	Model Answer	Exam: SUM 2025
Subject:	Applied Mathematics Common to All Branches	K-Scheme
		SUB CODE: 312301

Important Instructions to STUDENTS

1) *The model answer given here are prepared from the answers from the previously uploaded model answers by Board.*
2) *These model answers are not uploaded by the MSBTE official site but MSBTE study resources website prepared it for students. This model answer has question paper also inbuilt in it, no need to download it separate.*
3) *Please remembter that answers are not checked word to word but based on keywords which must be present in your answer*
4) *The model answer and the answer written by candidate may vary but the examiner may tryto assess the understanding level of the candidate*
5) *While assessing figures, examiner may give credit for principal components indicated in thefigure. The figures drawn bycandidate and model answer may vary. The examiner may give credit for anyequivalent figure drawn*
6) *For programming language papers, credit may be given to any other program based on equivalentconcept*
7) *Students are advised to prepare all the syllabus from recommended book and use these model answers for the purpose of tests.*

Q		ANSWER	Marking Scheme
1	a)	Q. $\int \sqrt{1+\cos 2x}\,dx$.	

→ Let; $I = \sqrt{1+\cos 2x}\,dx$.

using formula:-

$1+\cos\theta = 2\cos^2\left(\frac{\theta}{2}\right)$.

$\therefore 1+\cos 2x = 2\cos^2 x$.

$\therefore I = \int \sqrt{2\cos^2 x}\,dx$.

$\therefore I = \sqrt{2}\int \cos x\,dx$

$\therefore \boxed{I = \sqrt{2}\cdot \sin x + C.}$

1 b)

Question:

Evaluate the integral:

$$\int \left(\frac{1}{\sqrt{1-x^2}} - \cos x \right) dx$$

Solution:

Let

$$I = \int \left(\frac{1}{\sqrt{1-x^2}} - \cos x \right) dx$$

$$I = \int \frac{1}{\sqrt{1-x^2}}\, dx - \int \cos x \, dx$$

$$I = \sin^{-1} x - \sin x + C$$

1 c)

Q. $\int_2^4 \frac{dx}{2x+3}$.

→ Let; $I = \int_2^4 \frac{dx}{2x+3}$

$\therefore I = \left[\log(2x+3)\left(\frac{1}{2}\right) \right]_2^4$

$\therefore I = \frac{1}{2}\left[\log(2\times4+3) - \log(2\times2+3) \right]$

$\therefore I = \frac{1}{2}\left[\log(11) - \log(7) \right]$

$\therefore \boxed{I = \frac{1}{2} \log\left(\frac{11}{7}\right)}$

1	d)	**Q. Find Order & Degree of D.E.** $$\sqrt{\frac{dy}{dx}} = \sqrt[3]{\frac{d^2y}{dx^2}}.$$ → $\sqrt{\frac{dy}{dx}} = \sqrt[3]{\frac{d^2y}{dx^2}}.$ $\left(\frac{dy}{dx}\right)^{\frac{1}{2}} = \left(\frac{d^2y}{dx^2}\right)^{\frac{1}{3}}.$	1/2
		Taking 6^{th} power of Both side; $\left[\left(\frac{dy}{dx}\right)^{\frac{1}{2}}\right]^6 = \left[\left(\frac{d^2y}{dx^2}\right)^{\frac{1}{3}}\right]^6$	1/2
		$\left(\frac{dy}{dx}\right)^3 = \left(\frac{d^2y}{dx^2}\right)^2$ ↓ "Highest Order" derivative.	
		∴ Order of D.E. = "2". Degree of D.E. = "2".	1

1	e)	**Question:** Find approximate root of $x^3 - x - 1 = 0$ using Bisection Method (one iteration only). **Solution:** Evaluate function values: $$f(0) = -1, \quad f(1) = -1, \quad f(2) = 5$$ Root lies in $[1, 2]$. Midpoint: $$c = \frac{1+2}{2} = 1.5$$ Evaluate: $$f(1.5) = 0.875$$ **Final Answer:** Approximate root after one iteration: $\boxed{1.5}$	
1	f)	**Question:** Show that the root of $x^3 - 4x - 9 = 0$ lies in the interval $(2, 3)$. **Solution:** Evaluate the function at the endpoints: $$f(2) = 2^3 - 4 \times 2 - 9 = 8 - 8 - 9 = -9$$ $$f(3) = 3^3 - 4 \times 3 - 9 = 27 - 12 - 9 = 6$$ Since $f(2) < 0$ and $f(3) > 0$, the sign of the function changes between 2 and 3. **Conclusion:** Therefore, the root lies in the interval $\boxed{(2, 3)}$.	

