



SUMMER – 2022 EXAMINATION

Subject Name: Applied Mathematics

Model Answer

Subject Code:

22210

Important Instructions to examiners:

- 1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
- 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
- 3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills).
- 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
- 5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
- 6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
- 7) For programming language papers, credit may be given to any other program based on equivalent concept.
- 8) As per the policy decision of Maharashtra State Government, teaching in English/Marathi and Bilingual (English + Marathi) medium is introduced at first year of AICTE diploma Programme from academic year 2021-2022. Hence if the students in first year (first and second semesters) write answers in Marathi or bilingual language (English +Marathi), the Examiner shall consider the same and assess the answer based on matching of concepts with model answer.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.		Solve any <u>FIVE</u> of the following:	10
	a)	Find 'a' if $f(x) = ax + 10$ and $f(1) = 13$	02
	Ans	$f(x) = ax + 10$ $\therefore f(1) = a(1) + 10$ $\therefore 13 = a + 10$ $\therefore a = 3$	1 1
	b)	State whether the function $f(x) = \frac{x \cos x}{1 + \sin^2 x}$ is even or odd.	02
	Ans	$f(x) = \frac{x \cos x}{1 + \sin^2 x}$ $\therefore f(-x) = \frac{(-x) \cos(-x)}{1 + \sin^2(-x)}$ $= \frac{-x \cos x}{1 + \sin^2 x}$ $= -f(x)$ \therefore Given function is odd.	1 1



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1.	c)	Find $\frac{dy}{dx}$ if $y = x^e + e^x + e^e + \sqrt{x}$	02
	Ans	$y = x^e + e^x + e^e + \sqrt{x}$ $\therefore \frac{dy}{dx} = ex^{e-1} + e^x + \frac{1}{2\sqrt{x}}$	2
	d)	Evaluate : $\int (e^x + x^e + e^e) dx$	02
	Ans	$\int (e^x + x^e + e^e) dx$ $= e^x + \frac{x^{e+1}}{e+1} + e^e x + c$	2
e)	Evaluate : $\int \frac{\cos(\log x)}{x} dx$	02	
Ans	<p>Let $\log x = t$</p> $\frac{1}{x} dx = dt$ $= \int \cos t dt$ $= \sin t + c$ $= \sin(\log x) + c$	1	
f)	Find the area bounded by the curve $y = x^3$, x -axis and co-ordinates $x = 1$, $x = 3$.	02	
Ans	$\text{Area } A = \int_a^b y dx$ $= \int_1^3 x^3 dx$ $= \left[\frac{x^4}{4} \right]_1^3$ $= \frac{3^4}{4} - \frac{1^4}{4} = 20$	<p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p>	



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2.	g)	Separate into real and imaginary part for $\frac{1+i}{2-i}$	02
	Ans	$\frac{1+i}{2-i} = \frac{1+i}{2-i} \times \frac{2+i}{2+i}$ $= \frac{2+3i+i^2}{4-i^2}$ $= \frac{2+3i+(-1)}{4-(-1)}$ $= \frac{1+3i}{5}$ $= \frac{1}{5} + \frac{3}{5}i$	<p>½</p> <p>1</p> <p>½</p>

		Solve any <u>THREE</u> of the following:	12
	a)	Find maximum and minimum value of curve $x^3 - 9x^2 + 24x$.	04
		Let $y = x^3 - 9x^2 + 24x$	
		$\therefore \frac{dy}{dx} = 3x^2 - 18x + 24$	½
		$\therefore \frac{d^2y}{dx^2} = 6x - 18$	½
		Consider $\frac{dy}{dx} = 0$	
		$3x^2 - 18x + 24 = 0$	½
		$\therefore x = 2$ or $x = 4$	½
		at $x = 2 \quad \therefore \frac{d^2y}{dx^2} = 6(2) - 18 = -6 < 0$	
		$\therefore y$ is maximum at $x = 2$	
		$y_{\max} = (2)^3 - 9(2)^2 + 24(2) = 20$	1
		at $x = 4 \quad \therefore \frac{d^2y}{dx^2} = 6(4) - 18 = 6 > 0$	
		$\therefore y$ is minimum at $x = 4$	
		$y_{\min} = (4)^3 - 9(4)^2 + 24(4) = 16$	1



