Important Instructions to examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.
8) As per the policy decision of Maharashtra State Government, teaching in English/Marathi and Bilingual (English + Marathi) medium is introduced at first year of AICTE diploma Programme from academic year 2021-2022. Hence if the students in first year (first and second semesters) write answers in Marathi or bilingual language (English +Marathi), the Examiner shall consider the same and assess the answer based on matching of concepts with model answer.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.		**Solve any FIVE of the Following:**	**10**
	a)	If $f(x)=x^3-3x^2+5,$ find $f(0)+f(2)$	**02**
	Ans	$f(x)=x^3-3x^2+5$	
		$\therefore f(0)=0^3-3(0^2)+5=5$	½
		$f(2)=2^3-3(2^2)+5=1$	½
		$\therefore f(0)+f(2)=5+1=6$	1
		--	
	b)	Show that $f(x)=4x^4+3\cos x+x\sin x+1$ is an even function.	**02**
	Ans	$f(x)=4x^4+3\cos x+x\sin x+1$	
		$\therefore f(-x)=4(-x)^4+3\cos(-x)+(-x)\sin(-x)+1$	1
		$=4x^4+3\cos x+x\sin x+1$	
		$=f(x)$	
		$\therefore$ Given function is even.	1
		--	

Q. No	Sub Q. N.	Answer	Marking Scheme
1.	c)	Find $\frac{dy}{dx}$ if $y=e^{x}.\sin x.$	**02**
	Ans	$y=e^{x}.\sin x$	
		$\frac{dy}{dx}=e^{x}.\cos x+\sin x.e^{x}$	2
	d)	Evaluate $\int\frac{1}{3x+7}dx$	**02**
	Ans	$\int\frac{1}{3x+7}dx=\log\left(3x+7\right)\cdot\frac{1}{3}$	2
	e)	$\int\cos^{2}xdx$	**02**
	Ans	$\int\cos^{2}xdx=\int\frac{1+\cos 2x}{2}dx$	
		$=\frac{1}{2}\int(1+\cos 2x)dx$	1
		$=\frac{1}{2}\left(x+\frac{\sin 2x}{2}\right)+c$	1
	f)	Find the area under the curve $y=x^{2}$ from $x=0$ to $x=3$ with $x-axis.$	**02**
	Ans	Area $A=\int_{a}^{b}ydx=\int_{0}^{3}x^{2}dx$	
		$=\left[\frac{x^{3}}{3}\right]_{0}^{3}$	1
		$=\frac{3^{3}}{3}-0=9$	1
	h)	State the Trapezoidal rule of numerical integration.	**02**
		Trapezoidal rule	
	Ans	$\int_{a}^{b}f\left(x\right)dx=\frac{h}{2}\left[\left(y_{0}+y_{n}\right)+2\left(y_{1}+y_{2}+...+y_{n-1}\right)\right]$ where $h=\frac{b-a}{n}$	2

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.		**Solve any THREE of the following:**	**12**
	a)	Find $\frac{dy}{dx}$ if $x\sin y+y\sin x=0$	**04**
	Ans	$x\sin y+y\sin x=0$	
		$\therefore x\cos y\frac{dy}{dx}+\sin y+y\cos x+\sin x\frac{dy}{dx}=0$	2
		$\frac{dy}{dx}\left(x\cos y+\sin x\right)=-\sin y-y\cos x$	1
		$\frac{dy}{dx}=\frac{-\sin y-y\cos x}{x\cos y+\sin x}$	1
		--	
	b)	If $x=a\left(\theta-\sin\theta\right),\ y=a\left(1-\cos\theta\right)$ find $\frac{dy}{dx}$	**04**
	Ans	$\because x=a\left(\theta-\sin\theta\right)$	
		$\therefore \frac{dx}{d\theta}=a\left(1-\cos\theta\right)$	1½
		$\because y=a\left(1-\cos\theta\right)$	
		$\therefore \frac{dy}{d\theta}=a\left(\sin\theta\right)$	1½
		$\therefore \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$	
		$\therefore \frac{dy}{dx}=\frac{a\sin\theta}{a\left(1-\cos\theta\right)}$	
		$=\frac{\sin\theta}{\left(1-\cos\theta\right)}$	1
		--	
	c)	A manufacturer can sell x items at price of Rs.$\left(330-x\right)$ each. The cost of producing x items is Rs.$\left(x^2+10x+12\right)$. How many items must be sold so that his profit is maximum .	**04**

