



WINTER- 2024 EXAMINATION

Model Answer

Subject Code

Subject Name: Maths II

312301

Important Instructions to STUDENTS

- 1) The model answer given here are prepared from the answers from the previously uploaded model answers by Board.
- 2) These model answers are not uploaded by the MSBTE official site but MSBTE study resources website prepared it for students. This model answer has question paper also inbuilt in it, no need to download it separate.
- 3) Please remember that answers are not checked word to word but based on keywords which must be present in your answer
- 4) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate
- 5) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn
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- 7) Students are advised to prepare all the syllabus from recommended book and use these model answers for the purpose of tests.

Q.NO	SUB Q N	ANSWER	Marking Scheme
1.		Solve any five.	10 Marks
a)		Evaluate $\int \left( \frac{1}{1+x^2} + \cos x \right) dx$	
		$I = \int \frac{1}{1+x^2} + \cos x \cdot dx$	
		we know, $\int \frac{dx}{1+x^2} = \tan^{-1}(x) + C$	
		& $\int \cos x \cdot dx = \sin x + C$	
		$\therefore \int \frac{1}{1+x^2} + \cos x \, dx$	
		$= \tan^{-1}x + \sin x + C.$	
		$\int \frac{1}{1+x^2} + \cos x \, dx = \tan^{-1}x + \sin x + C.$	



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Marking  
Schemeb) Evaluate  $\int \sqrt{1 + \cos 2x} dx$ 

→  $\because 1 + \cos \theta = 2 \cos^2(\theta/2)$   
Using relation,  
 $1 + \cos 2x = 2 \cos^2 x.$

$$\begin{aligned} I &= \int \sqrt{1 + \cos 2x} dx \\ &= \int \sqrt{2 \cos^2 x} dx \\ &= \sqrt{2} \int \cos x dx \\ &= \sqrt{2} \cdot \sin x + C. \end{aligned}$$

$$\therefore \boxed{\int \sqrt{1 + \cos 2x} \cdot dx = \sqrt{2} \cdot \sin x + C.}$$

c) Evaluate  $\int_0^4 (4x - x^2) \cdot dx$ 

$$\begin{aligned} \rightarrow I &= \int_0^4 4x - x^2 \cdot dx \\ &= \left[ 4 \cdot \frac{x^2}{2} - \frac{x^3}{3} \right]_0^4 \\ &= \left[ 2x^2 - \frac{x^3}{3} \right]_0^4 \\ &= \left[ 2(4)^2 - \frac{4^3}{3} \right] - [0] = \frac{32}{3} \end{aligned}$$

$$\therefore \boxed{\int_0^4 (4x - x^2) dx = \frac{32}{3}}$$

d) Find order & degree of diff. eq<sup>n</sup>.  $\frac{d^2y}{dx^2} = \sqrt{y - \frac{dy}{dx}}$ 

→  $\frac{d^2y}{dx^2} = \sqrt{y - \frac{dy}{dx}}$ ; squaring both sides;

$$\left( \frac{d^2y}{dx^2} \right)^2 = y - \frac{dy}{dx}$$

$\therefore$  Higher order derivative is "2" & its power is "2".

$$\therefore \boxed{\text{Order} = 2}$$



e) Show that, root of eq<sup>n</sup>.  $x^3 - 2x - 5 = 0$ , lies between 2 & 3.

→ Given eq<sup>n</sup>.  $x^3 - 2x - 5 = 0$   
 $f(x) = x^3 - 2x - 5$

check:-  $f(2) = 2^3 - 2(2) - 5$

$$f(2) = 8 - 4 - 5$$

$$f(2) = -1.$$

&  $f(3) = 2x^3 - 2x - 5$

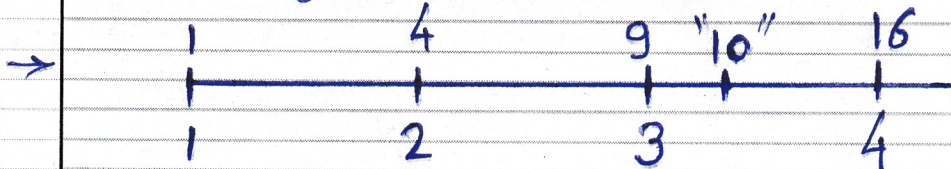
$$f(3) = 3^3 - 2(3) - 5$$

$$f(3) = 27 - 6 - 5$$

$$f(3) = 16.$$

∴  $f(2)$  is -ve &  $f(3)$  is +ve,  
 ∴ Root lies between 2 & 3.

f) Find approximate square root of number "10" using Bakhshali Interactive Method.



