cos B=-\frac{4}{5}$, then find $\sin(A+B)$. ′		
	Ans.	Given $\sin A=\frac{5}{13}$ $\cos B=-\frac{4}{5}$		
		 	1	
		$\therefore\cos A=\frac{12}{13}$ $\sin B=\frac{3}{5}$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3.		$\therefore$ A and B are obtuse		
		$\cos A=\frac{-12}{13}$, $\sin B=\frac{3}{5}$	1	
		$\therefore \sin(A+B)=\sin A.\cos B+\cos A.\sin B$		
		$=\frac{5}{13}.\frac{-4}{5}+\frac{-12}{13}.\frac{3}{5}$	1	
		$=\frac{-56}{65}$	1	04
		OR		
		Given $\sin A=\frac{5}{13}$		
		$\therefore \cos A=\pm\sqrt{1-\sin^2 A}$		
		$=\pm\sqrt{1-\frac{25}{169}}$		
		$=\pm\frac{12}{13}$		
		A and B are obtuse		
		$\therefore \cos A=-\frac{12}{13}$		
		and cosB=$\frac{-4}{5}$	1	
		$\sin B=\pm\sqrt{1-\cos^2 B}$		
		$=\pm\sqrt{1-\frac{16}{25}}$		
		$=\pm\frac{3}{5}$		
		$\sin B=\frac{3}{5}$ $\therefore$ B is obtuse	1	
		$\sin\left(A+B\right)=\sin A.\cos B+\cos A.\sin B$		
		$=\frac{5}{13}.\frac{-4}{5}+\frac{-12}{13}.\frac{3}{5}$	1	
		$=\frac{-56}{65}$	1	04
	(f)	Prove that $\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{8}{17}=\sin^{-1}\frac{84}{85}$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3.		$\sin A = \frac{4}{5}$		
		$\therefore \cos A = \sqrt{1-\sin^2 A}$		
		$= \sqrt{1-\frac{16}{25}}$		
		$= \frac{3}{5}$	1	
		and $\sin B = \frac{8}{17}$		
		$\therefore \cos B = \sqrt{1-\cos^2 B}$		
		$= \sqrt{1-\frac{64}{289}}$		
		$= \frac{15}{17}$	1	
		LHS=$A+B$		
		RHS=$\sin^{-1}\frac{84}{85}$		
		Consider,		
		$\sin(A+B) = \sin A.\cos B + \cos A.\sin B$		
		$= \frac{4}{5}.\frac{15}{17} + \frac{3}{5}.\frac{8}{17} = \frac{84}{85}$	1	
		$A+B = \sin^{-1}\frac{4}{5} + \sin^{-1}\frac{8}{17} = \sin^{-1}\frac{84}{85}\ A+B = \sin^{-1}\frac{84}{85}$	1	04
		OR		
		Let $\sin^{-1}\frac{4}{5} = A$, $\sin^{-1}\frac{8}{17}$ =B		
		$\therefore \sin A = \frac{4}{5}$, $\sin B = \frac{8}{17}$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3.		 	1	
		$\tan A=\frac{4}{3}$, $\tan B=\frac{8}{15}$ $A=\tan^{-1}\frac{4}{3}$, $B=\tan^{-1}\frac{8}{15}$ $\therefore \sin^{-1}\frac{4}{5}=\tan^{-1}\frac{4}{3}$, $\therefore \sin^{-1}\frac{8}{17}=\tan^{-1}\frac{8}{15}$	1	
		$\text{LHS}=\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{8}{17}$ $=\tan^{-1}\frac{4}{3}+\tan^{-1}\frac{8}{15}$ $=\tan^{-1}\left(\frac{\frac{4}{5}+\frac{8}{15}}{1-\frac{4}{5}.\frac{8}{15}}\right)$	1	
		$=\tan^{-1}\left(\frac{\frac{60+24}{45}}{\frac{45-32}{45}}\right)$ $=\tan^{-1}\left(\frac{84}{13}\right)$	½	
		$\text{RHS}=\sin^{-1}\frac{84}{85}$ Let $\sin^{-1}\frac{84}{85}=C$ $\therefore \sin C=\frac{84}{85}$ 		