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2.	b)	Find $\frac{dy}{dx}$ if $x^3 + y^3 + xy = 0$	04
	Ans	$x^3 + y^3 + xy = 0$ $\therefore 3x^2 + 3y^2 \frac{dy}{dx} + x \frac{dy}{dx} + y = 0$ $\frac{dy}{dx}(3y^2 + x) = -3x^2 - y$ $\frac{dy}{dx} = \frac{-3x^2 - y}{3y^2 + x}$	2 1 1
	c)	If $x = \sec \theta + \tan \theta$ and $y = \sec \theta - \tan \theta$ then show that $\frac{dy}{dx} = \frac{-y}{x}$	04
Ans	$x = \sec \theta + \tan \theta$ $\therefore \frac{dx}{d\theta} = \sec \theta \tan \theta + \sec^2 \theta$ $y = \sec \theta - \tan \theta$ $\therefore \frac{dy}{d\theta} = \sec \theta \tan \theta - \sec^2 \theta$ $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ $= \frac{\sec \theta \tan \theta - \sec^2 \theta}{\sec \theta \tan \theta + \sec^2 \theta}$ $= \frac{\sec \theta (\tan \theta - \sec \theta)}{\sec \theta (\tan \theta + \sec \theta)}$ $= \frac{-(\sec \theta - \tan \theta)}{(\tan \theta + \sec \theta)}$ $= \frac{-y}{x}$	1 1 1	
d)	Find Radius of curvature of curve $y = e^x$ at point $[0,1]$.	$y = e^x$	04



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2.	Ans	$\frac{dy}{dx} = e^x$ $\frac{d^2y}{dx^2} = e^x$ at point [0,1] $\frac{dy}{dx} = e^0 = 1$ $\frac{d^2y}{dx^2} = e^0 = 1$ $\rho = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ $= \frac{(1+1^2)^{\frac{3}{2}}}{1}$ $= 2.828$	<p>1</p> <p>1</p> <p>1</p> <p>1</p>
3.	Ans	<p>Solve any THREE of the following:</p> <p>a) Find the equation of tangent to the curve $y = x(x-2)$ at the point (2,0).</p> $y = x(x-2)$ $= x^2 - 2x$ $\therefore \frac{dy}{dx} = 2x - 2$ at point (2,0) $\therefore \frac{dy}{dx} = 2(2) - 2$ $\therefore m = 2$ Equation of tangent is $y - y_1 = m(x - x_1)$ $y - 0 = 2(x - 2)$ $y = 2x - 4$ $2x - y - 4 = 0$	<p>12</p> <p>04</p> <p>1</p> <p>1</p> <p>1</p>



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3.	b)	If $y = (\tan x)^x$ then find $\frac{dy}{dx}$.	04
	Ans	$y = (\tan x)^x$ $\therefore \log y = \log (\tan x)^x$ $\therefore \log y = x \log (\tan x)$ $\therefore \frac{1}{y} \frac{dy}{dx} = x \cdot \frac{1}{\tan x} \cdot \sec^2 x + \log (\tan x) \cdot 1$ $\therefore \frac{1}{y} \frac{dy}{dx} = \frac{x \sec^2 x}{\tan x} + \log (\tan x)$ $\therefore \frac{dy}{dx} = y \left[\frac{x \sec^2 x}{\tan x} + \log (\tan x) \right]$ $\therefore \frac{dy}{dx} = (\tan x)^x \left[\frac{x \sec^2 x}{\tan x} + \log (\tan x) \right]$	1 2 1
	c)	Find $\frac{dy}{dx}$ if $y = \cos^{-1} [4x^3 - 3x]$	04
	Ans	$y = \cos^{-1} [4x^3 - 3x]$ Put $x = \cos \theta \quad \therefore \theta = \cos^{-1} x$ $\therefore y = \cos^{-1} [4 \cos^3 \theta - 3 \cos \theta]$ $= \cos^{-1} [\cos 3\theta]$ $= 3\theta$ $= 3 \cos^{-1} x$ $\therefore \frac{dy}{dx} = 3 \left(\frac{-1}{\sqrt{1-x^2}} \right)$ $= \frac{-3}{\sqrt{1-x^2}}$	1 1 1 1