Nearest root is "9", whose sq. root is "3".

$$\therefore S = 10 \quad \& \quad N = 3.$$

$$\therefore \underline{d} = (S - N^2) = (10 - 3^2) = \underline{1}.$$

$$P = \frac{d}{2 \times N} = \frac{1}{2 \times 3}$$

$$P = \frac{1}{6}.$$

$$A = N + P = 3 + \frac{1}{6}$$

$$A = \frac{19}{6}.$$

$$\sqrt{S} \cong \left[ A - \frac{P^2}{2A} \right]$$

$$\sqrt{S} \cong \left[ \frac{19}{6} - \frac{(1/6)^2}{2 \times \frac{19}{6}} \right] \cong 3.1622$$

$$\therefore \sqrt{10} \cong 3.1622.$$



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g. A fair coin is tossed 8 times  
Find probability of getting  
exactly "2" heads?

$$\rightarrow n=8, \quad p=\frac{1}{2}, \quad q=\frac{1}{2};$$

To find  $P(2)$  using Binomial  
distribution;

$$P(r) = {}^n C_r p^r \cdot q^{n-r}$$

$$\therefore P(2) = {}^8 C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{(8-2)}$$

$$\therefore \boxed{P(2) = \frac{7}{62}}$$

2. Solve any three

12 Marks

g) Evaluate  $\int \frac{1}{\sqrt{1-x^2} (\sin^{-1} x)^2} dx$

$\rightarrow$  Put  $(\sin^{-1} x) = t$ . diff. w.r.t.  $x$ , we get,

$$\frac{1}{\sqrt{1-x^2}} = \frac{dt}{dx} \quad \therefore dx = \sqrt{1-x^2} \cdot dt.$$

$$I = \int \frac{1}{(\sqrt{1-x^2}) \cdot t^2 \cdot (\sqrt{1-x^2})} dt = \int \frac{dt}{t^2}$$

$$I = \int t^{-2} dt$$

$$I = \frac{t^{(-2+1)}}{(-2+1)} + C$$

$$I = \frac{t^{(-1)}}{(-1)} + C$$

$$I = \frac{-1}{t} + C$$

$$I = \frac{-1}{\sin^{-1} x} + C$$

$$\therefore \boxed{\int \frac{1}{\sqrt{1-x^2} (\sin^{-1} x)^2} dx = \frac{-1}{(\sin^{-1} x)} + C.}$$



b) Evaluate  $\int \frac{\cos x}{(\sin x + 1)(\sin x + 2)} dx$

→ Put,  $\sin x = t$  (diff. w.r.t.  $x$ ; we get)  
 $\cos x = \frac{dt}{dx} \therefore dx = \frac{dt}{\cos x}$

Now,  $I = \int \frac{\cos x}{(t+1)(t+2)} \times \frac{dt}{\cos x}$

$$I = \int \frac{1}{(t+1)(t+2)} dt.$$

Let's resolve into partial fraction,

$$\frac{1}{(t+1)(t+2)} = \frac{A}{t+1} + \frac{B}{t+2}$$

Multiplying both sides by  $[(t+1)(t+2)]$ ,

$$\boxed{1 = A(t+2) + B(t+1).}$$

Put  $t+1=0$

$$\therefore t = -1.$$

$$\therefore 1 = A(-1+2) + B(-1+1)$$

$$\therefore 1 = A(1) + B(0)$$

$$\therefore 1 = A.$$

$$\therefore \boxed{A = 1.}$$

Put,  $t+2=0$

$$\therefore t = -2.$$

$$\therefore 1 = A(-2+2) + B(-2+1)$$

$$\therefore 1 = 0 + (-1)B$$

$$\therefore -1 = B$$

$$\therefore \boxed{B = -1.}$$

Now,

$$I = \int \frac{1}{t+1} + \frac{(-1)}{t+2} dt$$

$$= \log(t+1) - \log(t+2) + C$$

$$= \log \left[ \frac{t+1}{t+2} \right] + C$$

$$\therefore \boxed{\int \frac{\cos x}{(\sin x + 1)(\sin x + 2)} dx = \log \left[ \frac{\sin x + 1}{\sin x + 2} \right] + C}$$



c) Evaluate  $\int e^x \cdot \sin x \, dx$ .