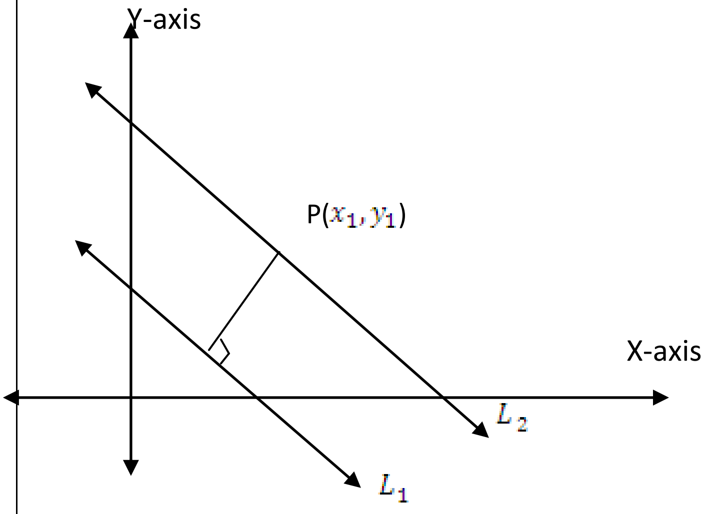
Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3.		$\therefore \tan C = \frac{84}{13}$		
		$C = \tan^{-1}\frac{84}{13}$		
		$\therefore \sin^{-1}\frac{84}{85} = \tan^{-1}\frac{84}{13}$		
		$\therefore$ RHS= $\tan^{-1}\frac{84}{13}$	½	04
		LHS=RHS		
4.		Attempt any four of the following:		16
	(a)	Prove that $\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A.\tan B}$		
	Ans.	$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$	½	
		$= \frac{sinAcosB + cosAsinB}{cosAcosB - sinAsinB}$	2	
		$= \frac{\frac{sinAcosB}{cosAcosB} + \frac{cosAsinB}{cosAcosB}}{\frac{cosAcosB}{cosAcosB} - \frac{sinAsinB}{cosAcosB}}$	1	
		$= \frac{tanA + tanB}{1 - tanAtanB}$	½	04
	(b)	**Prove that** $\cos 3A = 4\cos^3 A - 3\cos A$		
	Ans.	$cos3A = cos(2A + A)$		
		$= cos2AcosA - sin2AsinA$	1	
		$= (2\,cos^2 A - 1)cosA - 2sinAcosAsinA$	1	
		$= 2\,cos^3 A - cosA - 2\,sin^2 AcosA$		
		$= 2\,cos^3 A - cosA - 2\,(1 - cos^2 A)\,cosA$	1	
		$= 2\,cos^3 A - cosA - 2cosA + 2\,cos^3 A$		
		$= 4\,cos^3 A - 3cosA$	1	04

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4.	(c)	Without using calculator show that **$\sin 20^0.\sin 40^0.\sin 60^0.\sin 80^0=\frac{3}{16}$**		
	Ans.	$\sin 20^0.\sin 40^0.\sin 60^0.\sin 80^0 = \frac{1}{2}\sin 20^0(2\sin 40^0\sin 80^0)\sin 60^0$		
		$=\frac{1}{2}\sin 20^0(\sin 40^0-\sin 120^0)\sin 60^0$	1	
		$=\frac{1}{2}\sin 20^0\left(1-2\sin^2 20^0-\left(-\frac{1}{2}\right)\right)\sin 60^0$	1	
		$=\frac{1}{2}\sin 20^0\left(\frac{3}{2}-2\sin^2 20^0\right)\sin 60^0$		
		$=\frac{1}{2}\sin 20^0\frac{1}{2}(3-4\sin^2 20^0)\left(\frac{\sqrt{3}}{2}\right)$	½	
		$=\frac{\sqrt{3}}{8}(3\sin 20^0-4\sin^3 20^0)$		
		$=\frac{\sqrt{3}}{8}\sin(3\times 20^0)$	½	
		$=\frac{\sqrt{3}}{8}\sin(60^0)$		
		$=\frac{\sqrt{3}}{8}\frac{\sqrt{3}}{2}$		
		$=\frac{3}{16}$	1	04
		Note: give appropriate marks for another method.		
	(d)	Prove that **$\frac{\sin 8x-\sin 5x}{\cos 7x+\cos 6x}=\sin x+\cos x.\tan\frac{x}{2}$**		
	Ans.	$\frac{\sin 8x-\sin 5x}{\cos 7x+\cos 6x}=\sin x+\cos x.\tan\frac{x}{2}$		
		$=\frac{2\cos\frac{8x+5x}{2}.\sin\frac{8x-5x}{2}}{2\cos\frac{7x+6x}{2}.\cos\frac{7x-6x}{2}}$	1	