1	g)	**Q: An unbiased coin is tossed 5 times. Find Probability of getting exactly 3 heads?** → coin $p = \frac{1}{2}$; $q = 1 - \frac{1}{2} = \frac{1}{2}$ & $n = 5$ times. $r = 3$ [Exactly 3 heads]. using Binomial Distribution; $P(r) = {}^nC_r \cdot p^r \cdot q^{n-r}$. $\therefore P(3) = {}^5C_3 \times \left(\frac{1}{2}\right)^{(3)} \times \left(\frac{1}{2}\right)^{(5-3)}$. $\therefore$ $P(3) = 0.3125.$	
2	a)	**Q. Find value of $\int \frac{e^x(x+1)}{\cos^2(x \cdot e^x)} dx$.** → Put $x \cdot e^x = t$. diff. w.r.t. x. $\therefore (x \cdot e^x + e^x) dx = dt$. i.e.; $e^x[x+1] dx = dt$. $\therefore I = \int \frac{dt}{\cos^2 t}$ $\therefore I = \int \sec^2 t \, dt$ $\therefore I = \tan(t) + C$ $\therefore$ $I = \tan(x \cdot e^x) + C.$	1 1 1 1

2 b)

$$\int \frac{1}{x^2+3x+2}\,dx$$

Factor denominator: $x^2+3x+2=(x+1)(x+2)$

Partial fractions:

$$\frac{1}{(x+1)(x+2)}=\frac{A}{x+1}+\frac{B}{x+2}$$

Multiply both sides by $(x+1)(x+2)$:

$$1=A(x+2)+B(x+1)$$

Put $x=-2$, then

$$1=B(-1) \implies B=-1$$

Put $x=-1$, then

$$1=A(1) \implies A=1$$

Rewrite integral:

$$\int \frac{1}{x+1}\,dx-\int \frac{1}{x+2}\,dx$$

Integrate:

$$\log|x+1|-\log|x+2|+C=\log\left|\frac{x+1}{x+2}\right|+C$$

$$\boxed{\int \frac{1}{x^2+3x+2}\,dx=\log\left|\frac{x+1}{x+2}\right|+C}$$

2	c)	Q: Solve $\int \frac{dx}{5-4\cos x}$ → Let, $I = \int \frac{dx}{5-4\cos x}$. with standard substitution Put, $\tan(\frac{x}{2}) = t$, $dx = \frac{2dt}{(1+t^2)}$ & $\cos x = \frac{(1-t^2)}{(1+t^2)}$. $I = \int \frac{\frac{2dt}{(1+t^2)}}{5-4\left[\frac{1-t^2}{1+t^2}\right]}$ $\therefore I = \int \frac{\frac{2dt}{(1+t^2)}}{\frac{5(1+t^2)-4(1-t^2)}{(1+t^2)}}$. $\therefore I = 2\int \frac{dt}{5+5t^2-4+4t^2}$ $\therefore I = 2\int \frac{dt}{9t^2+1}$. $\therefore I = \frac{2}{9}\int \frac{dt}{t^2+\frac{1}{9}}$ $\therefore I = \frac{2}{9}\left(\frac{1}{\frac{1}{3}}\right)\tan^{-1}\left(\frac{t}{1/3}\right) + C$ $\therefore I = \frac{2}{3}\tan^{-1}(3t) + C$ $\therefore I = \frac{2}{3}\tan^{-1}\left[3\cdot\tan\left(\frac{x}{2}\right)\right] + C.$
2	d)	$$\int \frac{e^x}{(e^x-1)(e^x+1)}\,dx$$ Let $u = e^x$, then $du = e^x dx$, so $dx = \frac{du}{u}$ $$\int \frac{u}{(u-1)(u+1)} \cdot \frac{1}{u}\,du = \int \frac{1}{(u-1)(u+1)}\,du$$