→ Check LIATE Rule;  $\begin{matrix} e^x & \sin x \\ \downarrow & \downarrow \\ \text{Exp.} & \text{Trig.} \end{matrix}$

∴ Change Sequence;

$I = \int \sin x \cdot e^x \, dx$ ; Using Int<sup>n</sup> by parts;

$$I = \sin x \int e^x \, dx - \int \left[ \int e^x \, dx \cdot \frac{d}{dx} \sin x \right] dx$$

$$I = \sin x \cdot e^x - \int \cos x \cdot e^x \, dx + C$$

Again using Integration by parts;

$$I = \sin x \cdot e^x - \left[ \cos x \int e^x \, dx - \int \left[ \int e^x \, dx \cdot \frac{d}{dx} \cos x \right] dx \right] + C$$

$$I = \sin x \cdot e^x - \left[ \cos x \cdot e^x - \int e^x (-\sin x) \, dx \right] + C$$

$$I = \sin x \cdot e^x - \cos x \cdot e^x - \int e^x \cdot \sin x \, dx + C$$

$$I = \sin x \cdot e^x - \cos x \cdot e^x - I + C$$

$$I + I = e^x (\sin x - \cos x) + C$$

$$2I = e^x (\sin x - \cos x) + C$$

$$I = \frac{1}{2} \left[ e^x (\sin x - \cos x) \right] + C.$$

d) Solve  $\int \frac{1}{\sqrt{16 - 6x - x^2}} \cdot dx$

$$\rightarrow I = \int \frac{1}{\sqrt{-(x^2 + 6x - 16)}} \, dx$$

$$\text{Third term} = \left[ \frac{1}{2x} \times 6x \right]^2 = 9$$

$$I = \int \frac{1}{\sqrt{-(x^2 + 6x + 9 - 16 - 9)}} \, dx$$

$$I = \int \frac{1}{\sqrt{(5)^2 - (x+3)^2}} \, dx$$

$$\therefore \int \frac{1}{\sqrt{16 - 6x - x^2}} \cdot dx = \sin^{-1} \left( \frac{x+3}{5} \right) + C$$



Q.NO  
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ANSWER

Marking  
Scheme

3.

Solve any three

12 Marks

$$a) \int_0^{\pi/2} \frac{dx}{5+4\cos x}$$

$$\rightarrow \text{Put, } \tan\left(\frac{x}{2}\right) = t \therefore \cos x = \frac{(1-t^2)}{(1+t^2)} \text{ \& } dx = \frac{2 \cdot dt}{(1+t^2)}$$

$$\therefore \text{Limit change; } \left[ \begin{array}{c|c|c} x & 0 & \pi/2 \\ \hline t & 0 & \infty \end{array} \right]$$

\(\therefore\) Integral becomes;

$$I = \int_0^{\infty} \frac{2dt/(1+t^2)}{5+4\left(\frac{1-t^2}{1+t^2}\right)} dt.$$

$$I = \int_0^{\infty} \frac{2dt/(1+t^2)}{\frac{5}{1} + \frac{4-4t^2}{(1+t^2)}} dt$$

$$I = \int_0^{\infty} \frac{2dt/(1+t^2)}{\frac{5+5t^2+4-4t^2}{(1+t^2)}} dt$$

$$I = 2 \int_0^{\infty} \frac{dt}{t^2+9}$$

$$I = 2 \int_0^{\infty} \frac{dt}{(t)^2+(3)^2}$$

$$I = 2 \left[ \frac{1}{3} \cdot \tan^{-1}\left(\frac{t}{3}\right) \right]_0^{\infty}$$

$$I = \frac{2}{3} \left[ \tan^{-1}(\infty) - \tan^{-1}(0) \right]$$

$$I = \frac{2}{3} \left[ \frac{\pi}{2} - 0 \right]$$

$$\therefore \int_0^{\pi/2} \frac{dx}{5+4\cos x} = \frac{\pi}{3}$$



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ANSWER

Marking  
Scheme

$$b) \int_0^4 \frac{\sqrt[3]{x+5}}{\sqrt{x+5} + \sqrt[3]{9-x}} dx$$

→ using Property,  $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

$$I = \int_0^4 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5} + \sqrt[3]{9-x}} dx \quad \text{--- eq}^n \text{ ①}$$

$$I = \int_0^4 \frac{\sqrt[3]{(0+4-x)+5}}{\sqrt[3]{(0+4-x)+5} + \sqrt[3]{9-(0+4-x)}} dx$$

$$I = \int_0^4 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x} + \sqrt[3]{x+5}} dx \quad \text{--- eq}^n \text{ ②}$$

Adding eq<sup>n</sup>. ① & ②

$$I + I = \int_0^4 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5} + \sqrt[3]{9-x}} dx + \int_0^4 \frac{\sqrt[3]{9-x}}{\sqrt[3]{x+5} + \sqrt[3]{9-x}} dx$$

$$2I = \int_0^4 \frac{(\sqrt[3]{x+5} + \sqrt[3]{9-x})}{(\sqrt[3]{x+5} + \sqrt[3]{9-x})} dx$$

$$2I = \int_0^4 1 dx$$

$$2I = [x]_0^4$$

$$2I = [4 - 0]$$

$$I = \frac{4}{2} = 2$$

$$\therefore \int_0^4 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5} + \sqrt[3]{9-x}} dx = 2.$$



