MAHARASHTRA STATE BOARD OF TECHNICAL EDUCATION
(Autonomous)
(ISO/IEC -270001 - 2005 certified)

## WINTER -2019 EXAMINATION <br> Model Answer

## Important Instructions to examiners:

1) The answer should be examined by keywords and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language error such as grammatical, spelling errors should not be given more importance. (Not applicable for subject English and communication skill).
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figure drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In the some cases, the assumed constants values may vary and there may be some difference in the candidates answer and model answer.
6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidates understanding

| Q. <br> No. | Question and Model Answers | Marks |
| :--- | :--- | :--- | :--- |
| 1.A | Attempt any SIX of the following: | $\mathbf{1 2 M}$ |
| a) | Define Limit of eccentricity |  |
| Ans: | Limit of eccentricity: A load whose line of action does not coincide with the axis of a <br> member is called an eccentric load .The distance between the eccentric axis of the body and <br> the point of loading is called an eccentric limit ' e . The centrally located portion of a section <br> within which the load must act so as to produce only compressive stress is called a core or <br> kernel of section or limit of eccentricity. |  |


| b) | Write the formula for calculation of radius of curvature |  |
| :---: | :---: | :---: |
| Ans: | calculation of radius of curvature <br> Bending Equation <br> $\sigma / \mathrm{y}=\mathrm{M} / \mathrm{I}=\mathrm{E} / \mathrm{R} \quad$ or $\mathrm{R}=\mathrm{E} * \mathrm{I} / \mathrm{M}$ <br> Where, $\mathrm{M}=$ Bending moment <br> $\mathrm{E}=$ Modulus of elasticity I <br> $=$ Moment of Inertia <br> $\mathrm{R}=$ Radius of curvature | $1 \mathrm{M}$ $1 \mathrm{M}$ |
| c) | Define deflection of beam |  |
| Ans: | Figure: Elastic curve <br> The vertical Displacement of a point on a beam with respect to its original position before loading is called deflection of beam. It is denoted by "Y" | 2 M |
| d) | A cantilever of span ' $L$ ' carries a point load ' $w$ ' at 'L' from fixed end. State deflection at tree end in terms of El. |  |
|  | Maximum deflection $=\mathrm{Y} \max =\mathrm{YB}=\mathrm{WL} 3 /$ 3EI Where <br> $\mathrm{W}=$ Point load $\mathrm{L}=$ length (span) of beam(m) <br> $\mathrm{E}=$ modulus of elasticity $(\mathrm{KN} / \mathrm{m} 2)$ <br> $\mathrm{I}=$ moment of inertia of a beam m 4 | 2 M |
| e) | Define fixed beam |  |
| Ans: |  |  |



\begin{tabular}{|c|c|c|c|c|}
\hline \& 04

05 \& compressive as per the nature of external load.

\[
$$
\begin{aligned}
\text { Direct stress } & =\sigma 0 \\
& =\mathrm{P} / \mathrm{A}
\end{aligned}
$$

\] \& | compressive in nature and they both exist together in a member on either side of neutral axis or centroidal axis. |
| :--- |
| Bending stress $=\sigma b=M * y / I$ |
| Resultant stresses reach a higher value. Resultant stresses $=\sigma$ direct + $\sigma$ bending |
| Ecccentric load | \& | 1 M |
| :--- |
| mark |
| each | <br>

\hline b) \& \multicolumn{4}{|l|}{Define core of section . Sketch it Rectangular section} <br>

\hline \& \multicolumn{3}{|l|}{| Core of a section: Core of the section is that portion around the centroid in within which the line of action of load must act, so as to produce only compressive stress is called as core of the section. It is also defined as the region or area within which if load is applied, produces only compressive resultant stress. If Compressive load is applied, the there is no tension anywhere in thesection. $\begin{aligned} & e=\text { Core of section } e=d / 6 \text { or } e \\ & =b / 6 \end{aligned}$ |
| :--- |
| Rectangular section |} \& 2 M

1 M

1 M <br>
\hline c) \& \multicolumn{4}{|l|}{Explain steps involved in method of joint for calculation of forces in the member of frame} <br>

\hline Ans \& \multicolumn{3}{|l|}{| Calculation of forces in the member of frame by Method of joint, A truss is one of the major types of structures and is especially used in design of bridges and buildings. Step wise |
| :--- |
| 1. Examples of trusses |
| 2. Trusses, joint and forces |
| 3. UsingTrigonometry |
| 4. Draw a free body diagram |
| 5.Solve for reactionary forces of truss |
| 6.Locate a joint with only two members |} \& $1 / 2 \mathrm{M}$ for each step <br>