Use partial fractions:

$$\frac{1}{(u-1)(u+1)} = \frac{A}{u-1} + \frac{B}{u+1}$$

$$1 = A(u+1) + B(u-1)$$

$$1 = (A+B)u + (A-B)$$

Equating coefficients:

$$A + B = 0, \quad A - B = 1$$

Solving: $A = \frac{1}{2}, B = -\frac{1}{2}$

$$\int \left(\frac{1}{2(u-1)} - \frac{1}{2(u+1)} \right) du = \frac{1}{2} \log|u-1| - \frac{1}{2} \log|u+1| + C$$

$$= \frac{1}{2} \log \left| \frac{u-1}{u+1} \right| + C$$

Substitute $u = e^x$:

$$\int \frac{e^x}{(e^x-1)(e^x+1)}\, dx = \frac{1}{2} \log \left| \frac{e^x-1}{e^x+1} \right| + C$$

3 a)

Q. Solve $\int \tan^{-1}x \, dx$.

→ Let us use "1" as second function.

$I = \int \tan^{-1}x \cdot 1 \, dx$. Integrate by Parts;

$\therefore I = \tan^{-1}x \int 1\, dx - \int \left[\frac{d}{dx}(\tan^{-1}x) \int 1\, dx \right] dx$.

$\therefore I = x \cdot \tan^{-1}x - \int \left[\frac{1}{1+x^2} (x) \right] dx$

$\therefore I = x \cdot \tan^{-1}x - \frac{1}{2} \int \frac{2x}{1+x^2}\, dx$

$\therefore I = x \cdot \tan^{-1}x - \frac{1}{2} \log(1+x^2) + C$.

$\therefore \boxed{I = x \cdot \tan^{-1}x - \frac{1}{2} \log(1+x^2) + C.}$

3 b)

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x}\, dx \quad \cdots\cdots\cdots\cdots\cdots\cdots (1)$$

Using the identity $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$, we write:

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2} - x\right)}{\cos\left(\frac{\pi}{2} - x\right) + \sin\left(\frac{\pi}{2} - x\right)}\, dx$$

Now apply identities:

- $\cos\left(\frac{\pi}{2} - x\right) = \sin x$
- $\sin\left(\frac{\pi}{2} - x\right) = \cos x$

So equation (2) becomes:

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\cos x + \sin x}\, dx \quad \cdots\cdots\cdots\cdots\cdots\cdots (2)$$

Add equation (1) and equation (2):

$$2I = \int_0^{\frac{\pi}{2}} \left(\frac{\cos x}{\cos x + \sin x} + \frac{\sin x}{\cos x + \sin x} \right) dx$$

$$2I = \int_0^{\frac{\pi}{2}} \frac{\cos x + \sin x}{\cos x + \sin x}\, dx = \int_0^{\frac{\pi}{2}} 1\, dx$$

$$2I = [x]_0^{\frac{\pi}{2}} = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

$$\boxed{\int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x}\, dx = \frac{\pi}{4}}$$

3 c)

Q. Form D.E. of $y = Ae^{2x} + Be^{-2x}$.

→ diff. w.r.t. x,

$$\therefore \frac{dy}{dx} = A \cdot \frac{d}{dx}[e^{2x}] + B \frac{d}{dx}[e^{-2x}]$$

$$\therefore \frac{dy}{dx} = 2A \cdot e^{2x} - 2Be^{-2x}$$

Again diff. w.r.t. x,

$$\frac{d^2y}{dx^2} = 2A \frac{d}{dx}(e^{2x}) - 2B \frac{d}{dx}(e^{-2x})$$

$$\therefore \frac{d^2y}{dx^2} = 4(Ae^{2x} + Be^{-2x})$$

But; $y = Ae^{2x} + Be^{-2x}$

$$\therefore \frac{d^2y}{dx^2} = 4y$$

$$\therefore \boxed{\frac{d^2y}{dx^2} - 4y = 0}$$ is the required D.E.