Q.NO	SUB QN	ANSWER	Marking Scheme
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c) Solve  $(2xy + y^2)dx + (x^2 + 2xy + \sin y)dy = 0$

→ check eq<sup>n</sup>. for EXACT;

$$M = 2xy + y^2 \quad \& \quad N = x^2 + 2xy + \sin y$$

$$\frac{dM}{dy} = 2x(1) + 2y \quad \Bigg| \quad \frac{dN}{dx} = 2x + 2y(1) + 0$$

$$\frac{dM}{dy} = 2x + 2y \quad \Bigg| \quad \frac{dN}{dx} = 2x + 2y$$

∴  $\frac{dM}{dy} = \frac{dN}{dx}$  Given eq<sup>n</sup>. is exact.

$$\text{sol}^n :- \int M dx + \int N dy = C$$

↓  
(terms free from x)

$$\int 2xy + y^2 dx + \int \sin y dy = C$$

$$2(y) \left( \frac{x^2}{2} \right) + y^2 x + (-\cos y) = C$$

$$\therefore \boxed{x^2 y + xy^2 - \cos y = C.}$$

d) Using Bisection method find  $x^3 - x - 1 = 0$

→ step(I) To find root lies between,

$$f(x) = x^3 - x - 1$$

$$f(0) = 0 - 0 - 1$$

$$\underline{\underline{f(0) = -1.}}$$

$$f(1) = 1 - 1 - 1$$

$$\underline{\underline{f(1) = -1.}}$$

$$f(2) = 2^3 - 2 - 1$$

$$\underline{\underline{f(2) = 5.}}$$

Root lies between 1 & 2

Step II

(P.T.O.)



Step II Iterations.

-VE (a)	+VE (b)	$x = \frac{a+b}{2}$	$f(x) = x^3 - x - 1$	sign
a=1	b=2	$x = \frac{1+2}{2}$	$= (1.5)^3 - (1.5) - 1$	
	↓	<u><math>x = 1.5</math></u>	<u><math>= 0.875</math></u>	+VE
a=1	b=1.5	$x = \frac{1+1.5}{2}$	$= (1.25)^3 - 1.25 - 1$	
	↓	<u><math>x = 1.25</math></u>	<u><math>= -0.296</math></u>	-VE
a=1.25	b=1.5	$x = \frac{1.25+1.5}{2}$		
		<u><u><math>x = 1.375</math></u></u>		

Approximate root of eq<sup>n</sup>. is  $x = 1.375$ .

4

solve any three

12 Marks

a) Find root of  $x^3 + 2x^2 - 8 = 0$   
using Regula falsi method.

→ Step I] To find root lies between;

$$f(x) = x^3 + 2x^2 - 8$$

$$f(0) = 0 + 0 - 8$$

$$\underline{\underline{f(0) = -8}}$$

$$f(1) = 1^3 + 2(1)^2 - 8$$

$$\underline{\underline{f(1) = -5}}$$

$$f(2) = 2^3 + 2(2)^2 - 8$$

$$\underline{\underline{f(2) = 8}}$$

∴ Root lies between 1 & 2.

(10 T/O)



## Step II Iterations

-VE (a)	+VE (b)	$x = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$	$f(x) = x^3 + 2x - 8$	Sign
$a=1$ $f(a) = -5$	$b=2$ $f(b) = 8$	$x = \frac{1(8) - 2(-5)}{8 - (-5)}$	$= (1.384)^3 + 2(1.384) - 8$	
↓		<u><math>x = 1.384</math></u>	<u><math>= -1.518</math></u>	-VE
$a=1.384$ $f(a) = -1.518$	$b=2$ $f(b) = 8$	$x = \frac{(1.384 \times 8) - (2 \times -1.518)}{8 - (-1.518)}$	$= (1.482)^3 + 2(1.482) - 8$	
↓		<u><math>x = 1.482</math></u>	<u><math>= -0.352</math></u>	-VE
$a=1.482$ $f(a) = -0.352$	$b=2$ $f(b) = 8$	$x = \frac{(1.482 \times 8) - (2 \times -0.352)}{8 - (-0.352)}$		
		<u><u><math>x = 1.503</math></u></u>		

Approximate root of eq<sup>n</sup> is  $x = 1.503$

b) Using Newton-Raphson Method find root of eq<sup>n</sup>.  $x^4 - x - 9 = 0$ . Perform 3 iterations.