Q. No.	Sub Q.N.	Answer	Marking Scheme
2.	Ans	Let number of items be x	
		Selling price $=(330-x)x=330x-x^2$	½
		Cost price $=\left(x^2+10x+12\right)$	
		Profit $=$ Selling price $-$ Cost price	
		$\therefore\ p=330x-x^2-\left(x^2+10x+12\right)$	
		$=330x-x^2-x^2-10x-12$	
		$=320x-2x^2-12$	½
		$\dfrac{dp}{dx}=320-4x$	1
		$\dfrac{d^2p}{dx^2}=-4$	1
		$\therefore$ profit is maximum	
		Let $\dfrac{dp}{dx}=0$	
		$\therefore 320-4x=0$	
		$\therefore x=80$	1
		--	
	d)	Find the radius of curvature for $y=x^3+3x^2+2$ at $(1,2)$.	**04**
	Ans	$y=x^3+3x^2+2$	
		$\therefore \dfrac{dy}{dx}=3x^2+6x$	1
		$\therefore \dfrac{d^2y}{dx^2}=6x+6$	1
		at $(1,2)$	
		$\dfrac{dy}{dx}=3(1)^2+6(1)=9$	½
		$\therefore \dfrac{d^2y}{dx^2}=6(1)+6=12$	½
		$\therefore$ Radius of curvature is $\rho=\dfrac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.		$=\frac{\left[1+(9)^2\right]^{\frac{3}{2}}}{12}$	
		$=61.878$	1
3.		**Solve any <u>THREE</u> of the following:**	**12**
	a)	Find the equation of tangent to the curve $2x^2-xy+3y^2=18$ at $(3,1)$.	**04**
	Ans	$2x^2-xy+3y^2=18$	
		$\therefore 4x-\left(x\frac{dy}{dx}+y.1\right)+6y\frac{dy}{dx}=0$	1
		$\therefore 4x-x\frac{dy}{dx}-y+6y\frac{dy}{dx}=0$	
		$\therefore (6y-x)\frac{dy}{dx}=y-4x$	
		$\therefore \frac{dy}{dx}=\frac{y-4x}{6y-x}$	1
		at $(3,1)$	
		$\therefore \frac{dy}{dx}=\frac{1-4(3)}{6(1)-3}$	
		$\therefore \frac{dy}{dx}=\frac{-11}{3}$	
		$\therefore$ slope of tangent , $m=\frac{-11}{3}$	½
		Equation of tangent at $(3,1)$ is	
		$y-1=\frac{-11}{3}(x-3)$	½
		$\therefore 3y-3=-11x+33$	
		$\therefore 11x+3y-36=0$	1
	b)	Find $\frac{dy}{dx}$ if $y=x^{\sin x}+(\tan x)^x$	**04**

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	Ans	Let $u = x^{\sin x}$	
		$\therefore \log u = \log x^{\sin x}$	
		$\therefore \log u = \sin x \log x$	½
		$\therefore \frac{1}{u}\frac{du}{dx} = \sin x \cdot \frac{1}{x} + \log x.\cos x$	1
		$\therefore \frac{1}{u}\frac{du}{dx} = \frac{\sin x}{x} + \cos x \log x$	
		$\therefore \frac{du}{dx} = u\left(\frac{\sin x}{x} + \cos x \log x\right)$	
		$\therefore \frac{du}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + \cos x \log x\right)$	½
		Let $v = (\tan x)^x$	
		$\therefore \log v = \log(\tan x)^x$	
		$\therefore \log v = x \log(\tan x)$	
		$\therefore \frac{1}{v}\frac{dv}{dx} = x \cdot \frac{1}{\tan x}.\sec^2 x + \log(\tan x) \cdot 1$	1
		$\therefore \frac{1}{v}\frac{dv}{dx} = \frac{x \sec^2 x}{\tan x} + \log(\tan x)$	
		$\therefore \frac{dv}{dx} = v\left[\frac{x \sec^2 x}{\tan x} + \log(\tan x)\right]$	
		$\therefore \frac{dv}{dx} = (\tan x)^x \left[\frac{x \sec^2 x}{\tan x} + \log(\tan x)\right]$	½
		$\therefore \frac{dy}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + \cos x \log x\right) + (\tan x)^x \left[\frac{x \sec^2 x}{\tan x} + \log(\tan x)\right]$	½
		--	
	c)	Find $\frac{dy}{dx}$ if $y = \log(xe^x)$	**04**
	Ans	$y = \log(xe^x)$	
		$\frac{dy}{dx} = \frac{1}{xe^x}(xe^x + e^x)$	04
		--	