3	d)	Q. Show that root of eq^n $x^3-4x+1=0$ in (1,2) & find it by Newton-Raphson metho[d] performing two iterations. → Let, $f(x)=x^3-4x+1$ & $f'(x)=3x^2-4$ Step I) To find root lies between, $f(0)=0^3-4(0)+1=+1$ Put $x=1$; $f(1)=1^3-4(1)+1=-2$ Put $x=2$; $f(2)=2^3-4(2)+1=+1$ ∴ Root lies bet^n 1 & 2. Step II) Iterations:- Iteration I):- $x_1=x_0-\frac{f(x_0)}{f'(x_0)}$ $\therefore x_1=2-\frac{1}{3(2)^2-4}$ $\therefore x_1=1.875.$ Iteration II):- $x_2=x_1-\frac{f(x_1)}{f'(x_1)}$ $\therefore x_2=1.875-\frac{(1.875)^3-4(1.875)+1}{3(1.875)-4}$ $\therefore x_2=1.861.$ ∴ from two iterations, approximate root of eq^n is $x_2=1.861.$	

4 a)

Solve $x^3 - 6x + 3 = 0$ using **Bisection Method** (3 iterations)

Step 1: Find initial interval $[a, b]$

Define

$$f(x) = x^3 - 6x + 3$$

Check signs:

x	$f(x)$	Sign
0	3	$+$
1	$1 - 6 + 3 = -2$	$-$

Since $f(0) > 0$ and $f(1) < 0$, root lies in $[0, 1]$.

Step 2: Perform three iterations of bisection

Iteration	a	b	$x = \frac{a+b}{2}$	Sign of $f(x)$	New Interval
1	0	1	0.5	$+$	$[0.5, 1]$
2	0.5	1	0.75	$-$	$[0.5, 0.75]$
3	0.5	0.75	0.625	$-$	

Conclusion:

The root of the equation

$$x^3 - 6x + 3 = 0$$

after three iterations is approximately

$$\boxed{x \approx 0.625}$$

4	b)	

Q: Solve using Jacobis Iterative method.

$10x+y+2z=13,\ 3x+10y+z=14,\ 2x+3y+10z=15$

Carry out two iterations.

→ From first equation,

$10x = 13 - y - 2z \quad \therefore x = \frac{13 - y - 2z}{10}$ — eqn. (4)

From second equation,

$10y = 14 - 3x - z \quad \therefore y = \frac{14 - 3x - z}{10}$ — eqn. (5)

From third equation,

$10z = 15 - 2x - 3y \quad \therefore z = \frac{15 - 2x - 3y}{10}$ — eqn. (6)

First iteration :-

Putting $x_0 = 0,\ y_0 = 0,\ z_0 = 0$ in R.H.S. of eqn. (4), (5), (6) we get,

$x_1 = \frac{13 - y_0 - 2z_0}{10} \quad \therefore x_1 = \frac{13-0-0}{10} \quad \therefore x_1 = 1.3$

$y_1 = \frac{14 - 3x_0 - z_0}{10} \quad \therefore y_1 = \frac{14-0-0}{10} \quad \therefore y_1 = 1.4$

$z_1 = \frac{15 - 2x_0 - 3y_0}{10} \quad \therefore z_1 = \frac{15-0-0}{10} \quad \therefore z_1 = 1.5$

Second iteration :-

Putting $x_1 = 1.3,\ y_1 = 1.4\ \&\ z_1 = 1.5$ in eqn. (4), (5) & (6)

$x_2 = \frac{13 - y_1 - 2z_1}{10} \quad \therefore x_2 = \frac{13 - (1.4) - 2(1.5)}{10} \quad \therefore x_2 = 0.86$

$y_2 = \frac{14 - 3x_1 - z_1}{10} \quad \therefore y_2 = \frac{14 - 3(1.3) - (1.5)}{10} \quad \therefore y_2 = 0.86$

$z_2 = \frac{15 - 2x_1 - 3y_1}{10} \quad \therefore z_2 = \frac{15 - 2(1.3) - 3(1.4)}{10} \quad \therefore z_2 = 0.82$

After two iterations,

approximate solution is :-

$x = 0.86$

$y = 0.86$

$z = 0.82$

4 c) **Solve the foloowing system of equation by Gtauss-seidel method 20x-y+2z=17,3x+20y-z=-18,2x-3y+20z=25**