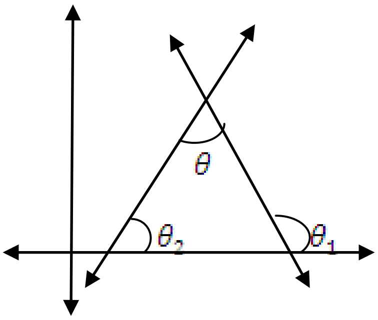
Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4.		$\frac{2\cos\frac{13x}{2}.\sin\frac{3x}{2}}{2\cos\frac{13x}{2}.\cos\frac{x}{2}}$		
		$=\frac{\sin\frac{3x}{2}}{\cos\frac{x}{2}}$		
		$=\frac{\sin\left(x+\frac{x}{2}\right)}{\cos\frac{x}{2}}$	1	
		$=\frac{\sin x\cos\frac{x}{2}+\cos x\sin\frac{x}{2}}{\cos\frac{x}{2}}$	1	
		$=\sin x+\cos x.\tan\frac{x}{2}$	1	04
	(e)	Prove that		
		$\mathbf{\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{13}=\tan^{-1}\left(\frac{2}{9}\right)}$		
	Ans.	$\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{13}=\tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{13}}{1-\frac{1}{7}\frac{1}{13}}\right)$	2	
		$=\tan^{-1}\left(\frac{\frac{13+7}{91}}{\frac{91-1}{91}}\right)$	1	
		$=\tan^{-1}\left(\frac{20}{90}\right)$	½	
		$=\tan^{-1}\left(\frac{2}{9}\right)$	½	04
	(f)	Prove that		
		$\mathbf{\sin^{-1}\frac{3}{5}-\cos^{-1}\frac{5}{13}=\cos^{-1}\frac{56}{65}}$		
	Ans.	$Let\ \sin^{-1}\frac{3}{5}=A$ $\cos^{-1}\frac{5}{13}=B$		
		$\therefore sinA=\frac{3}{5}$ $\therefore cosA=\frac{5}{13}$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4.		$cosA = \sqrt{1-\sin^2 A}$ $= \sqrt{1-\frac{9}{25}}$ $= \frac{4}{5}$ $sinA = \sqrt{1-\cos^2 A}$ $= \sqrt{1-\frac{25}{169}}$ $= \frac{12}{13}$	2	
		$L.H.S = A - B$ $Consider\ \ cos(A-B) = cosAcosB + sinAsinB$ $= \frac{4}{5}.\frac{5}{13} + \frac{3}{5}.\frac{12}{13}$ $= \frac{20}{65} + \frac{36}{65}$ $\therefore cos(A-B) = \frac{56}{65}$	1	
		$A - B = \cos^{-1}\left(\frac{56}{65}\right)$ $\therefore \sin^{-1}\frac{3}{5} - \cos^{-1}\frac{5}{13} = \cos^{-1}\frac{56}{65}$	1	04
		OR *Note : This example can be solved by another method , see question no.3 (f) on page.no.15.*		
5.		Attempt any Four of the following:		
	(a)	Prove that $\boldsymbol{\frac{sin4\theta + sin2\theta}{1 + cos2\theta + cos4\theta} = tan2\theta}$		
	Ans.	$\frac{sin4\theta + sin2\theta}{1 + cos2\theta + cos4\theta} = \frac{2sin2\theta cos2\theta + sin2\theta}{1 + cos4\theta + cos2\theta}$	1	
		$= \frac{2sin2\theta cos2\theta + sin2\theta}{2\cos^2 2\theta + cos2\theta}$	1	
		$= \frac{sin2\theta(2cos2\theta + 1)}{cos2\theta(2cos2\theta + 1)}$	1	
		$= tan2\theta$	1	04