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	d)	Evaluate $\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx$	**04**
	Ans	$\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx$	
		Let $\sqrt{x}=t$	½
		$\therefore \frac{1}{2\sqrt{x}}dx=dt$	1
		$\therefore \frac{1}{\sqrt{x}}dx=2dt$	½
		$=\int \cos t2dt$	½
		$=2\int \cos tdt$	
		$=2\sin t+c$	1
		$=2\sin\sqrt{x}+c$	½
4.		**Solve any <u>THREE</u> of the following:**	**12**
	a)	Evaluate $\int \frac{1}{x^2+3x+2}dx$	**04**
	Ans	$\int \frac{1}{x^2+3x+2}dx$	
		$\int \frac{1}{(x+1)(x+2)}dx$	½
		Consider $\frac{1}{(x+1)(x+2)}=\frac{A}{x+1}+\frac{B}{x+2}$	½
		$\therefore 1=A(x+2)+B(x+1)$	
		Put $x=-1$	
		$\therefore 1=A$	1
		Put $x=-2$	
		$\therefore 1=B(-1)$	1
		$\therefore B=-1$	
		$\int \frac{1}{(x+1)(x+2)}dx=\int\left(\frac{1}{x+1}+\frac{-1}{x+2}\right)dx$	
		$=\log(x+1)-\log(x+2)+c$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	b)	Evaluate $\int \frac{dx}{5+4\cos x}$	**04**
	Ans	$\int \frac{dx}{5+4\cos x}$	
		Put $\tan\frac{x}{2}=t$, $dx=\frac{2dt}{1+t^2}$, $\cos x=\frac{1-t^2}{1+t^2}$	
		$I=\int \frac{\frac{2dt}{1+t^2}}{5+4\left(\frac{1-t^2}{1+t^2}\right)}$	1
		$=\int \frac{2dt}{5\left(1+t^2\right)+4\left(1-t^2\right)}$	
		$=2\int \frac{dt}{5+5t^2+4-4t^2}$	
		$=2\int \frac{dt}{t^2+9}$	1
		$=2\int \frac{dt}{t^2+3^2}$	½
		$=2\frac{1}{3}\tan^{-1}\frac{t}{3}+c$	1
		$=\frac{2}{3}\tan^{-1}\left(\frac{\tan\frac{x}{2}}{3}\right)+c$	½
		--	
	c)	Evaluate $\int \frac{1-\tan x}{1+\tan x}dx$	**04**
	Ans	$=\int \frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}dx$	1
		$=\int \frac{\cos x-\sin x}{\cos x+\sin x}dx$	1
		$=\log\left(\cos x+\sin x\right)+c$	2
		--	