Given System of Equations:

$$\begin{aligned} 1)\quad & 20x - y + 2z = 17 \\ 2)\quad & 3x + 20y - z = -18 \\ 3)\quad & 2x - 3y + 20z = 25 \end{aligned}$$

Initial Guess (Iteration 0):

$$x_0 = 0, \quad y_0 = 0, \quad z_0 = 0$$

Iteration 1:

$$x_1 = \frac{1}{20}(17 + y_0 - 2z_0) = \frac{1}{20}(17 + 0 - 0) = 0.850$$

$$y_1 = \frac{1}{20}(-18 - 3x_1 + z_0) = \frac{1}{20}(-18 - 3(0.850) + 0) = \frac{-20.550}{20} = -1.028$$

$$z_1 = \frac{1}{20}(25 - 2x_1 + 3y_1) = \frac{1}{20}(25 - 2(0.850) + 3(-1.028)) = \frac{1.216}{20} = 0.061$$

Iteration 2:

$$x_2 = \frac{1}{20}(17 + y_1 - 2z_1) = \frac{1}{20}(17 - 1.028 - 2(0.061)) = \frac{15.850}{20} = 0.793$$

$$y_2 = \frac{1}{20}(-18 - 3x_2 + z_1) = \frac{1}{20}(-18 - 3(0.793) + 0.061) = \frac{-20.318}{20} = -1.016$$

$$z_2 = \frac{1}{20}(25 - 2x_2 + 3y_2) = \frac{1}{20}(25 - 2(0.793) + 3(-1.016)) = \frac{1.366}{20} = 0.068$$

Iteration 3:

$$x_3 = \frac{1}{20}(17 + y_2 - 2z_2) = \frac{1}{20}(17 - 1.016 - 2(0.068)) = \frac{15.848}{20} = 0.792$$

$$y_3 = \frac{1}{20}(-18 - 3x_3 + z_2) = \frac{1}{20}(-18 - 3(0.792) + 0.068) = \frac{-20.308}{20} = -1.015$$

$$z_3 = \frac{1}{20}(25 - 2x_3 + 3y_3) = \frac{1}{20}(25 - 2(0.792) + 3(-1.015)) = \frac{1.367}{20} = 0.068$$

$$\boxed{\begin{aligned} x_3 &= 0.792 \\ y_3 &= -1.015 \\ z_3 &= 0.068 \end{aligned}}$$

4	d)	Q: If 20% bolts produced by a machine are defective, determine probability that out of 4 bolts drawn - (i) Exactly one is defective. (ii) At most two are defective. (iii) At least one is defective. Solve by Binomial Distribution.?	

→ $p = 20\% = \frac{20}{100} \therefore p = 0.2$

$q = 1 - p \therefore q = 1 - 0.2 \therefore q = 0.8.$

$n = 4.$

(i) Exactly one is defective.

$$P(r) = {}^{n}C_{r} \cdot p^{r} \cdot q^{n-r}$$

$$\therefore P(1) = {}^{4}C_{1} \cdot (0.2)^{1} \cdot (0.8)^{4-1}$$

$$\therefore P(1) = 0.4096.$$

(ii) At most '2' :-

$$P(0) = {}^{4}C_{0} \times (0.2)^{0} \times (0.8)^{4-0}$$

$$P(0) = 0.4096.$$

$$P(2) = {}^{4}C_{2} \times (0.2)^{2} \times (0.8)^{4-2}$$

$$P(2) = 0.1536.$$

$$P(\text{At most "2"}) = P(0) + P(1) + P(2)$$

$$= 0.4096 + 0.4096 + 0.1536$$

$$P(\text{At most "2"}) = 0.9728.$$

4	e)	**A random variable has a poisson distribution such that p(2)=p(3) Find P(5)**	

Given $P(2) = P(3)$, 1

$$\frac{e^{-m}m^2}{2!} = \frac{e^{-m}m^3}{3!}$$

Cancel e^{-m},

$$\frac{m^2}{2} = \frac{m^3}{6}$$

Multiply both sides by 6,

$$3m^2 = m^3$$

Divide both sides by m^2, 1

$$3 = m$$

Then,

$$P(5) = \frac{e^{-3}3^5}{5!} = \frac{e^{-3} \times 243}{120} = \frac{243}{120}e^{-3} = 2.025 \times e^{-3}$$ 1

Numerically,

$$P(5) \approx 2.025 \times 0.04979 = 0.1008$$ 1

5 a) i)

Q: Solve $\int \frac{dx}{3 - 2\sin^2 x}$.