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5.	(b)	**Prove that** $\boldsymbol{cosC + cosD = 2cos\left(\frac{C+D}{2}\right)cos\left(\frac{C-D}{2}\right)}$		
		Since $cos(A+B) + cos(A-B) = 2cosA.cosB \qquad -(1)$	1	
		$Put \qquad A+B=C \qquad A-B=D$		
		solve simultaneously		
		$\therefore A = \frac{C+D}{2}$	1	
		$\therefore \ B = \frac{C-D}{2}$	1	
		From (1) $\quad cosC + cosD = 2cos\left(\frac{C+D}{2}\right)cos\left(\frac{C-D}{2}\right)$	1	04
	(c)	**If x and y are both positive, then show that**		
		$\boldsymbol{\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)}$		
	Ans.	Let $\tan^{-1}x = A \qquad \tan^{-1}y = B$		
		$\therefore tanA = x \qquad tanB = y$	1	
		$\therefore \tan^{-1}x - \tan^{-1}y = A - B$ -(1)		
		$\therefore \tan^{-1}\left(\frac{x-y}{1+xy}\right) = \tan^{-1}\left(\frac{\text{tanA} - \text{tanB}}{\text{1+tanAtanB}}\right)$	1	
		$= \tan^{-1}(\tan(A-B))$	1	
		$= A - B$		
		$= \tan^{-1}x - \tan^{-1}y$ from(1)	1	04
		$\therefore \tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$		
	(d)	**Show that the perpendicular distance between two parallel lines**		
		ax+by+$\boldsymbol{c_1 = 0}$ **and ax+by+**$\boldsymbol{c_2 = 0}$ **is** $\boldsymbol{\left\lvert\frac{c_1 - c_2}{\sqrt{a^2+b^2}}\right\rvert}$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5.	Ans.	Let $L_1=ax+by+c_1=0$ and $L_2=ax+by+c_2=0$ $Let\ (x_1,y_1)$ be a point on the line L_2 $ax_1+by_1+c_2=0$ $\therefore\ ax_1+by_1=-c_2$	1	
				