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	d)	Evaluate: $\int \frac{\log x}{x(\log x+2)(\log x+3)}\,dx$	**04**
	Ans	$\int \frac{\log x}{x(\log x+2)(\log x+3)}\,dx$	
		Put $\log x = t$ $\therefore \frac{1}{x}dx = dt$	1
		$\int \frac{t}{(t+2)(t+3)}\,dt$	
		consider $\frac{t}{(t+2)(t+3)} = \frac{A}{t+2} + \frac{B}{t+3}$	½
		$\therefore t = A(t+3) + B(t+2)$	
		Put $t = -2$ $A = -2$	½
		Put $t = -3$ $B = 3$	½
		$\therefore \frac{t}{(t+2)(t+3)} = \frac{-2}{t+2} + \frac{3}{t+3}$	
		$\therefore \int \frac{t}{(t+2)(t+3)}\,dt = \int \left(\frac{-2}{t+2} + \frac{3}{t+3} \right) dt$	
		$= -2\log(t+2) + 3\log(t+3) + c$	1
		$= -2\log(\log x+2) + 3\log(\log x+3) + c$	½
	e)	Evaluate $\int_0^7 \frac{\sqrt[3]{x}}{\sqrt[3]{x} + \sqrt[3]{7-x}}\,dx$	**04**
	Ans	Let $I = \int_0^7 \frac{\sqrt[3]{x}}{\sqrt[3]{x} + \sqrt[3]{7-x}}\,dx$ --------------------(1)	
		$\therefore I = \int_0^7 \frac{\sqrt[3]{7-x}}{\sqrt[3]{7-x} + \sqrt[3]{7-(7-x)}}\,dx$	½
		$\therefore I = \int_0^7 \frac{\sqrt[3]{7-x}}{\sqrt[3]{7-x} + \sqrt[3]{x}}\,dx$ --------------------(2)	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.		add (1) and (2)	
		$\therefore I+I=\int_0^7 \frac{\sqrt[3]{x}}{\sqrt[3]{x}+\sqrt[3]{7-x}}\,dx+\int_0^7 \frac{\sqrt[3]{7-x}}{\sqrt[3]{7-x}+\sqrt[3]{x}}\,dx$	
		$\therefore 2I=\int_0^7 \frac{\sqrt[3]{x}+\sqrt[3]{7-x}}{\sqrt[3]{x}+\sqrt[3]{7-x}}\,dx$	½
		$\therefore 2I=\int_0^7 1\,dx$	
		$\therefore 2I=\left[x\right]_0^7$	1
		$\therefore 2I=7-0$	
		$\therefore I=\frac{7}{2}$	1
		--	
		Solve any TWO of the following:	**12**
5.	a)	Find the area of the circle $x^2+y^2=25$ by using definite integration.	**06**
	Ans	$x^2+y^2=25$	
		$\therefore y^2=25-x^2$	
		$\therefore y=\sqrt{25-x^2}$	½
		$At\ \ y=0,\quad 25-x^2=0$	½
		$\therefore x=5$	
		$\therefore A=4\int_a^b ydx$	½
		$=4\int_0^5 \sqrt{25-x^2}\,dx$	
		$=4\int_0^5 \sqrt{5^2-x^2}\,dx$	½
		$=4\left[\frac{x}{2}\sqrt{5^2-x^2}+\frac{5^2}{2}\sin^{-1}\left(\frac{x}{5}\right)\right]_0^5$	1
		$=4\left[0+\frac{25}{2}\sin^{-1}(1)\right]-\left[0+\frac{25}{2}\sin^{-1}(0)\right]$	1 1
		$=4\left[\frac{25}{2}\cdot\frac{\pi}{2}\right]$	
		$=25\pi$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	b)	Attempt the following:	**06**
	i)	Find the order and degree of the D.E.	**03**
	Ans	$\frac{d^2y}{dx^2}=\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}$	
		Squaring both sides	
		$\left(\frac{d^2y}{dx^2}\right)^2=\left[1+\left(\frac{dy}{dx}\right)^2\right]^3$	
		$\text{order}=2$	1½
		$\text{degree}=2$	1½
		--	
	ii)	Solve the D.E.	**03**
		$x\sqrt{1-y^2}\,dx+y\sqrt{1-x^2}\,dy=0$	
	Ans	$x\sqrt{1-y^2}\,dx+y\sqrt{1-x^2}\,dy=0$	
		$\therefore x\sqrt{1-y^2}\,dx=-y\sqrt{1-x^2}\,dy$	
		$\frac{x}{\sqrt{1-x^2}}dx=-\frac{y}{\sqrt{1-y^2}}dy$	1
		$\therefore \int\frac{x}{\sqrt{1-x^2}}dx=-\int\frac{y}{\sqrt{1-y^2}}dy$	
		$\therefore \int\frac{-2x}{\sqrt{1-x^2}}dx=-\int\frac{-2y}{\sqrt{1-y^2}}dy$	1
		$2\sqrt{1-x^2}=-2\sqrt{1-y^2}+c$	1
		--	
	c)	The velocity of a particle is given by $\mathrm{V}=t^2-6t+7$. Find distance covered in 3 seconds.	**06**
	Ans	$V=t^2-6t+7$	
		$\therefore \frac{dx}{dt}=t^2-6t+7$	1
		$\therefore dx=\left(t^2-6t+7\right)dt$	1

SUMMER – 2022 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** **Subject Code:** 22201