\hline
\end{tabular}

\begin{tabular}{|c|c|c|}
\hline \& \begin{tabular}{l}
7. Determine the unknown forces of the joint. \(\sum \mathrm{F}_{\mathrm{x}}=0 \sum \mathrm{~F}_{\mathrm{y}}=0\) \\
8. Calculation will give you a negative or a positive number designating the real direction of forces. Full analysis of a simple truss by the method of joint
\end{tabular} \& \\
\hline Q. 2 \& Attempt any THREE of the following: \& 12 M \\
\hline a) \& A rectangular column is 250 mm wide and 100 mm thick. It carries a load of 200 kN at an eccentricity of 100 mm in the plane bisecting thickness. Find the maximum and minimum intensities of stress in section. \& \\
\hline Ans: \& \[
\begin{aligned}
\& \mathrm{A}=250 \times 100=25 \times 103 \mathrm{~mm}^{2} \\
\& \mathrm{P}=200 \mathrm{kN} \\
\& \mathrm{Iyy}=100 \times 2503 / 12=130.208 \times 10^{6} \mathrm{~mm} 4 \\
\& \mathrm{Zxx}=\mathrm{Iyy} / \mathrm{Y}=130.208 \times 106 / 125=1.041 \times 10^{6} \mathrm{~mm} 3 \\
\& \mathrm{M}=\mathrm{P} \times \mathrm{e}=200 \times 103 \times 100=20 \times 10^{6} \mathrm{~N}-\mathrm{mm} . \\
\& \text { Direct stress }=6 \mathrm{~d}=\mathrm{P} / \mathrm{A}=200 \times 10^{3} / 25 \times 10^{3}=8 \mathrm{~N} / \mathrm{mm}^{2} \\
\& \text { Bending stress } 6 \mathrm{~b}=\mathrm{M} / \mathrm{Z}=20 \times 106 / 1.041 \times 10^{6}=19.21 \mathrm{~N} / \mathrm{mm}^{2} \\
\& \text { Maximum stress }=6 \mathrm{~d}+6 \mathrm{~b}=8+19.21=27.21 \mathrm{~N} / \mathrm{mm}^{2}(\text { Comp. }) \\
\& \text { Minimum stress }=6 \mathrm{~d}-6 \mathrm{~b}=8-19.21=(-) 11.21 \mathrm{~N} / \mathrm{mm}^{2} \text { (tensile) }
\end{aligned}
\] \& \begin{tabular}{l}
1 M \\
1 M \\
1 M \\
1 M
\end{tabular} \\
\hline b) \& (b) A hollow C.I. column of external diameter 300 mm and-internal diameter \(\mathbf{2 5 0} \mathbf{~ m m}\) carries an axial load ' \(w\) ' \(\mathbf{k N}\) and a load of \(100 \mathbf{k N}\) at an eccentricity \(\mathbf{1 2 5}\) mm . Calculate the maximum value of ' \(\mathbf{w}\) ' so as to avoid the tensile stresses, \& \\
\hline Ans: \& ```
Axial load = W,
eccentric load \(\mathrm{P}=100 \mathrm{kN}\). , eccentricity e \(=125 \mathrm{~mm}\)
Area \(\mathrm{A}=\pi *\left(\mathrm{D} 1^{2}-\mathrm{D} 2^{2}\right) / 4=\pi \mathrm{x}\left(300^{2}-250^{2}\right) / 4=21.59 \times 10^{3} \mathrm{~mm}^{2}\).
Moment of Inertia \(\mathrm{I}=\pi^{*} \mathrm{D}^{4} / 64=\pi\left(300^{4}-250^{4}\right) / 64=205.86 \times 10^{6} \mathrm{~mm}^{4}\)
\(\mathrm{Z}=\mathrm{I} / \mathrm{ymax}=205.86 \times 106 / 150=1.3724 \times 10^{6} \mathrm{~mm}^{3}\)
\(\mathrm{M}=\mathrm{P} \times \mathrm{e}=100 \times 10^{3} \times 125=12.5 \times 106 \mathrm{~N}-\mathrm{mm}\).
For No tension, 6d = 6b
Direct stress \(=(\mathrm{W} / \mathrm{A})+(\mathrm{P} / \mathrm{A})\)
\(6 \mathrm{~d}=(\mathrm{W} / \mathrm{A})+\left(100 \times 10^{3} / 21.59 \times 10^{3}\right)\)
Bending stress \(6 \mathrm{~b}=\mathrm{M} / \mathrm{Z}=12.5 \times 106 / 1.3724 \times 10^{6}=9.108 \mathrm{~N} / \mathrm{mm}^{2}\)
For no tension, \(6 \mathrm{~d}=6 \mathrm{~b}\)
\(\left(\mathrm{W} / 21.59 \times 10^{3}\right)+4.63=9.108\)
\(\mathrm{W}=4.47 * 21.59 \times 10^{3}=\)
\(=96.64 \times 10^{3} \mathrm{~N}\)
``` \& \begin{tabular}{l}
1 M \\
1 M \\
1 M \\
1 M
\end{tabular} \\
\hline c) \& A chimney having diameter \(\mathbf{4 m}\) and 50 m height. It is subjected to a horizontal wind pressure of 1.5 kPa normal to chimney. Find maximum bending stress in chimney. \& \\
\hline \& \begin{tabular}{l}
\[
\begin{aligned}
\& \mathrm{h}=50 \mathrm{~m}, \\
\& \mathrm{p}=1.5 \mathrm{kN} / \mathrm{m}^{2}, \\
\& \mathrm{D}=4 \mathrm{~m}