		now the perpendicular distance from (x_1,y_1) on L_1 is $p=\left\lvert\frac{ax_1+by_1+c_1}{\sqrt{a^2+b^2}}\right\rvert$	1	
		$=\left\lvert\frac{-c_2+c_1}{\sqrt{a^2+b^2}}\right\rvert$	1	
		$==\left\lvert\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right\rvert$ OR $=\left\lvert\frac{c_2-c_1}{\sqrt{a^2+b^2}}\right\rvert$	1	04
	(e)	**Find the acute angle between the lines 3x − y+4=0 and 2x+y− 3=0**		
	Ans.	For 3x − y+4=0 $slope=m_1=\frac{-a}{b}=\frac{-3}{-1}=3$	1	
		For 2x+y − 3=0 $slope=m_2=\frac{-a}{b}=\frac{-2}{1}=-2$	1	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5.		$\tan\theta=\left\lvert\frac{m_1-m_2}{1+m_1m_2}\right\rvert$		
		$=\left\lvert\frac{3+2}{1+3.(-2)}\right\rvert$	1	
		$=1$		
		$\theta=\tan^{-1}1$		
		$\therefore \theta=45^0 \ or \ \frac{\pi}{4}$	1	04
	(f)	**Find the equation of line passing through the point of intersection of lines $x+y=0$ and $2x-y=9$ and through the point $(2,5)$**		
	Ans.	$x+y=0$		
		$\underline{2x-y=9}$		
		$3x \quad = 9$		
		$\therefore x=3$	1	
		$\therefore y=-3$	1	
		$\therefore$ Point of intersection $=(3,-3)$		
		$\therefore$ equation is		
		$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$		
		$\therefore \frac{y-5}{-3-5}=\frac{x-2}{3-2}$	1	
		$\therefore 8x+y-21=0$	1	04
		OR		
		$\therefore$ Point of intersection $=(3,-3)$	2	
		$\therefore$ Slope=m=$\frac{y_2-y_1}{x_2-x_1}=\frac{-3-5}{3-2}=-8$	1	
		$\therefore$ Equation is $y-y_1=m(x-x_1)$		
		$y-5=-8(x-2)$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5.		$\therefore 8x+y-21=0$	1	04
6.		Attempt any Four of the following:		
	(a)	**If m_1 and m_2 are the slpes of the two lines, then prove that the angle between two lines is $\theta=\tan^{-1}\left\|\frac{m_1-m_2}{1+m_1\,m_2}\right\|$**		
	Ans.	Let θ_1 = Angle of inclination of L_1 θ_2 = Angle of inclination of L_2 slope of L_1 is $m_1=\tan\theta_1$ slope of L_2 is $m_2=\tan\theta_2$  	1	
		from fig. $\theta=\theta_1-\theta_2$	1	
		$\therefore \tan\theta=\tan(\theta_1-\theta_2)$ $=\frac{\tan\theta_1-\tan\theta_2}{1+\tan\theta_1\tan\theta_2}$	1	
		$=\frac{m_1-m_2}{1+m_1\,m_2}$ $\therefore \theta=\tan^{-1}\left(\frac{m_1-m_2}{1+m_1\,m_2}\right)$ For angle to be acute. $\theta=\tan^{-1}\left\|\frac{m_1-m_2}{1+m_1\,m_2}\right\|$	1	04
	(b)	**Find the equation of the lines passing through the point of intersection of lines $2x+3y=13$ and $5x-y=7$ and perpendicular to the line $3x-y+17=0$**		
	Ans.	$2x+3y=13 \quad -(1)$ $5x-y=7 \quad -(2)$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6.				
		$2x+3y=13$		
		$\underline{15x-3y=21}$		
		$\therefore 17x \quad = 34$		
		$\therefore x=2$	1	
		$y=3$	1	
		$\therefore$ point of intersection=(2,3)		
		slope of the line $3x-y+17=0$ is		
		$m_1=\frac{-a}{b}=\frac{-3}{-1}=3$		
		$\therefore$ slope of the required line is,		
		$m=-\frac{1}{m_1}=-\frac{1}{3}$	½	
		$\therefore$ equation is		
		$y-y_1=m(x-x_1)$		
		$y-3=-\frac{1}{3}(x-2)$	1	
		$\therefore x+3y-11=0$	½	04
	(c)	Find the range and coefficient of range of the following data:		

Age (in years)	10-19	20-29	30-39	40-49	50-59	60-69	70-79
Frequency	03	61	223	137	53	19	04

		Range=Upper boundary of the last class − lower boundary of first class		
		$=79.5-9.5$	1	
		$=70$	1	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6.		$\text{coefficient of range} = \dfrac{\text{Range}}{\text{sum of the highest and lowest value}}$		
		$= \dfrac{70}{79.5+9.5}$	1	
		$= \dfrac{70}{89}$ or 0.787	1	04
	(d)	Find mean deviation from median for the following data:		

Class interval	0−10	10−20	20−30	30−40	40−50
Freauency	5	8	15	16	6

Ans.

Class	x_i	f_i	$f_i x_i$	$D_i = \lvert x_i - \bar{x}\rvert$	$f_i D_i$
0−10	5	5	25	22	110
10−20	15	8	120	12	96
20−30	25	15	375	2	30
30−40	35	16	560	8	12
40−50	45	6	270	18	108
		50	1350		472

1

$\bar{x} = \dfrac{\sum f_i x_i}{N} = \dfrac{1350}{50} = 27$ 1

$\text{M.D.} = \dfrac{\sum f_i D_i}{N}$

$= \dfrac{472}{50}$ 1

$= 9.44$ 1 04

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6.	e)	Find variance and coefficient of variance for the following data:		

Class Interval	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	3	5	9	15	20	16	10	2

Ans.