→ Divide numerator & denominator by "$\cos^2 x$".

$$\therefore I = \int \frac{\frac{dx}{\cos^2 x}}{\frac{3 - 2\sin^2 x}{\cos^2 x}}$$

$$\therefore I = \int \frac{\sec^2 dx}{3\sec^2 x - 2\tan^2 x}.$$

$$\therefore I = \int \frac{\sec^2 x \, dx}{3[1 + \tan^2 x] - 2\tan^2 x}$$

$$\therefore I = \int \frac{\sec^2 x \, dx}{3 + \tan^2 x}$$

Put: $\tan x = t$ diff. w.r.t. x

$\sec^2 x \, dx = dx$

$$\therefore I = \int \frac{dt}{(\sqrt{3})^2 + t^2}$$

$$\therefore I = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{t}{\sqrt{3}}\right) + C$$

$$\therefore \boxed{I = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{t}{\sqrt{3}}\right) + C.}$$

5 a) ii)

Problem:

$$\mathbf{I} = \int \frac{\mathbf{x+1}}{\mathbf{(x-1)^2}}\,\mathbf{dx}$$

Let $\mathbf{t = x - 1}$, so $\mathbf{x = t + 1}$, and $\mathbf{dx = dt}$.

Then,

$$\mathbf{I} = \int \frac{\mathbf{(t+1)+1}}{\mathbf{t^2}}\,\mathbf{dt} = \int \frac{\mathbf{t+2}}{\mathbf{t^2}}\,\mathbf{dt}$$

Now split the integrand:

$$\mathbf{I} = \int \left(\frac{\mathbf{t}}{\mathbf{t^2}} + \frac{\mathbf{2}}{\mathbf{t^2}}\right)\mathbf{dt} = \int \left(\frac{\mathbf{1}}{\mathbf{t}} + \mathbf{2t^{-2}}\right)\mathbf{dt}$$

Now integrate term by term:

$$\mathbf{I} = \int \frac{\mathbf{1}}{\mathbf{t}}\,\mathbf{dt} + \int \mathbf{2t^{-2}}\,\mathbf{dt} = \log|\mathbf{t}| - \frac{\mathbf{2}}{\mathbf{t}} + \mathbf{C}$$

Substitute back $\mathbf{t = x - 1}$:

$$\mathbf{I} = \log|\mathbf{x-1}| - \frac{\mathbf{2}}{\mathbf{x-1}} + \mathbf{C}$$

{ NOTE : This problem can be solved using Partial Fraction Method also, Answer remains same.

5	b) i)	Let $\mathbf{I} = \int_2^5 \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\,dx$ ...(1) Using the property: $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$ Here, $a = 2, \quad b = 5 \quad \Rightarrow \quad a+b = 7$ Substitute $x \to 7-x$ into equation (1): $$I = \int_2^5 \frac{\sqrt{7-x}}{\sqrt{7-(7-x)}+\sqrt{7-x}}\,dx$$ $$\Rightarrow I = \int_2^5 \frac{\sqrt{7-x}}{\sqrt{x}+\sqrt{7-x}}\,dx \quad ...(2)$$ Now, add equations (1) and (2): $$I + I = \int_2^5 \left(\frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}} + \frac{\sqrt{7-x}}{\sqrt{x}+\sqrt{7-x}}\right)dx$$ $$2I = \int_2^5 \frac{\sqrt{x}+\sqrt{7-x}}{\sqrt{x}+\sqrt{7-x}}\,dx$$ $$2I = \int_2^5 1\,dx$$ $$2I = [x]_2^5 = 5 - 2 = 3$$ $$I = \frac{3}{2}$$	

5	b ii	**Let**	

$$\mathbf{I} = \int_0^4 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\,dx \quad ...(1)$$

We use the property:

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

Here,

$$a = 0, \quad b = 4$$

$$\Rightarrow a+b-x = 0+4-x = 4-x$$

Now apply the property:

$$\mathbf{I} = \int_0^4 \frac{\sqrt[3]{(4-x)+5}}{\sqrt[3]{(4-x)+5}+\sqrt[3]{9-(4-x)}}\,dx$$

$$= \int_0^4 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+5}}\,dx \quad ...(2)$$

Add equations (1) and (2):

$$I + I = \int_0^4 \left[\frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}} + \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+5}}\right] dx$$

$$= \int_0^4 1\,dx$$

$$= 4$$

$$\Rightarrow 2I = 4$$

$$\Rightarrow \boxed{\mathbf{I = 2}}$$

5	c i	**Let** $\mathbf{I} = \int_0^{\frac{\pi}{2}} \log(\tan \mathbf{x})\, \mathbf{dx} \quad \mathbf{...(1)}$ **Using the identity:** $\tan\left(\frac{\pi}{2} - x\right) = \cot x \quad \Rightarrow \quad \log\left(\tan\left(\frac{\pi}{2} - x\right)\right) = \log(\cot x)$ $= \log\left(\frac{1}{\tan x}\right) = -\log(\tan x) \quad (\text{since } \log(1/a) = -\log(a))$ **Now apply the property:** $\mathbf{I} = \int_0^{\frac{\pi}{2}} \log(\tan(\frac{\pi}{2} - \mathbf{x}))\, \mathbf{dx}$ $\mathbf{I} = \int_0^{\frac{\pi}{2}} -\log(\tan \mathbf{x})\, \mathbf{dx} = -\mathbf{I} \quad \mathbf{...(2)}$ **Add equations (1) and (2):** $\mathbf{I} + \mathbf{I} = \mathbf{0} \quad \Rightarrow \quad \mathbf{2I} = \mathbf{0}$ $\boxed{\mathbf{I} = \mathbf{0}}$	

5	c ii	$$I = \int_0^{\frac{\pi}{2}} \sin x \cdot \cos x \, dx$$ $$\cos x = t$$ $$-\sin x \, dx = dt \quad \Rightarrow \quad \sin x \, dx = -dt$$	
6	a) i)		

x	0	$\frac{\pi}{2}$
$t = \cos x$	$\cos 0 = 1$	$\cos \frac{\pi}{2} = 0$

$$I = \int_1^0 t \cdot (-dt)$$

$$= \int_0^1 t \, dt$$

$$= \left[\frac{t^2}{2} \right]_0^1$$

$$= \frac{1^2}{2} - \frac{0^2}{2}$$

$$= \frac{1}{2}$$

6 a) i)

$$(1 + x^2) \, dy - (1 + y^2) \, dx = 0$$

Rewriting:

$$(1 + x^2) \, dy = (1 + y^2) \, dx$$

$$\frac{dy}{1 + y^2} = \frac{dx}{1 + x^2}$$

Integrating both sides:

$$\int \frac{dy}{1 + y^2} = \int \frac{dx}{1 + x^2}$$

$$\tan^{-1}(y) = \tan^{-1}(x) + C$$

$$\tan^{-1}(y) - \tan^{-1}(x) = C$$

6	a) ii)	$$(x^2 + 6xy - y^2)\,dx + (3x^2 - 2xy + y^2)\,dy = 0$$ $$M = x^2 + 6xy - y^2, \quad N = 3x^2 - 2xy + y^2$$ $$\therefore \frac{\partial M}{\partial y} = 0 + 6x - 2y = 6x - 2y$$ $$\therefore \frac{\partial N}{\partial x} = 6x - 2y$$ $$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow \text{Given eq. is "Exact"}$$ $$\int M\,dx + \int (\text{terms free from } x \text{ only in } N)\,dy = C$$ $$= \int (x^2 + 6xy - y^2)\,dx + \int y^2\,dy$$ $$= \int x^2\,dx + \int 6xy\,dx - \int y^2\,dx + \int y^2\,dy$$ $$= \frac{x^3}{3} + 3x^2y - xy^2 + \frac{y^3}{3} = C$$ $$\boxed{\frac{x^3}{3} + 3x^2y - xy^2 + \frac{y^3}{3} = C}$$	