\end{aligned}
\] \\
Assum i) Density of masonary \(Y=22 \mathrm{kN} / \mathrm{m}^{3}\), and Coefficient of Wind pressure \(\mathrm{C}=1\)
\[
\begin{aligned}
\mathrm{A} \& =\pi * \mathrm{D}^{2} / 4=\pi * 4^{2} / 4 \\
\& =12.56 \mathrm{~mm} 2
\end{aligned}
\] \\
ii)WindForce
\[
\mathrm{P}=\mathrm{C} \times \mathrm{p} \times \mathrm{D} \times \mathrm{h}=1 \times 1.5 \times 4 \times 50=300 \mathrm{kN}
\] \\
iii) Self wtofchimney
\[
\begin{aligned}
\mathrm{W} \& =\mathrm{Y} \times \text { volume }=\mathrm{Y} \times \mathrm{A} \times \mathrm{h}=22 \times 12.56 \times 50 \\
\& =13.82 \times 10^{3} \mathrm{~N}
\end{aligned}
\]
\end{tabular} \& 1 M

1 M <br>
\hline
\end{tabular}

\begin{tabular}{|c|c|c|}
\hline \& \begin{tabular}{l}
iv) Direct stress \(=6 \mathrm{~d}=\mathrm{W} / \mathrm{A}=13.82 \times 10^{3} / 12.56=1.099 \times 10^{3} \mathrm{~N} / \mathrm{mm}^{2}\) \\
Moment of Inertia \(I=\pi\left(D^{4}\right) / 64=\pi 4^{4} / 64=12.566 \mathrm{~mm}^{4}\) \\
Bending stress \(6 \mathrm{~b}=\mathrm{M} / \mathrm{Z}\)
\[
\begin{aligned}
\& =\mathrm{M}^{*} \mathrm{Y} / \mathrm{I}=\mathrm{Pe} *(\mathrm{~d} / 2) / \mathrm{I}=300 *(50 / 2) *(4 / 2) / 12.56 \\
\& =1193.66 \mathrm{~N} / \mathrm{mm}^{2} \\
6 \max =60+6 \mathrm{~b} \& =2292 \mathrm{~N} / \mathrm{mm}^{2}(\text { Comp. }) \quad 6 \mathrm{~min}=60-6 \mathrm{~b}=94.66 \mathrm{~N} / \mathrm{mm} 2 \text { (Comp.) }
\end{aligned}
\]
\end{tabular} \& 1 M

1 M <br>
\hline d) \& A simply supported beam carries a u.d.I. of $4 \mathrm{kN} / \mathrm{m}$ over entire span 4 m . Find deflection at mid span in terms of EI. \& <br>
\hline Ans \& Maximum deflection at mid span in terms

$$
\text { Deflection at Centre } \quad \mathrm{Ymax}=\frac{5 w L^{4}}{384 E I}
$$ \& $1 M$

$1 M$
$1 M$
$1 M$ <br>
\hline e) \& A cantilever of span 2 m carries 10 kN load at free end. Find deflection at free end if $\mathrm{El}=15 \times 10^{\mathbf{3}} \mathrm{kN} . \mathrm{m} 2$ \& <br>
\hline
\end{tabular}

| Ans: | $\begin{aligned} & \mathrm{El}=15 \times 10 \mathrm{kN} . \mathrm{m} 2 \\ & \mathrm{~L}=2 \mathrm{~m} \\ & \mathrm{~W}=10 \mathrm{kN} \end{aligned}$ <br> Deflection at free end $\mathrm{YB}=-\mathrm{WL}^{3} / 3 \mathrm{EI}$ $\begin{aligned} & =-10 \times 2^{3} / 3 \times 15 \times 10^{3} \\ & =-1.77 \mathrm{~mm} \end{aligned}$ | 2 M <br> 2 M |
| :---: | :---: | :---: |
| f) | Write Clapeyron's moment theorem for a beam with different M.I. giving meaning of each term. |  |
| Ans: | The clapeyron's theorem of three moment is applicable to two span continuous beams .It state that " For any two consecutive spans of continuous beam subjected to an external loading and having different moment of inertia, the support moments <br> $\mathrm{M}_{\mathrm{A}}, \mathrm{M}_{\mathrm{B}}$ and $\mathrm{M}_{\mathrm{C}}$ at supports $\mathrm{A}, \mathrm{B}$ and C respectively are given by following equation <br> If the moment of inertia is not constant then claperon's theorem can be stated in the form of following equation. $M_{A} \frac{L_{1}}{I_{1}}+2 M_{B}\left(\frac{L_{1}}{I_{1}}+\frac{L_{2}}{I_{2}}\right)+M_{C} \frac{L_{2}}{I_{2}}=-\left[\frac{6 A_{1} X_{1}}{L_{1} I_{1}}+\frac{6 A_{2} X_{2}}{L_{2} I_{2}}\right]$ <br> Where, $L_{1}$ and $L_{2}$ are length of span $A B$ and $B C$ respectively. <br> $\mathrm{I}_{1}$ and $\mathrm{I}_{2}$ are moment of inertia of span AB and BC respectively. <br> $A_{1}$ and $A_{2}$ are area of simply supported $B M D$ of span $A B$ and $B C$ respectively. | 1 M |


|  | $\mathrm{X}_{1}$ and $\mathrm{X}_{2}$ are distances of centroid of simply supported BMD from A and C respectively |  |
| :---: | :---: | :---: |