→ Step I. To find root lies between

$$f(x) = x^4 - x - 9 \quad \therefore f'(x) = 4x^3 - 1$$

$$f(0) = (0) - (0) - 9$$

$$\underline{\underline{f(0) = -9}}$$

$$f(1) = 1^4 - 1 - 9$$

$$\underline{\underline{f(1) = -9}}$$

$$f(2) = 2^4 - 2 - 9$$

$$\underline{\underline{f(2) = 5}}$$





Q.NO	SUB QN	ANSWER	Marking Scheme
		<p>Let, initial root <math>x_0 = 2</math></p> <p>Iteration I) <math>x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}</math></p> $x_1 = 2 - \frac{2^4 - 2 - 9}{4(2)^3 - 1}$ $x_1 = 2 - \frac{5}{31}$ <div style="border: 1px solid black; padding: 5px; display: inline-block;"><math>x_1 = 1.838</math></div> <p>Iteration II)</p> $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$ $x_2 = 1.838 - \frac{1.838^4 - 1.838 - 9}{4(1.838)^3 - 1}$ <div style="border: 1px solid black; padding: 5px; display: inline-block;"><math>x_2 = 1.813</math></div> <p>Iteration III)</p> $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$ $= 1.813 - \frac{(1.813)^4 - 1.813 - 9}{4(1.813)^3 - 1}$ <div style="border: 1px solid black; padding: 5px; display: inline-block;"><math>x_3 = 1.813</math></div>	
		<p>c) Solve by Gauss-Seidal method,</p> $\begin{aligned} 5x - 2y + 3z &= 18 \\ x + 7y - 3z &= 22 \\ 2x - y + 6z &= 22 \end{aligned}$ <p>→ Step I) Formation of equation,</p> <p>from eq<sup>n</sup> ① <math>x = \frac{1}{5}(18 + 2y - 3z)</math></p> <p>from eq<sup>n</sup> ② <math>y = \frac{1}{7}(22 - x + 3z)</math></p> <p>from eq<sup>n</sup> ③ <math>z = \frac{1}{6}(22 - 2x + y)</math></p>	





Q.NO	SUB QN	ANSWER	Marking Scheme
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Step II) Iterations :-

Iteration I using  $y=0$  &  $z=0$

$$x = \frac{1}{5}(18 + 0 - 0) = 3.600$$

$$x = 3.600$$

using  $x=3.6$  &  $z=0$

$$y = \frac{1}{7}[22 - 3.6 + 3(0)]$$

$$y = 2.628$$

using  $x=3.6$  &  $y=2.628$ ,

$$z = \frac{1}{6}[22 - 2(3.6) + 2.628]$$

$$z = 2.904$$

Iteration II using  $y=2.628$ ,  $z=2.904$

$$x = \frac{1}{5}[18 + 2(2.628) - 3(2.904)]$$

$$x = 2.909$$

using  $x=2.909$ ,  $z=2.904$

$$y = \frac{1}{7}[22 - 2.909 + 3(2.904)]$$

$$y = 3.972$$

using  $x=2.909$ ,  $y=3.972$

$$z = \frac{1}{6}[22 - 2(2.909) + 3.972]$$

$$z = 3.359$$

Iteration III using  $y=3.972$ ,  $z=3.359$

$$x = \frac{1}{5}[18 + 2(3.972) - 3(3.359)]$$

$$x = 3.1734$$

using  $x=3.1734$ ,  $z=3.359$ ,

$$y = \frac{1}{7}[22 - 3.1734 + 3(3.359)]$$

$$y = 4.129$$

using  $y=4.129$ ,  $x=3.1734$ ,

$$z = \frac{1}{6}[22 - 2(3.1734) + 4.129]$$

$$z = 3.297$$





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Scheme

d) If 20% of bolts produced by a machine are defective, determine probability that out of 4 bolts drawn -

(i) One is defective

(ii) at the most two are defective.

→ Probability of defective bolts  $P = \frac{20}{100} = 0.2$

$$q = 1 - p \therefore q = 1 - 0.2 \quad q = 0.8$$

$$n = 4: P(r) = {}^n C_r \cdot p^r \cdot q^{n-r}$$

(i) One defective,  $n = 4, r = 1.$

$$P(1) = {}^4 C_1 \cdot (0.2)^1 \cdot (0.8)^{4-1}$$

$$P(1) = \underline{\underline{0.4096.}}$$

(ii) Probability of getting 2 defective in 4.

$$= P(0) + P(1) + P(2)$$

$$= {}^4 C_0 (0.2)^0 (0.8)^{4-0} + {}^4 C_1 (0.2)^1 (0.8)^{4-1}$$

$$+ {}^4 C_2 (0.2)^2 (0.8)^{4-2}$$

$$= \underline{\underline{0.9728.}}$$

e) If probability that an electric motor is defective is 0.01. What is probability that sample of 300 electric motors will contain exactly 5 defective motors?