6	b i)	Q. Solve D.E. $\frac{dy}{dx} + y\tan x = \cos^2 x$. → std. form:- $\frac{dy}{dx} + Py = Q$. $\therefore P = \tan x$ & $Q = \cos^2 x$ $I.F. = e^{\int P dx} = e^{\int \tan x\, dx} = e^{\log(\sec x)}$ $\therefore I.F. = \sec x$. Solution of Linear differential equation, $y \times (I.F.) = \int Q \times (I.F.)\, dx + C$ $\therefore y(\sec x) = \int \cos^2 x\, (\sec x)\, dx + C$ $\therefore y \sec x = \int \cos^2 x \left(\frac{1}{\cos x}\right) dx + C$ $\therefore y \sec x = \int \cos x\, dx + C$ $\therefore$ $\boxed{y \cdot \sec x = \sin x + C.}$ is the required general sol^r.

6	b ii)	verify whether $y = \cos x$ is a solution of the differential equation: $$\frac{d^2y}{dx^2} + y = 0$$	
		Given: $$y = \cos x$$ $$\frac{dy}{dx} = -\sin x$$ $$\frac{d^2y}{dx^2} = -\cos x$$ Substitute into the equation: $$\frac{d^2y}{dx^2} + y = -\cos x + \cos x = 0$$ $$y = \cos x \text{ is a solution of } \frac{d^2y}{dx^2} + y = 0$$	

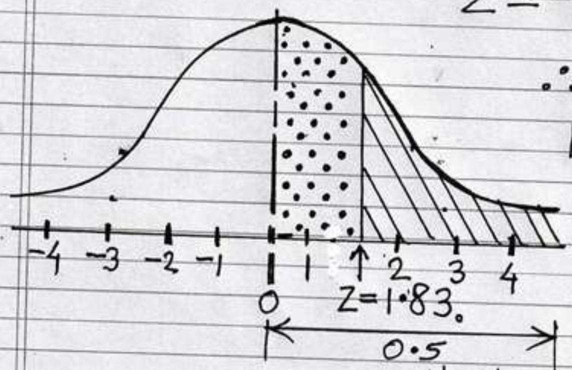
6 c)

Q. A factory manufactured 2000 electric bulbs with avarage life of 2040 Hrs & S.D. of 60 Hrs. Assuming Normal distribution find number of bul[b]s having life (i) More than 2150 Hrs & (ii) less than 1960 Hrs.

Given = area under normal curve)
Z=0 to Z=1.83 is 0.4667
Z=0 to Z=1.33 is 0.4082.

→ $n = 2000$, $\bar{x} = 2040$ Hrs, S.D. $(\sigma) = 60$ Hrs.

(i) Case (i) Life more than 2150 Hrs :-



$$Z = \frac{a - \bar{x}}{\sigma} = \frac{2150 - 2040}{60}$$

$\therefore \boxed{Z = +1.833.}$

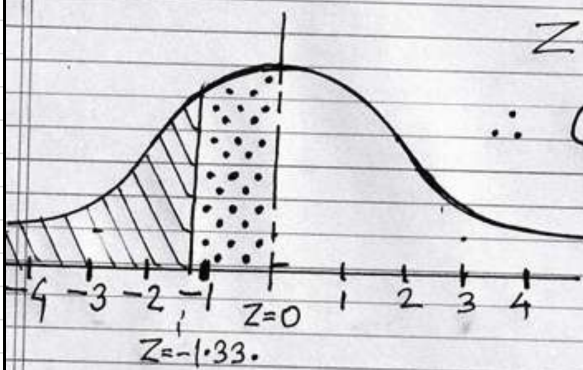
P(More than 2150 Hrs)
= P(more than Z=1.83)
= 0.5 − 0.4667

$\therefore \boxed{P = 0.0333}$

Number of bulbs = P × n
= 2000 × 0.0333

Nos = 66.6 ≅ 67 bulbs.

(ii) Less than 1960 Hrs :-



$$Z = \frac{a - \bar{x}}{\sigma} = \frac{1960 - 2040}{60}$$

$\therefore \boxed{Z = -1.33}$

P(less than 1960 Hrs)
= P(less than Z = −1.33)
P = 0.5 − 0.4082

$\boxed{P = 0.0918.}$

No. of bulbs = P × n = 0.0918 × 2000

Nos. = 183.6 ≅ 184 bulbs.