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4.		Solve any <u>THREE</u> of the following:	12
	a)	Evaluate : $\int \frac{dx}{4\cos^2 x + 9\sin^2 x}$	04
	Ans	$\int \frac{dx}{4\cos^2 x + 9\sin^2 x}$ $= \int \frac{\frac{dx}{\cos^2 x}}{4\cos^2 x + 9\sin^2 x}$ $= \int \frac{\sec^2 x dx}{4 + 9\tan^2 x}$ <p>Put $\tan x = t$ $\sec^2 x dx = dt$</p> $= \int \frac{dt}{4 + 9t^2}$ $\int \frac{1}{2^2 + (3t)^2} dt \quad \text{or} \quad \frac{1}{9} \int \frac{dt}{\left(\frac{2}{3}\right)^2 + t^2}$ $= \frac{1}{2} \tan^{-1}\left(\frac{3t}{2}\right) \frac{1}{3} + c \quad = \frac{1}{9} \frac{1}{\frac{2}{3}} \tan^{-1}\left(\frac{t}{\frac{2}{3}}\right) + c$ $= \frac{1}{6} \tan^{-1}\left(\frac{3 \tan x}{2}\right) + c \quad = \frac{1}{6} \tan^{-1}\left(\frac{3}{2} \tan x\right) + c$	<p>½</p> <p>½</p> <p>1</p> <p>½</p>
	b)	Evaluate : $\int \frac{dx}{5 + 4\cos x}$	04
	Ans	$\int \frac{dx}{5 + 4\cos x}$	



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4.	b)	$\text{Put } \tan \frac{x}{2} = t, \quad dx = \frac{2dt}{1+t^2}, \quad \cos x = \frac{1-t^2}{1+t^2}$ $I = \int \frac{2dt}{5+4\left(\frac{1-t^2}{1+t^2}\right)}$ $= \int \frac{2dt}{5(1+t^2)+4(1-t^2)}$ $= 2 \int \frac{dt}{5+5t^2+4-4t^2}$ $= 2 \int \frac{dt}{t^2+9}$ $= 2 \int \frac{dt}{t^2+3^2}$ $= 2 \frac{1}{3} \tan^{-1} \frac{t}{3} + c$ $= \frac{2}{3} \tan^{-1} \left(\frac{\tan \frac{x}{2}}{3} \right) + c$	<p>1</p> <p>½</p> <p>1</p> <p>1</p> <p>½</p>
	c)	Evaluate $\int x \cdot \tan^{-1} x \, dx$	04
	Ans	$\int x \cdot \tan^{-1} x \, dx$ $= \tan^{-1} x \int x \, dx - \int \left(\int x \, dx \right) \frac{d}{dx} (\tan^{-1} x) \, dx + c$ $= \frac{x^2}{2} \tan^{-1} x - \int \frac{x^2}{2} \frac{1}{x^2+1} \, dx + c$ $= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{x^2}{x^2+1} \, dx + c$ $= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{(x^2+1)-1}{x^2+1} \, dx + c$ $= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \left(1 - \frac{1}{x^2+1} \right) \, dx + c$	<p>1</p> <p>1</p> <p>1</p>



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4.		$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} (x - \tan^{-1} x) + c$	1
	d)	<p>Evaluate : $\int \frac{dx}{\sqrt{13-6x-x^2}}$</p>	04
	Ans	$\int \frac{dx}{\sqrt{13-6x-x^2}}$ $= \int \frac{dx}{\sqrt{-(x^2+6x-13)}}$ <p>Third term = $\left(\frac{1}{2} \times 6\right)^2 = 9$</p> $= \int \frac{dx}{\sqrt{-(x^2+6x+9-9-13)}}$ $= \int \frac{dx}{\sqrt{-((x+3)^2-22)}}$ $= \int \frac{dx}{\sqrt{(\sqrt{22}^2-(x+3)^2)}}$ $= \sin^{-1} \left(\frac{x+3}{\sqrt{22}} \right) + c$	1 1 1 1
	e)	<p>Evaluate : $\int_0^{\frac{\pi}{2}} \frac{\tan x}{\tan x + \cot x} dx$</p>	04
	Ans	<p>Let $I = \int_0^{\frac{\pi}{2}} \frac{\tan x}{\tan x + \cot x} dx$ -----(1)</p> $= \int_0^{\frac{\pi}{2}} \frac{\tan \left(\frac{\pi}{2} - x \right)}{\tan \left(\frac{\pi}{2} - x \right) + \cot \left(\frac{\pi}{2} - x \right)} dx$	1