| Q. 3 | Attempt any FOUR of the following | 16 M |
| a) | State any two advantages and two disadvantages of fixed beam over simply supported beam |  |
| Ans. | Advantages <br> 1) Fixed beam is more stiff, strong and stable than simply supported beam. <br> 2) For the same span and loading a fixed beam has lesser value of bending moment as compared to simply supported beam. <br> 3) For the same span and loading fixed beam has lesser value of deflection as compared to simply supported beam. <br> Disadvantages. <br> 1) A little sinking of one support over ,the other induces additional moment at each end <br> 2) Extra care has to be taken to achieve correct fixity at the ends. <br> 3) Due to end fixity temperature stresses induced due to variation in temperature | $2 \mathrm{M}$ $2 \mathrm{M}$ |
| b) | Fixed beam of span 6 meter caries a point load of 100 KN at 4 meter from left support calculate fixed end moment |  |
| Ans. | $\begin{aligned} & \mathrm{MA}=\mathrm{wab}^{2} / \mathrm{L}^{2} \\ & =100^{*} 4^{*}(2)^{2} / 6^{2} \\ & =-\mathbf{4 4 . 4 4} \mathbf{~ K N} / \mathbf{M} \end{aligned}$ $\begin{aligned} & \mathrm{MB}=\mathrm{wba}{ }^{2} / \mathrm{L}^{2} \\ & =100^{*} 2^{*}(4)^{2} / 6^{2} \\ & =-\mathbf{8 8 . 8 9} \mathbf{K N} / \mathbf{M} \end{aligned}$ | 1M <br> 1M <br> 1 M <br> 1M |
| c) | Calculates Maximum Deflection at A Beam shown in the fig. use Macaulay's Method $\mathrm{E}=2 \times 108 \mathrm{KN} / \mathrm{m} 2 \& \mathrm{I}=0.733 \times 10-4 \mathrm{~m} 4$ |  |
| Ans: | $\mathrm{RA}=\mathrm{wb} / \mathrm{l}$ |  |


|  | $\begin{aligned} & =75^{*} 4.5 / 6=\mathbf{5 6 . 2 5} \mathbf{K N} . \\ & \mathrm{RB}=\mathrm{wa} / \mathrm{l} \\ & =75^{*} 1.5 / 6=\underline{\mathbf{1 8 . 7 5} \mathbf{K N} .} \end{aligned}$ <br> Consider a section x - x at a distance " x " from ' A ' in portion CB. $\text { EI d²y } / \mathrm{dx}^{2}=\mathrm{Mx}=56.25 \mathrm{x}-75(\mathrm{x}-1.5)$ <br> Integrating w.t.r. x $\text { EI dy } / \mathrm{dx}=56.25 \mathrm{x}^{2} / 2+\text { C1-75 }(x-1.5)^{2} / 2$ <br> Again integrating wrtx $\text { EI } y=56.25^{2} \mathrm{x}^{3} / 6+\text { C1x+C2-75 * }(\mathrm{x}-1.5)^{3} / 6$ <br> At $x=0 \& Y=0$ $\mathrm{C} 2=0$ <br> At $x=6 \quad Y=0$ $\begin{aligned} & 0=56.25(6)^{3} / 6+c 1^{*} 6+0-75(6-1.50)^{3} / 6 \\ & =2025+c 1^{*} 6-1139.06 \\ & C 1 * 6=885.93 \\ & C 1 * 6=885.93 \\ & \mathbf{C 1}=-\mathbf{1 4 7 . 6 5} \\ & \text { EIY }=56.25^{2}\left(x^{3} / 6\right)-147.65 x-75(x-1.5)^{3} / 6 \end{aligned}$ <br> E dy $/ \mathrm{dx}=56.25 \mathrm{x}^{2} / 2-147.65-75(\mathrm{x}-1.5)^{2} / 2$ <br> dy/dx=0 $\begin{aligned} & 0=28.125 x^{2}-147.5-37.5(x-1.5)^{2} \\ & 0=28.125 x^{2}-147.5-37.5\left(x^{2} 3 x+2.25\right) \\ & 0=-28.125 x^{2}-147.5-37.5 x^{2}+112.5 x-84.37 \\ & 0=-9.375 x^{2}+112.5 x-231.80 \end{aligned}$ <br> By solving above equation $\begin{aligned} & X=112.5_{+-} \sqrt{ }\left(112.5^{2}\right)-4^{*} 9.375^{*} 231.78 / 2^{*} 9.375 \\ & X=112.5_{+-} \sqrt{ }(3964.5) / 18.75 \\ & X=112.5_{+-} 62.96 / 18.75 \\ & \mathbf{X}=\mathbf{2 . 6 4 m \quad X}=\mathbf{9 . 3 5 m} \\ & \text { PUT X=2.64m } \\ & E I Y=56.25^{*} 2.64 / 6-147.5^{*} 2.64-75(2.64-1.5)^{3} / 6 \\ & E I Y=172.49-389.4-18.51 \\ & Y=-235.43 / E I \\ & Y=-235.42 / 2^{*} 10^{8 *} 0.733^{*} 10^{-4} \\ & Y_{\text {max }}=\mathbf{- 0 . 0 1 6 M} \\ & Y \text { max }=-\mathbf{1 6} \mathbf{~ m m} . ~(d o w n w a r d ~ d e f l e c t i o n) . ~ \end{aligned}$ | 1M <br> 1M <br> 1 M <br> 1 M |
| :---: | :---: | :---: |
| d) | Find the slope at free end of beam as shown in figure. |  |