Class	x_i	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
0-5	2.5	3	7.5	6.25	18.75
5-10	7.5	5	37.5	56.25	281.25
10-15	12.5	9	112.5	156.25	1406.25
15-20	17.5	15	262.5	306.25	4593.75
20-25	22.5	20	450	506.25	10125
25-30	27.5	16	440	756.25	12100
30-35	32.5	10	325	1056.25	10562.5
35-40	37.5	2	75	1406.25	2812.5
		80	1710		41900

1

$$\text{Mean} = \frac{\sum f_i x_i}{N} = \frac{1710}{80} = 21.375$$

`1

$$\text{S.D.} = \sqrt{\frac{\sum f_i x_i^2}{N} - \left(\frac{\sum f_i x_i}{N}\right)^2}$$

$$= \sqrt{\frac{41900}{80} - \left(\frac{1710}{80}\right)^2}$$

$$= 8.177$$

1

$$\text{Coefficient of variance} = C.V. = \frac{S.D.}{Mean} \times 100$$

$$\therefore C.V. = 38.25$$

1

04

<table>
<tr><th>Que. No.</th><th>Sub. Que.</th><th>Model answers</th><th>Marks</th><th>Total Marks</th></tr>
<tr><td>6.</td><td></td><td>
<table>
<tr><td>Class</td><td>x_i</td><td>f_i</td><td>d_i</td><td>$f_i d_i$</td><td>d_i^2</td><td>$f_i d_i^2$</td></tr>
<tr><td>0-5</td><td>2.5</td><td>3</td><td>-3</td><td>-9</td><td>9</td><td>27</td></tr>
<tr><td>5-10</td><td>7.5</td><td>5</td><td>-2</td><td>-10</td><td>4</td><td>20</td></tr>
<tr><td>10-15</td><td>12.5</td><td>9</td><td>-1</td><td>-9</td><td>1</td><td>9</td></tr>
<tr><td>15-20</td><td>$\underline{17.5}$ a</td><td>15</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>20-25</td><td>22.5</td><td>20</td><td>1</td><td>20</td><td>1</td><td>20</td></tr>
<tr><td>25-30</td><td>27.5</td><td>16</td><td>2</td><td>32</td><td>4</td><td>64</td></tr>
<tr><td>30-35</td><td>32.5</td><td>10</td><td>3</td><td>30</td><td>9</td><td>90</td></tr>
<tr><td>35-40</td><td>37.5</td><td>2</td><td>4</td><td>8</td><td>16</td><td>32</td></tr>
<tr><td></td><td></td><td>80</td><td>4</td><td>62</td><td></td><td>262</td></tr>
</table>

Mean=a+c $\bar{d}$

$\bar{d}$ = Mean=A+c $\bar{d}$

$$\bar{d} = \frac{\sum f_i d_i}{N} = \frac{62}{80} = 0.775$$

Mean=a+c $\bar{d} = 17.5 + 5(0.775) = 21.375$

$$\text{S.D.}= \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2} \times h$$

$$= \sqrt{\frac{262}{80} - \left(\frac{62}{80}\right)^2} \times 5$$

$$= 8.177$$

$$\text{Coefficient of variance} = C.V. = \frac{S.D.}{Mean} \times 100$$

$$= \frac{8.177}{21.375} \times 100$$

$$\therefore C.V. = 38.25$$
</td><td>1
1
1
1</td><td>04</td></tr>
</table>

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6.		*Note: Students may take any another value for A in the above/below example. So the above table and corresponding values vary accordingly. But the final answer will be the same.*		
	(f)	The two sets of observations are given below: Set-I Set-II $\bar{X} = 82.5$ $\bar{Y} = 48.75$ $\sigma_x = 7.3$ $\sigma_y = 8.35$ Which of the two sets is more consistent?		
		$C.V.(I) = \frac{\sigma}{\bar{x}} \times 100 = \frac{7.3}{82.5} \times 100 = 8.848$	1½	
		$C.V.(II) = \frac{\sigma}{\bar{x}} \times 100 = \frac{8.35}{48.75} \times 100 = 17.12$	1½	
		$\therefore C.V.(I) < C.V.(II)$ Set I is more consistent	1	04
		Important Note *In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.*		