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.		$\therefore \int dx = \int \left(t^2 - 6t + 7\right) dt$	
		$\therefore x = \frac{t^3}{3} - 3t^2 + 7t + c$	1
		Initially $x = 0$ when $t = 0$	
		$\therefore c = 0$	1
		$\therefore x = \frac{t^3}{3} - 3t^2 + 7t$	1
		Distance covered in 3 sec,	
		$\therefore x = \frac{(3)^3}{3} - 3(3)^2 + 7(3)$	
		$\therefore x = 3$	1
		--	
6.		**Solve any TWO of the following:**	**12**
	a)	Using Trapezoidal rule calculate the approximate value of	**06**
	i)	$\int_0^4 e^x dx$ from given data:	**03**

x	0	1	2	3	4
y	1	2.72	7.39	20.09	54.60

	Ans	$\int_a^b f(x)\,dx = \frac{h}{2}\left[\left(y_0 + y_n\right) + 2\left(y_1 + y_2 + ... + y_{n-1}\right)\right]$	
		$a = 0, b = 4$ and $h = 1$	1
		$\therefore \int_0^4 e^x dx = \frac{1}{2}\left[\left(1 + 54.60\right) + 2\left(2.72 + 7.39 + 20.09\right)\right]$	1
		$= 58$	1
		--	
	ii)		
		Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's one third rule given by	**03**

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.			

x	0	0.25	0.5	0.75	1
y	1	0.9412	0.8	0.64	0.5

Ans

Let $y=\frac{1}{1+x^2}$ $a=0,b=1$ and $n=4$

$\therefore h=\frac{b-a}{n}=\frac{1-0}{4}=\frac{1}{4}$ — 1

Using Simpson's $1/3^{rd}$ rule

$$\int_a^b f(x)dx=\frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+...+y_{n-1})+2(y_2+y_4+...+y_{n-2})\right]$$

$$\therefore\int_0^1 f(x)dx=\frac{1/4}{3}\left[(1+0.5)+4(0.9412+0.64)+2(0.8)\right]$$ — 1

$$\therefore\int_0^1\frac{1}{1+x^2}dx=0.7854$$ — 1

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b) Evaluate $\int_0^6\frac{1}{1+x}dx$ taking $h=1$ by using Simpson's one third rule. — **06**

Ans

Let $y=\frac{1}{1+x}$, $h=1$

x	0	1	2	3	4	5	6
y	1	0.5	0.33	0.25	0.2	0.17	0.14

2

Using Simpson's $1/3^{rd}$ rule

$$\int_a^b f(x)dx=\frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+...+y_{n-1})+2(y_2+y_4+...+y_{n-2})\right]$$

$$\therefore\int_0^6 f(x)dx=\frac{1}{3}\left[(1+0.14)+4(0.5+0.25+0.17)+2(0.33+0.2)\right]$$ — 2

$$\therefore\int_0^6\frac{1}{1+x}dx=1.96$$ — 2

SUMMER – 2022 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** **Subject Code:** 22201

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	c)	Evaluate $\int_0^{\pi} \sin x dx$ using Simpson's $3/8^{th}$ rule. Divide the interval $[0,\pi]$ into 6 equal parts.	**06**
	Ans	Here $n=6$	
		$y=\sin x$ $a=0,\quad b=\pi$	
		$\therefore h=\frac{\pi-0}{6}=\frac{\pi}{6}$	1

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$y=\sin x$	0	0.5	0.866	1	0.866	0.5	0

			2
		Using Simpson's $3/8^{th}$ rule.	
		$\int_a^b f(x)dx=\frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2+y_4...+y_{n-1})+2(y_3+y_6+...+y_{n-3})\right]$	
		$\therefore \int_0^{\pi} \sin xdx=\frac{3\left(\frac{\pi}{6}\right)}{8}\left[(0+0)+3(0.5+0.866+0.866+0.5)+2(1)\right]$	1
		$\therefore \int_0^{\pi} \sin xdx=2.002$	2
		OR	

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$y=\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0

			2
		$\therefore \int_0^{\pi} \sin xdx=\frac{3\left(\frac{\pi}{6}\right)}{8}\left[(0+0)+3\left(\frac{1}{2}+\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}+\frac{1}{2}\right)+2(1)\right]$	1
		$=2.002$	2
		--	

Q. No.	Sub Q. N.	Answer	Marking Scheme
		Important Note *In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.* --	