| Ans: | $\boldsymbol{\theta} \mathrm{A}$ - slope at free end of beam due to udl. $=w L^{3} / 6 E I$ <br> $\boldsymbol{\theta}$ B-slope at free end of beam due to point load $\begin{gathered} =\mathrm{wL}^{2} / 2 \mathrm{EI} \\ \boldsymbol{\theta} \max =\boldsymbol{\theta} \mathrm{A}+\boldsymbol{\theta} \mathrm{B} \end{gathered}$ $\begin{aligned} & =\mathrm{wL}^{3} / 6 \mathrm{EI}+\mathrm{wL}^{2} / 2 \mathrm{EI} \\ & =5^{*} 4^{3} / 6 \mathrm{EI}+8^{*} 4^{2} / 2 \mathrm{EI} \\ & =53.33 / \mathrm{EI}+64 / \mathrm{EI} \\ & =117.33 / \mathrm{EI} \end{aligned}$ | 1 M <br> 1 M <br> 2 M |
| :---: | :---: | :---: |
| e) | State Any Four Assumptions Made In Analysis Of Simple Frame |  |
| Ans: | 1) The frame is perfect one i.e. the relation $n=2 j-3$ always satisfy <br> 2) All the member are hinged or pin jointed at the end <br> 3) The loads are acting only at the joint <br> 4) The self weight of member is neglected. | $\begin{gathered} 1 \mathrm{M} \\ \text { EACH } \end{gathered}$ |
| f) | Determine the forces in member AB and BC use method of section Let us consider the equilibrium of truss to right of section. |  |
| Ans : |  |  |


|  | =21.21KN.(COMP) <br> let us consider the equilibrium of the truss to right of section 2-2 <br> $\sum \mathrm{fy}=0$ <br> $0=-$ FAB Sin $45+$ FCB <br> FCB=21.21 Sin45 <br> FBC=15KN (TENS). | 2 M <br> 2 M |
| :---: | :---: | :---: |
| Q. 4 | Attempt any FOUR of the following | 16 M |
| a) | Write the step wise procedure for analysis of continuous beam |  |
| Ans : | Step 1 to draw bending moment diagram <br> 1) Assume the continuous beam as a series of simply supported beam and draw the usual $\boldsymbol{\mu}$ diagram due to vertical loads <br> 2) Calculate $6 \mathrm{a} \overline{\mathrm{x}} / \mathrm{L}$ (calculate $6 \mathrm{a} \overline{\mathrm{x}} / \mathrm{L}$ for varying moment of Inertia) <br> 3) Apply the CLAPEYRON THEOREM , three moment and find the unknown fixed end moment draw the $\boldsymbol{\mu}$ diagram. <br> 4) Superimpose the $\boldsymbol{\mu}$ diagram over $\boldsymbol{\mu}$ diagram and draw the net bending moment diagram <br> Step to draw a SF diagram <br> 1) Calculate the reaction of simply supported beam <br> 2) Calculate the reaction due to difference of fixed end moment <br> 3) Superimposed reaction due to above two cases and find the reaction of continuous beam <br> 4) Knowing the support reactions draw SF Diagram as usual. | 2 M <br> 2 M |
| b) | Find d support moment of a continuous beam as shown in figure use clapeyron's theorem. |  |
| Ans : | GIVEN DATA:- <br> Span $A C=6 m$ \& CD $=3 \mathrm{~m}$ <br> Known moments are MA=MD=0 <br> A)Assume The AC \& CD As A Simply Supported Beam draw $\boldsymbol{\mu}$ diagram $\mathrm{BM}_{\mathrm{B}}=\mathrm{Wab}_{1} \mathrm{~b}_{1} / \mathrm{l}_{1}$ |  |







|  |  | for sketch |
| :---: | :---: | :---: |
| c) | A truss is loaded as shown in fig. Determine the nature and magnitude of forces in the member CD, FD and FG. |  |
| Ans : | 1) Calculate Support Reactions <br> Taking moment at B $\begin{aligned} & \mathrm{R}_{\mathrm{F}} \times 1+3 \times 0.5-2 \times 1.5-4 \times\left[0.5+\frac{0.5}{2}\right]-5 \times 0.5 \\ & \mathrm{R}_{\mathrm{F}}=7 \mathrm{kN}(\uparrow) \end{aligned}$ <br> Now $R_{F}+R_{B V}=4+5+2$ <br> Now $\begin{aligned} & \mathrm{R}_{\mathrm{BV}}=11-7=4 \mathrm{kN} .(\uparrow) \\ & \mathrm{R}_{\mathrm{BH}}=3 \mathrm{kN} .(\longleftarrow) \end{aligned}$ <br> 2) Calculate Slope ( $\theta$ ) | 1 M sketch |

$$
\begin{aligned}
& \text { angle } \mathrm{CAF}=\text { angle } \mathrm{AFC}=\theta \\
& \begin{array}{c}
\tan \theta=\left(\frac{0.5}{0.25}\right) \\
{[\boldsymbol{\theta}=\mathbf{6 3 . 4 3 6}]^{0}}