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4.		$= \int_0^{\frac{\pi}{2}} \frac{\cot x}{\cot x + \tan x} dx \text{ -----(2)}$ <p>Adding equation (1) and (2)</p> $\therefore 2I = \int_0^{\frac{\pi}{2}} \frac{\tan x}{\tan x + \cot x} dx + \int_0^{\frac{\pi}{2}} \frac{\cot x}{\cot x + \tan x} dx$ $= \int_0^{\frac{\pi}{2}} \frac{\tan x + \cot x}{\tan x + \cot x} dx$ $= \int_0^{\frac{\pi}{2}} 1 dx$ $= [x]_0^{\frac{\pi}{2}}$ $= \frac{\pi}{2} - 0$ $2I = \frac{\pi}{2}$ $\therefore I = \frac{\pi}{4}$	<p>1</p> <p>1</p> <p>1</p>
5.	a) i)	<p>Solve any TWO of the following:</p> <p>Evaluate : $\int_{-1}^1 \frac{1}{1+x^2} dx$</p> $\int_{-1}^1 \frac{1}{1+x^2} dx$ $= [\tan^{-1} x]_{-1}^1$ $= \tan^{-1} 1 - \tan^{-1} (-1)$ $= \frac{\pi}{4} + \frac{\pi}{4}$ $= \frac{\pi}{2} \text{ or } 90^\circ$	<p>12</p> <p>03</p> <p>1</p> <p>1</p> <p>1</p>



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5.		$\left(\frac{d^2y}{dx^2}\right)^4 = 1 + \left(\frac{dy}{dx}\right)^2$ <p>order = 2 degree = 4</p>	<p>1½</p> <p>1½</p>
	ii)	<p>Find integrating factor of D.E. $x \frac{dy}{dx} - y = x^2$.</p> <p>Ans $x \frac{dy}{dx} - y = x^2$</p> $\frac{dy}{dx} - \frac{y}{x} = x$ <p>Comparing with $\frac{dy}{dx} + Py = Q$</p> $P = -\frac{1}{x}, \quad Q = x$ <p>Integrating factor = $e^{\int P dx}$</p> $= e^{\int -\frac{1}{x} dx}$ $= e^{-\log x}$ $= e^{\log x^{-1}}$ $= \frac{1}{x}$	<p>03</p> <p>1</p> <p>1</p> <p>1</p>
	c)	<p>Solve the DE $L \frac{dI}{dt} + RI = E$, given $I = 0$ when $t = 0$ and L, E, R are constants.</p> <p>Ans $L \frac{dI}{dt} + RI = E$</p> $\therefore \frac{dI}{dt} + \frac{R}{L} I = \frac{E}{L}$ <p>Comparing with $\frac{dy}{dx} + Py = Q$</p>	<p>06</p>