→  $P = 0.01, n = 300, m = 0.01 \times 300 = 3$

$$P(\text{Exactly } 5) = P(5) = \frac{e^{-m} \cdot m^r}{r!}$$

$$P(5) = \frac{e^{-3} (3)^5}{5!}$$

$$P(5) = \underline{\underline{0.10082.}}$$



3. Solve any two

12 Marks

(i) Evaluate  $\int \frac{dx}{3-2\sin^2 x}$

→ Divide Numerator & Denominator  $\cos^2 x$

$$\therefore I = \int \frac{dx/\cos^2 x}{\frac{3-2\sin^2 x}{\cos^2 x}}$$

$$I = \int \frac{dx/\cos^2 x}{\frac{3}{\cos^2 x} - \frac{2\sin^2 x}{\cos^2 x}} = \int \frac{\sec^2 x dx}{3\sec^2 x - 2\tan^2 x}$$

$$I = \int \frac{\sec^2 x dx}{3(1+\tan^2 x) - 2\tan^2 x} = \int \frac{\sec^2 x dx}{3 + \tan^2 x}$$

Put,  $\tan x = t$ . Diff. w.r.t.  $x$ ,  
 $\sec^2 x dx = dt$

$$\therefore I = \int \frac{dt}{(\sqrt{3})^2 + t^2} = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{t}{\sqrt{3}}\right) + C$$

$$\therefore \int \frac{dx}{3-2\sin^2 x} = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{\tan x}{\sqrt{3}}\right) + C$$

(ii) Evaluate  $\int \frac{1 - \tan x}{1 + \tan x} dx$

$$\rightarrow I = \int \frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}} dx = \int \frac{\frac{\cos x - \sin x}{\cos x}}{\frac{\cos x + \sin x}{\cos x}} dx$$

$$I = \int \frac{\cos x - \sin x}{\cos x + \sin x} dx = \int f(x) = \cos x + \sin x$$

$$f'(x) = -\sin x + \cos x = \cos x - \sin x$$

$$\int \frac{f'(x)}{f(x)} dx = \log(f(x)) + C$$

$$I = \log(\cos x + \sin x) + C.$$



$$\int_0^1 \frac{dx}{x^2-x+1}$$

$$\rightarrow \text{Third term} = \left[ \frac{1}{2x} \times (-x) \right]^2 = \frac{1}{4}$$

$$I = \int_0^1 \frac{dx}{x^2 - x + \frac{1}{4} + 1 - \frac{1}{4}}$$

$$I = \int_0^1 \frac{dx}{\left(x - \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$I = \left[ \frac{2}{\sqrt{3}} \tan^{-1} \left( \frac{x - \frac{1}{2}}{\frac{\sqrt{3}}{2}} \right) \right]_0^1$$

$$I = \frac{2}{\sqrt{3}} \left[ \tan^{-1} \left( \frac{2(1 - 1/2)}{\sqrt{3}} \right) - \tan^{-1} \left( \frac{2(0 - 1/2)}{\sqrt{3}} \right) \right]$$

$$I = \frac{2}{\sqrt{3}} \left\{ \tan^{-1} \left( \frac{1}{\sqrt{3}} \right) - \tan^{-1} \left( -\frac{1}{\sqrt{3}} \right) \right\}$$

$$I = \frac{2}{\sqrt{3}} \left( \frac{\pi}{6} + \frac{\pi}{6} \right) = \frac{4\pi}{6\sqrt{3}} = \frac{2\pi}{3\sqrt{3}}$$

$$\boxed{\int_0^1 \frac{dx}{x^2-x+1} = \frac{2\pi}{3\sqrt{3}}}$$

(ii) Evaluate  $\int_0^{\pi/2} \sin^3 x \cdot \cos x \cdot dx$

$\rightarrow$  Put,  $\sin x = t$ ; diff. w.r.t.  $x$ .  $\cos x dx = dt$

$$I = \int_0^{\pi/2} t^3 dt = \int_0^{\pi/2} t^3 \cdot \cos x \cdot \frac{dt}{\cos x}$$