\end{array} \\
& \text { Now angle CFD }=180-\theta-\theta \\
& \quad=180-63.43-63.43 \\
& \text { Angle } \mathbf{C F D}=\mathbf{5 3 . 1 3}^{\mathbf{0}}
\end{aligned}
$$

3) Take the section $1-1$ passing through members CD, FD, FG

Consider $\mathrm{F}_{\mathrm{CD}}, \mathrm{F}_{\mathrm{FD}} \& \mathrm{~F}_{\mathrm{FG}}$ as tensile and consider equilibrium of all forces to the left of section 1-1
a) $\sum \mathrm{M}_{\mathrm{F}}=0 \quad$ clockwise +ve and anticlockwise -ve moment

$$
\begin{aligned}
-2 & \times 0.5+\mathrm{F}_{\mathrm{CD}} \times 0.5=0 \\
-1 & +\mathrm{F}_{\mathrm{CD}} \times 0.5=0 \\
\mathrm{~F}_{\mathrm{CD}} & =-\frac{1}{0.5} \\
\mathbf{F}_{\mathrm{CD}} & =\mathbf{2} \mathbf{~ k N} \text { ( + ve sign indicate Tension) }
\end{aligned}
$$

b) $\sum M_{D}=0$

$$
\begin{aligned}
&-2 \times(0.5+0.25)+7 \times \frac{0.5}{2}-\mathrm{F}_{\mathrm{FG}} \times 0.5=0 \\
&-1.5+1.75-0.5 \mathrm{~F}_{\mathrm{FG}}=0 \\
& \mathrm{~F}_{\mathrm{FG}}=\frac{-0.25}{0.5} \\
& \mathbf{F}_{\mathrm{FG}}=\mathbf{0 . 5} \mathbf{k N}(\text { Tensile })
\end{aligned}
$$

$$
\begin{aligned}
& \sum \mathrm{F}_{\mathrm{x}}=0 \text { gives } \\
& \quad+3+\mathrm{F}_{\mathrm{CD}}+\mathrm{F}_{\mathrm{FG}}+\mathrm{F}_{\mathrm{FD}} \cos \theta=0 \\
& \quad+3-1+0.5+\mathrm{F}_{\mathrm{FD}} \cos 63.43^{\circ}=0 \\
& \quad \mathrm{~F}_{\mathrm{FD}}=\frac{-2.5}{\cos 63.43}
\end{aligned}
$$

$$
\mathbf{F}_{\mathrm{FD}}=-5.59 \mathrm{kN}(- \text { ve sign indicate compression })
$$

## OR

The problem can be solved using Method of joints
Consider joint A

$\sum \mathrm{F}_{\mathrm{y}}=0$
$-2+\mathrm{F}_{\mathrm{AC}} \sin 63.43^{0}=0$
$\mathrm{F}_{\mathrm{AC}}=\mathbf{2 . 2 3 6} \mathbf{k N}$ (Tension)
$\sum \mathrm{F}_{\mathrm{x}}=0$
$\mathrm{F}_{\mathrm{AF}}+\mathrm{F}_{\mathrm{AC}} \cos 63.43^{0}=0$
$\mathrm{F}_{\mathrm{AF}}+2.236 \cos 63.43^{0}=0$
$\mathrm{F}_{\mathrm{AF}}=\mathbf{- 1} \mathbf{k N}$ (-ve sign indicate Compression )
Consider joint C


Calculate maximum deflection of beam if $\mathrm{I}=8 \times 10^{7} \mathrm{~mm}^{4}$

$$
\mathrm{E}=2 \times 10^{5} \mathrm{~mm}^{2}
$$

Using macullays method
Calculate support reactions
$\sum \mathrm{M}_{\mathrm{A}}=0$ clockwise +ve and anticlockwise -ve moment
$-R_{B} \times 4+10 \times 6=0$
$\mathrm{R}_{\mathrm{B}}=15 \mathrm{kN}$
$\sum \mathrm{f}_{\mathrm{y}}=0$
$\mathrm{R}_{\mathrm{A}}+\mathrm{R}_{\mathrm{B}}=10$
$\mathrm{R}_{\mathrm{A}}+15=10$
$R_{A}=-5 \mathrm{kN}(-$ ve sign indicate downward reaction $)$
EI d ${ }^{2} y / d^{2}=M$
-Differential equation
Consider x from free end of overhang and Considering right side of section (Anti clock wise +ve and Clockwise -ve sign convension )

EI d ${ }^{2} y / d x^{2}=\left.10 \mathrm{x}|+15(\mathrm{x}-2)|_{\mathrm{X}=2 \mathrm{~m}}\right|_{\mathrm{x}=6 \mathrm{~m}}$
EI d ${ }^{2} y / d x^{2}=-10 x \left\lvert\,+15 \frac{(x-2)^{2}}{2}\right.$
--------- Slope equation

| Ely $=-5 \frac{x^{3}}{3}+\mathrm{C}_{1} \mathrm{x}+\mathrm{C}_{2}$ | $15 \frac{(x-2)^{3}}{6} \quad-\cdots-----$ - Deflection equation |
| :---: | :---: | :---: |

Calculate the constants of integration using boundry condition i) At $x=2 m y=0$ putting in Deflection equation

$$
\begin{align*}
& \operatorname{EI}(0)=-\frac{5}{3}(2)^{3}+C_{1} \times 2+C_{2}+\frac{15(2-2)^{3}}{6} \\
& 0=-13.333+2 \mathrm{C}_{1}+\mathrm{C}_{2} \\
& 2 \mathrm{C}_{1}+\mathrm{C}_{2}=+13.333 \tag{I}
\end{align*}
$$

ii) At $\mathrm{x}=6 \mathrm{~m}, \mathrm{y}=0$ putting in Deflection equation
$\mathrm{EI}(0)=-\frac{5}{3}(6)^{3}+6 \mathrm{C}_{1}+\mathrm{C}_{2}+\frac{15(6-2)^{3}}{6}$
$=-360+6 C_{1}+C_{2}+160$