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5.		$\therefore P = \frac{R}{L} \text{ and } Q = \frac{E}{L}$ $IF = e^{\int \frac{R}{L} dt} = e^{\frac{R}{L}t}$ $\therefore \text{Solution is}$ $I \cdot IF = \int Q \cdot IF dt + c$ $I \cdot e^{\frac{R}{L}t} = \int \frac{E}{L} e^{\frac{R}{L}t} dt + c$ $= \frac{E}{L} \int e^{\frac{R}{L}t} dt + c$ $= \frac{E}{L} \frac{e^{\frac{R}{L}t}}{\frac{R}{L}} + c$ $= \frac{E}{R} e^{\frac{R}{L}t} + c$ <p>when $t = 0, I = 0$</p> $\therefore c = \frac{-E}{R}$ $\therefore I \cdot e^{\frac{R}{L}t} = \frac{E}{R} e^{\frac{R}{L}t} - \frac{E}{R}$ $\therefore I = \frac{E}{R} \left(1 - e^{-\frac{R}{L}t} \right)$	<p>2</p> <p>1</p> <p>1</p> <p>1</p>
6.		<p>Solve any TWO of the following:</p> <p>a) i) Express $z = \frac{-1}{2} + i \frac{\sqrt{3}}{2}$ in polar form.</p> <p>Ans $z = \frac{-1}{2} + i \frac{\sqrt{3}}{2}$ Comparing with $Z = x + iy$</p> $\therefore x = \frac{-1}{2}, y = \frac{\sqrt{3}}{2}$ $r = \sqrt{x^2 + y^2}$ $r = \sqrt{\left(\frac{-1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 1$	<p>12</p> <p>03</p> <p>1</p>



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6.		$= 3L^{-1} \left[\frac{1}{\left(s - \frac{3}{2}\right)} \right]$ $= 3e^{\frac{3}{2}t}$	1
	ii)	<p>Find $L^{-1} \left[\frac{3s+2}{s^2+16} \right]$</p> $L^{-1} \left[\frac{3s+2}{s^2+16} \right]$ $= L^{-1} \left[\frac{3s+2}{s^2+16} \right]$ $= L^{-1} \left[\frac{3s}{s^2+16} + \frac{2}{s^2+16} \right]$ $= 3L^{-1} \left(\frac{s}{s^2+16} \right) + \frac{1}{2} L^{-1} \left(\frac{4}{s^2+16} \right)$ $= 3 \cos 4t + \frac{1}{2} \sin 4t$	03 1 1+1
	c)	<p>Solve using Laplace transform.</p> $\frac{dy}{dt} + 3y = 1 + e^t, \text{ given that } y(0) = -1$ $\frac{dy}{dt} + 3y = 1 + e^t$ $\therefore L \left\{ \frac{dy}{dt} + 3y \right\} = L \{ 1 + e^t \}$ $\therefore sL(y) - y(0) + 3L(y) = \frac{1}{s} + \frac{1}{s-1}$ $\therefore sL(y) + 1 + 3L(y) = \frac{s-1+s}{s(s-1)}$ $\therefore (s+3)L(y) = \frac{2s-1}{s(s-1)} - 1$	06 1



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		$\therefore (s+3)L(y) = \frac{2s-1-s^2+s}{s(s-1)}$ $\therefore L(y) = \frac{-s^2+3s-1}{s(s-1)(s+3)}$ $\therefore y = L^{-1}\left(\frac{-s^2+3s-1}{s(s-1)(s+3)}\right)$ <p>Let $\frac{-s^2+3s-1}{s(s-1)(s+3)} = \frac{A}{s} + \frac{B}{s-1} + \frac{C}{s+3}$</p> $\therefore -s^2+3s-1 = A(s-1)(s+3) + Bs(s+3) + Cs(s-1)$ <p>Put $s = 0$</p> $\therefore -1 = A(0-1)(0+3)$ $\therefore A = \frac{1}{3}$ <p>Put $s = 1$</p> $\therefore -1^2+3(1)-1 = B(1)(1+3)$ $B = \frac{1}{4}$ <p>Put $s = -3$</p> $-(-3)^2+3(-3)-1 = C(-3)(-3-1)$ $-19 = 12C$ $C = \frac{-19}{12}$ $\frac{-s^2+3s-1}{s(s-1)(s+3)} = \frac{1}{3} + \frac{1}{4} + \frac{-19}{s+3}$ $\therefore y = L^{-1}\left\{\frac{1}{3} + \frac{1}{4} + \frac{-19}{s+3}\right\}$ $\therefore y = \frac{1}{3} + \frac{1}{4}e^t - \frac{19}{12}e^{-3t}$	<p>1</p> <p>1</p> <p>1</p> <p>2</p>



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		<p style="text-align: center;"><u>Important Note</u></p> <p><i>In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.</i></p> <p>-----</p>	