Limit Change,

$x$	0	$\pi/2$
$t$	$\sin 0 = 0$	$\sin(\pi/2) = 1$

$$I = \int_0^1 t^3 dt = \left[ \frac{t^4}{4} \right]_0^1 = \frac{1}{4}$$

$$\boxed{\int_0^{\pi/2} \sin^3 x \cdot \cos x \cdot dx = \frac{1}{4}}$$





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5 (i) Evaluate  $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

→ Let,  $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$  — (1)

using property,

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin [0 + \frac{\pi}{2} - x]}}{\sqrt{\sin [0 + \frac{\pi}{2} - x]} + \sqrt{\cos [0 + \frac{\pi}{2} - x]}} dx$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$
 — (2)

Adding eq<sup>n</sup> (1) & (2)

$$I + I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx + \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$2I = \int_0^{\pi/2} \frac{(\sqrt{\sin x} + \sqrt{\cos x})}{(\sqrt{\sin x} + \sqrt{\cos x})} dx$$

$$2I = \int_0^{\pi/2} 1 dx$$

$$2I = [x]_0^{\pi/2}$$

$$2I = \frac{\pi}{2} - 0$$

$$I = \frac{\pi}{4}$$

$$\therefore \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx = \frac{\pi}{4}$$





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ii Evaluate  $\int_2^5 \frac{\sqrt{x}}{\sqrt{7-x} + \sqrt{x}} dx$

→ Let,  $I = \int_2^5 \frac{\sqrt{x}}{\sqrt{7-x} + \sqrt{x}} dx$  — (1)

using property,

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$I = \int_2^5 \frac{\sqrt{2+5-x}}{\sqrt{7-(2+5-x)} + \sqrt{2+5-x}} dx$$

$$\therefore I = \int_2^5 \frac{\sqrt{7-x}}{\sqrt{x} + \sqrt{7-x}} dx$$
 — (2)

Adding eq<sup>n</sup> (1) & (2)

$$I + I = \int_2^5 \frac{\sqrt{x}}{\sqrt{7-x} + \sqrt{x}} dx + \int_2^5 \frac{\sqrt{7-x}}{\sqrt{x} + \sqrt{7-x}} dx$$

$$2I = \int_2^5 \frac{(\sqrt{x} + \sqrt{7-x})}{(\sqrt{7-x} + \sqrt{x})} dx$$

$$2I = \int_2^5 1 dx$$

$$2I = [x]_2^5$$

$$2I = [5 - 2]$$

$$2I = 3$$

$$I = \frac{3}{2}$$

$$\therefore \int_2^5 \frac{\sqrt{x}}{\sqrt{7-x} + \sqrt{x}} dx = \frac{3}{2}$$





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6.

a  
i

Form D.E. if  $y = ax^2 + b$

→  $y = ax^2 + b$  Diff. w.r.t.  $x$

$$\frac{dy}{dx} = a(2x) + 0$$

$$\therefore a = \frac{1}{2x} \cdot \frac{dy}{dx} \text{ Again diff. w.r.t. } x$$

$$\frac{d^2y}{dx^2} = 2a \quad (1)$$

$$\frac{d^2y}{dx^2} = 2 \left( \frac{1}{2x} \right) \frac{dy}{dx}$$

$$\therefore \frac{d^2y}{dx^2} - \frac{1}{x} \cdot \frac{dy}{dx} = 0$$

multiplying by " $x$ "

$$x \cdot \frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$$

ii) Solve =  $\sec^2 x \cdot \tan y \, dx + \sec^2 y \cdot \tan x \, dy = 0$

→  $\sec^2 x \cdot \tan y \, dx = -\sec^2 y \cdot \tan x \, dy$

$$\frac{\sec^2 x}{\tan x} \, dx = -\frac{\sec^2 y}{\tan y} \, dy$$

Integrating both sides,

$$\int \frac{\sec^2 x}{\tan x} \, dx = -\int \frac{\sec^2 y}{\tan y} \, dy$$

$$\log(\tan x) = -\log(\tan y) + \log C$$

$$\log[\tan x] + \log[\tan y] = \log C$$

$$\log(\tan x \cdot \tan y) = \log C$$

$$\tan x \cdot \tan y = C$$



(i) Solve D.E.  $x \cdot \frac{dy}{dx} + y = x^3$   
→ Divide each term by  $x$ ,

$$\frac{x \cdot \frac{dy}{dx} + y}{x} = \frac{x^3}{x}$$

$$\frac{dy}{dx} + \left(\frac{1}{x}\right)y = x^2$$

Eq<sup>n</sup>. in form  $\frac{dy}{dx} + py = Q$

∴ Integrating Factors I.F. =  $e^{\int p dx}$

$$\text{I.F.} = e^{\int \frac{1}{x} dx}$$

$$\text{I.F.} = e^{\log x}$$

$$\therefore \boxed{\text{I.F.} = x}$$

Solution of equation,

$$y \times \text{I.F.} = \int Q \times \text{I.F.} dx + C$$

$$y \cdot x = \int x^2 \times x dx + C$$

$$x \cdot y = \int x^3 dx + C$$

$$\boxed{x \cdot y = \frac{x^4}{4} + C}$$

(ii) Show that,  $(3x^2 + 6xy^2)dx + (6x^2y + 4y^2)dy = 0$   
is an exact D.E.