$6 C_{1}+C_{2}=+200$
Solving two simultaneous equation

$$
\begin{aligned}
& C_{1}=46.67 \\
& C_{2}=-80
\end{aligned}
$$

Putting values if $\mathrm{C}_{1} \& \mathrm{C}_{2}$ \& rewriting slope \& deflection equation
EI $\frac{d y}{d x}=-5 x^{2}+46.67 \left\lvert\,+15 \frac{(x-2)^{2}}{2}------\right.$ Final Slope equation

|  | EI $y=-5 \frac{x^{8}}{3}+46.67 x-80 \left\lvert\,+\frac{15(x-2)^{3}}{6}------------\right.$ Final Deflection equation <br> Maximum deflection will be their where slope is zero. <br> Putting $\frac{d y}{d x}=0$ in slope equation to get distance x where deflection level is maximum. $\begin{aligned} 0= & -5 x^{2}+46.67+7.5(x-2)^{2} \\ 0= & -5 x^{2}+46.67+7.5\left(x^{2}-4 x+4\right) \\ 0= & -5 x^{2}+46.67+7.5 x^{2}-30 x+30 \\ & 2.5 x^{2}-30 x+76.67=0 \end{aligned}$ <br> Solving quadratic equation <br> $x=8.3 \mathrm{~m}, x=3.69 \mathrm{~m}$ deflection is maximum putting in deflection equation. <br> Hence deflection is maximum at $x=3.69 \mathrm{~m}$ $\text { EI ymax }=-\frac{5}{3}(3.69)^{3}+46.67 x(3.69)-80+\frac{15}{6}(3.69-2)^{3}$ <br> EI ymax $=-83.739+172.212-80+12.067$ $\operatorname{ymax}=\frac{20.54}{E I}$ <br> Putting values of $\mathrm{E}=2 \times 10^{5} \mathrm{~N} / \mathrm{mm}^{2}, \mathrm{I}=8 \times 10^{7} \mathrm{~mm}^{4}$ <br> $\mathrm{EI}=16 \times 10^{3} \mathrm{kN} . \mathrm{m}^{2}$ $\begin{aligned} \operatorname{ymax} & =\frac{20.54}{16 \times 10^{\mathrm{s}}} \\ \operatorname{ymax} & =1.2837 \times 10^{-3} \mathrm{~m} \\ \operatorname{ymax} & =1.2837 \mathrm{~mm} \end{aligned}$ <br> (distance $x$ can be considered from left support and problem can be solved if student solves problem by considering $x$ from left support appropriate marks shall be given to the students accordingly.) | 1 M |
| :---: | :---: | :---: |
| b) | Draw SFD and BMD for beam show in fig. |  |
| Ans : | 1) Calculate Reactions of a beam by considering beam as a simply supported beam $\begin{aligned} & \sum M_{A}=0 \\ & -R_{D} \times 6+3 \times 1.5+3 \times 3+5 \times 6 \times \frac{6}{2}=0 \\ & \mathbf{R}_{\mathrm{D}}=\mathbf{1 7 . 2 5} \mathbf{~ k N} \\ & \mathrm{R}_{\mathrm{A}}+\mathrm{R}_{\mathrm{D}}=3+3+5 \times 6 \\ & \mathrm{R}_{\mathrm{A}}=36-17.25 \\ & \mathbf{R}_{\mathrm{A}}=\mathbf{1 8 . 7 5} \mathbf{k N} \end{aligned}$ <br> 2) Calculate Simply Supported BM $\begin{aligned} \mathrm{m}_{\mathrm{A}} & =\mathrm{M}_{\mathrm{D}}=0 \\ \mathrm{~m}_{\mathrm{B}} & =18.75 \times 1.5-5 \times 1.5 \times \frac{1.5}{2} \\ \mathrm{~m}_{\mathrm{B}} & =28.125-5.625 \end{aligned}$ | 1 M |

$\mathrm{m}_{\mathrm{B}}=\mathbf{2 2 . 5 \mathrm { kN } . \mathrm { m } \text { (Sagging) }}$
$\mathrm{m}_{\mathrm{C}}=18.75 \times 3-3 \times 1.5 \times-5 \times 3 \times \frac{3}{2}$
$\mathrm{m}_{\mathrm{C}}=56.25-4.5-22.5$
$\mathbf{m}_{\mathrm{C}}=29.25 \mathrm{kN} . \mathrm{m}$ (Sagging)
3) Calculate Fixed end moments
$\mathrm{M}_{\mathrm{A}}=\frac{W 1 a 1 b 1^{\mathrm{z}}}{L^{2}}-\frac{W 2 a 2 b 2^{\mathrm{z}}}{L^{z}}-\frac{W L^{2}}{12}$
$\mathrm{M}_{\mathrm{A}}=\frac{-3 \times 1.5 \times(4.5)^{\mathrm{z}}}{6^{\mathrm{x}}}-\frac{3 \times 3 \times 3^{\mathrm{x}}}{6^{\mathrm{x}}}-\frac{5 \times 6^{\mathrm{x}}}{12}$
$\mathrm{M}_{\mathrm{A}}=-2.531-2.25-15$
$\mathrm{M}_{\mathrm{A}}=\mathbf{- 1 9 . 7 8 1} \mathbf{k N} . \mathrm{m}$
$\mathrm{M}_{\mathrm{D}}=\frac{W 1 a 1^{z^{2}} b 1}{L^{z}}-\frac{W 2 a 2^{z} b 2}{L^{2}}-\frac{W L^{x}}{12}$
$\mathrm{M}_{\mathrm{D}}=\frac{-3 x(1.5)^{2} x 4.5}{6^{2}}-\frac{3 x 3^{2} x 3}{6^{2}}-\frac{5 x 6^{2}}{12}$
$\mathrm{M}_{\mathrm{D}}=-0.844-2.25-15$
$\mathrm{M}_{\mathrm{D}}=\mathbf{- 1 8 . 0 9 4} \mathrm{kN} . \mathrm{m}$
