314320

12526

03 Hours / 70 Marks Seat No. | | | | | | | | |

Instructions – (1) All Questions are *Compulsory.*

(2) Answer each next main Question on a new page.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Assume suitable data, if necessary.

(6) Use of Non-programmable Electronic Pocket Calculator is permissible.

(7) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Solve any <u>FIVE</u> of the following : **10**

a) If f $(x, y) = x^3 + y^3 + 3xy$ find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$

b) If f $(x, y) = x^3y + \cos x + \sin y$ find $\frac{\partial^2 f}{\partial x \partial y}$

c) Find the rank of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$

d) Find the eigen value of matrix $A = \begin{bmatrix} 10 & -9 \\ 6 & -5 \end{bmatrix}$

e) Find the projection of $\bar{a}$ on $\bar{b}$ if $\bar{a} = 2\bar{i} + \bar{j} + \bar{k}$ and $\bar{b} = \bar{i} + 3\bar{j} + \bar{k}$

f) Show that the vectors $2\bar{i} + 3\bar{j} + \bar{k}$ and $4\bar{i} - 2\bar{j} - 2\bar{k}$ are perpendicular to each other

P.T.O.

Marks

g) Given

x	0	$\pi/8$	$\pi/4$
$y\ (x) = \tan x$	0	0.4141	1.0

Find $\int_0^{\pi/4} \tan x\ dx$ using trapezoidal rule.

2. Solve any THREE of the following : 12

a) Examine the function $f\ (x,\ y) = x^2 - xy + 2y^2 - 5x + 6y - 9$ for maxima or minima

b) Reduce the given matrix to normal form and hence find its rank

$$A = \begin{bmatrix} 1 & 4 & -7 \\ 2 & -2 & -3 \\ 3 & 2 & -10 \end{bmatrix}$$

c) Examine the following linear system of equation for consistency and solve it if consistent.

$x + 2y - 3z = -1\ ;\ 4x - 2y + 6z = 8$

$15x - 3y + 9z = 21$

d) A force $\bar{F} = 2\bar{i} + \bar{j} - \bar{k}$ act on a point $3\bar{i} - \bar{j} - 2\bar{k}$. Find the moment of the force about point $4\bar{i} - \bar{j} + \bar{k}$

3. Solve any THREE of the following : 12

a) Divide 120 into three parts such that sum of product of two taken at a time is maximum

b) Find the inverse of matrix by elementary transformation

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

c) Determine the value of λ for which the system of equation is consistent and also find solution

$3x + 2y + 4z = 3$

$x + y + z = \lambda$

$5x + 4y + 6z = 15$

d) Find extreme value of f $(x,\ y) = x^2 + y^2$ subjected to the condition $x + y = 4$ using Lagrange undetermined multiplier.

Marks

4. Solve any THREE of the following : **12**

a) Find a vector of magnitude 7 units and perpendicular to
$\bar{a} = 2\bar{i} + \bar{j} - 3\bar{k}$,
$\bar{b} = \bar{i} - 2\bar{j} + \bar{k}$

b) Calculate the volume of parallelopiped whose co terminus edges are $\bar{i} - \bar{j} + \bar{k}$; $2\bar{i} + 3\bar{j} - \bar{k}$ and $-\bar{i} - \bar{j} + 5\bar{k}$

c) Find the angle between the vectors $2\bar{i} + 3\bar{j} + \bar{k}$ and $\bar{i} - 3\bar{j} + 4\bar{k}$

d) Calculate $\int_1^5 \frac{dx}{x+2}$ using Simpson $\frac{1}{3}$ rule.
Divide [1, 5] in 4 equal parts.

e) Solve the following LPP by graphical method
Maximize $z = 5x_1 + 7x_2$
subjected to $x_1 + x_2 \leqslant 4$;
$3x_1 + 8x_2 \leqslant 4$
$x_1 \geqslant 0$ and $x_2 \geqslant 0$
Also find feasible region and optimum solution

5. Solve any TWO of the following : **12**

a) Find the eigen value and eigen vector for
Matrix A = $\begin{bmatrix} 10 & -9 \\ 6 & -5 \end{bmatrix}$

b) Find y'(0) and y" (0) from the following data at $x = 0$

x	0	1	2	3	4	5
y	4	8	15	7	6	2

c) ABC television company operates two assembly lines L_1 and line L_2. Each line is used to assemble the components of three types of T.V : colour standard and economy. The daily expected production is as follows.

T.V / Machine	L_1	L_2
Colour	3	1
Standard	1	1
Economy	2	6

Marks

The daily running cost for two lines are ₹6000 for line L_1 and 4000 for line L_2. It is given that company must produce atleast 24 colour, 16 standard and 48 economy TV sets. Formulate the above problem as LPP to minimize the total cost and solve it graphically.

6. Solve any TWO of the following : **12**

a) i) Test the consistency of the following equation and solve them if possible:

$3x + 3y + 2z = 1;\ x + 2y = 4;\ 10y + 3z = -2$

ii) Find the value of 'k' for which the following equations are consistent. $3x_1 - 2x_2 + 2x_3 = 3;\ x_1 + kx_2 - 3x_3 = 0$ and $4x_1 + x_2 + 2x_3 = 7$

b) i) Evaluate $\int_2^7 \frac{1}{x}\,dx$ by Trapezoidal rule by dividing into 5 equal parts.

ii) Find $f'(0)$ if

x	0	1	2	3	4	5
$f(x)$	3	6	11	18	27	38

c) Solve the following by simplex method

Maximize $z = 3x + 2y$

$x + y \leqslant 4$

$x - y \leqslant 2$

$x,\ y \geqslant 0$