$$\rightarrow M = (3x^2 + 6xy^2) \quad \& \quad N = (6x^2y + 4y^2)$$

$$\frac{dM}{dy} = 0 + 6x(2y) \quad \& \quad \frac{dN}{dx} = 6y(2x) + 0$$

$$\frac{dM}{dy} = 12xy \quad \& \quad \frac{dN}{dx} = 12xy$$

$$\therefore \frac{dM}{dy} = \frac{dN}{dx}$$

∴  $\boxed{\text{Given eq<sup>n</sup>. is exact.}}$



6 C. In a sample of 1000 cases, mean of certain test is 14 & S.D. is 2.5. Assuming distribution to be normal find -

(i) How many students score bet<sup>n</sup> 12 & 15?

(ii) How many students score above 18?

Given -

$$A(0.8) = 0.2881$$

$$A(0.4) = 0.1554$$

$$A(1.6) = 0.4452.$$

$$\rightarrow N = 1000, \bar{x} = 14, \sigma = 2.5.$$

Case I] No. of students scoring bet<sup>n</sup> 12 & 15.

$$a_1 = 12$$

$$a_2 = 15$$

$$z_1 = \frac{a_1 - \bar{x}}{\sigma}$$

$$z_2 = \frac{a_2 - \bar{x}}{\sigma}$$

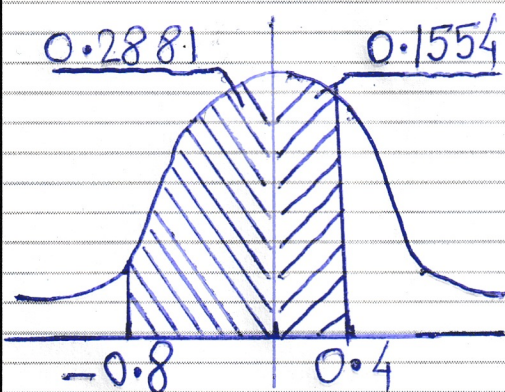
$$z_1 = \frac{12 - 14}{2.5}$$

$$z_2 = \frac{15 - 14}{2.5}$$

$$\underline{\underline{z_1 = -0.8}}$$

$$\underline{\underline{z_2 = +0.5}}$$

$$\therefore P(\text{bet}^n 12 \& 15) = A(-0.8 \leq z \leq 0.4)$$



$$= 0.2881 + 0.1554$$

$$= 0.4435$$

$\therefore$  No. of students

$$= 1000 \times 0.4435$$

$$= 443.5$$

$$\underline{\underline{\approx 444 \text{ students}}}}$$

Case II] Score above 18 i.e.  $a = 18$

$$z = \frac{a - \bar{x}}{\sigma}$$

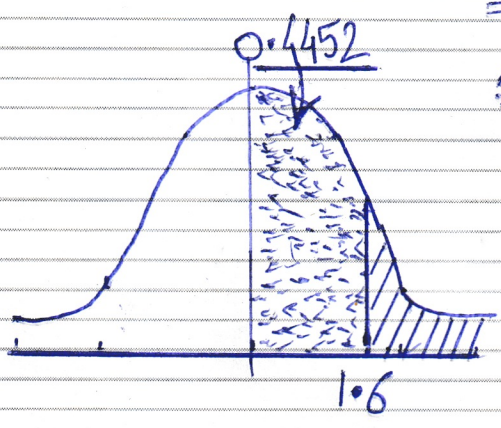
$$z = \frac{18 - 14}{2.5} \quad \therefore z = +1.6$$





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$$\begin{aligned}
 P(\text{Above } 18) &= P(Z \text{ above } 1.6) \\
 &= A(\text{right to } 1.6) \\
 &= 0.5 - 0.4452 \\
 &= 0.0548 \\
 \text{No. of students} &= 0.0548 \times 1000 \\
 &= 54.8 \\
 &\approx \underline{\underline{55 \text{ students}}}
 \end{aligned}$$



Case I  
Students scoring bet<sup>n</sup> 12 & 15 are = 444

Case II  
Students scoring above 18 are = 55.