4) Draw final BMD by drawing S. S. BMD \& Fixed diagram \& Super imposing each other.

5) Calculate Support Reactions for fixed beam
$\sum_{\mathrm{MA}}=0$

$$
3 \times 1.5+3 \times 3+5 \times 6 \times \frac{6}{2}-19.78+18.09-R_{D} \times 6=0
$$

$$
R_{D}=16.960 \mathrm{kN}
$$

$R_{A}+R_{D}-3-3-5 \times 6=0$
$\mathrm{R}_{\mathrm{A}}=36-16.96$
$\mathrm{R}_{\mathrm{A}}=19.04 \mathrm{kN}$
6) Shear Force Calculations

|  |  | 1 M |
| :---: | :---: | :---: |
| c) | Draw SFD and BMD for beam show in fig. by claperon's thermo of three moments. |  |
| Ans : | 1) Using Claperon's Three moment thermo for span $A B \& B C$ $M_{A} \frac{L_{1}}{I_{1}}+2 M_{B}\left(\frac{L_{1}}{I_{1}}+\frac{L_{2}}{I_{2}}\right)+M_{C} \frac{L_{2}}{I_{2}}+M_{C} \frac{L_{2}}{I_{2}}=-\left[\frac{6 A_{1} X_{1}}{L_{1} I_{1}}+\frac{6 A_{2} X_{2}}{L_{2} I_{2}}\right]$ $\mathrm{M}_{\mathrm{A}}=\mathrm{M}_{\mathrm{c}}=0$ <br> as it is simply supported <br> Putting values of $L_{1} \& L_{2}, I_{1} \& I_{2}$ in above equation $\begin{equation*} \mathrm{M}_{\mathrm{A}} \frac{6}{2 I}+2 \mathrm{M}_{\mathrm{B}} \frac{6}{21}+\frac{4}{I}+\mathrm{M}_{\mathrm{C}} \frac{4}{I}=-\frac{6 A_{1} X_{1}}{2 I x 6}+\frac{6 A_{2} X_{2}}{I x 4} \tag{I} \end{equation*}$ <br> 2) Calculate Simply Supported BM \& Draw S. S. BMD $\begin{aligned} & \mathrm{m}_{\mathrm{AB}}=\frac{P L}{4} \\ & \mathrm{~m}_{\mathrm{AB}}=\frac{100 x 6}{4} \\ & \mathrm{~m}_{\mathrm{AB}}=150 \mathrm{kN} \cdot \mathrm{~m} \\ & \mathrm{~m}_{\mathrm{BC}}=\frac{w L^{2}}{8} \end{aligned}$ | 1 M |

$m_{B C}=\frac{25 x 4^{2}}{8}$
$\mathrm{m}_{\mathrm{BC}}=50 \mathrm{kN} . \mathrm{m}$

$A_{1}=\frac{1}{2} \times 6 \times 150$
$\mathrm{A}_{1}=450 \mathrm{kN} . \mathrm{m}_{2}$
$\mathrm{A}_{2}=\frac{2}{3} \times 4 \times 50$
$\mathrm{A}_{2}=133.33 \mathrm{kN} . \mathrm{m}_{2}$
$X_{1}=\frac{6}{2}$
$\mathrm{X}_{1}=3 \mathrm{~m}$
$\mathrm{X}_{2}=\frac{4}{2}$
$\mathrm{X}_{2}=2 \mathrm{~m}$
Putting all values in equation (I)

$$
\begin{aligned}
& 0+0+2 \mathrm{M}_{\mathrm{B}}[7]=-\left[\frac{6 x 450 \times 3}{12}+\frac{6 \times 133.33 \times 2}{4}\right] \\
& 14 \mathrm{M}_{\mathrm{B}}=-[675+400] \\
& \mathbf{M}_{\mathbf{B}}=-76.785 \mathrm{kN} . \mathrm{m}
\end{aligned}
$$

3) Draw final BMD

4) Calculate Support Reactions

Consider Span AB
Taking moment at B

$$
\begin{aligned}
& \sum_{\mathrm{MA}}=0 \\
& \mathrm{R}_{\mathrm{A}} \times 6-100 \times 3+76.78=0 \\
& \mathbf{R}_{\mathrm{A}}=\mathbf{3 7 . 2 0 3} \mathbf{~ k N} . \mathrm{m}
\end{aligned}
$$

Consider Span BC
Taking moment at B

$$
\begin{aligned}
& \sum_{\mathrm{MB}}=0 \\
& -\mathbf{R}_{\mathrm{C}} \mathrm{x} 4+25 \times 4 \times \frac{4}{2}-76.78=0 \\
& \mathbf{R}_{\mathrm{C}}=\mathbf{3 0 . 8 0 5} \mathbf{~ k N . m}
\end{aligned}
$$

|  | $\begin{aligned} & \sum_{\mathrm{mB}}=0 \\ & \mathrm{R}_{\mathrm{A}}+\mathrm{R}_{\mathrm{B}}+\mathrm{R}_{\mathrm{C}}-100-25 \times 4=0 \\ & 37.203+\mathrm{R}_{\mathrm{B}}+30.805=200 \\ & \mathbf{R}_{\mathbf{B}}=\mathbf{1 3 1 . 9 9} \mathbf{k N} . \mathbf{m} \end{aligned}$ <br> 5) Shear Force Calculations <br> Shear Force at A just Left $=0$ $\text { Just Right }=37.203 \mathrm{kN}$ <br> Shear Force at D just Left $=+37.203$ $\begin{aligned} \text { Just Right } & =+37.203-100 \\ & =-62.797 \mathrm{kN} \end{aligned}$ <br> Shear Force at B just Left $=+37.203-100$ $=-62.797 \mathrm{kN}$ $\begin{aligned} \text { Just Right }= & -62.797+131.99 \\ & =69.193 \mathrm{kN} \end{aligned}$ <br> Shear Force at C just Left $=+37.203-100+131.99-25 \times 4$ $=-30.805 \mathrm{kN}$ $\text { Just Right }=-30.805+30.805=0$ | 1 M |
| :---: | :---: | :---: |

