 12425

WINTER- 2024 EXAMINATION

Model Answer

Subject Name: Basic Maths (M-I) — **Subject Code:** 311302

Important Instructions to STUDENTS

1) *The model answer given here are prepared from the answers from the previously uploaded model answers by Board.*
2) *These model answers are not uploaded by the MSBTE official site but MSBTE study resources website prepared it for students. This model answer has question paper also inbuilt in it, no need to download it separate.*
3) *Please remembter that answers are not checked word to word but based on keywords which must be present in your answer*
4) *The model answer and the answer written by candidate may vary but the examiner may tryto assess the understanding level of the candidate*
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7) *Students are advised to prepare all the syllabus from recommended book and use these model answers for the purpose of tests.*

Q.NO	SUB Q N	ANSWER	Marking Scheme
1.		Attempt any Five	10 Marks
	a)	Find x if $\log_2(x+2)=3$.	
	→	$\log_2(x+2)=3$	
		Exp. Form: $2^3=(x+2)$	
		$\therefore 8 = x+2$	
		$\therefore 8-2 = x$ $\quad \therefore \boxed{x=6}$	
	b)	If $\sin A = 0.4$ find value of $\sin 3A$.	
	→	$\sin 3A = 3\sin A - 4\sin^3 A$	
		$\therefore \sin 3A = 3(0.4) - 4(0.4)^3$	
		$\therefore \boxed{\sin 3A = 0.944}$	
	c)	Find acute angle between lines whose slopes are $\sqrt{3}$ & $\frac{1}{\sqrt{3}}$.	
	→	$m_1=\sqrt{3}$ & $m_2 = 1/\sqrt{3}$	
		$\theta = \tan^{-1}\left\lvert\frac{m_1-m_2}{1+m_1 \cdot m_2}\right\rvert$	
		$\theta = \tan^{-1}\left\lvert\frac{\sqrt{3}-\frac{1}{\sqrt{3}}}{1+(\sqrt{3})(\frac{1}{\sqrt{3}})}\right\rvert = \frac{\pi}{6}^c$ or 30°	

Scheme

d) If $f(x) = 3x^2 - 5x + 7$, Show that, $f(-1) = 3f(1)$

→ L.H.S. = $F(-1)$
$= 3(-1)^2 - 5(-1) + 7$
$= 3 + 5 + 7$
$= 15.$

R.H.S. = $3F(1)$
$= 3[3(1)^2 - 5(1) + 7]$
$= 3[3 - 5 + 7]$
$= 15$

∴ $\boxed{\text{L.H.S.} = \text{R.H.S. Prooved.}}$

e) Find $\frac{dy}{dx}$ if, $y = a^x + x^a + e^a + \log_a x$

→ $y = a^x + x^a + e^a + \log_a x$

diff. w.r.t. x;

$$\frac{dy}{dx} = a^x \cdot \log a + a x^{a-x} + 0 + \frac{1}{x \cdot \log a}$$

$$\therefore \boxed{\frac{dy}{dx} = a^x \cdot \log a + a \cdot x^{(a-x)} + \frac{1}{x \cdot \log a}}$$

f) Find $\frac{dy}{dx}$, if, $y = e^x \cdot \sin^{-1} x$.

→ $y = e^x \cdot \sin^{-1} x$ diff. w.r.t. x,

$$\frac{dy}{dx} = e^x \cdot \frac{d}{dx}[\sin^{-1} x] + \sin^{-1} x \cdot \frac{d}{dx}[e^x]$$

$$\therefore \frac{dy}{dx} = e^x \left[\frac{1}{\sqrt{1-x^2}}\right] + \sin^{-1} x \, [e^x]$$

$$\therefore \boxed{\frac{dy}{dx} = e^x \left[\frac{1}{\sqrt{1-x^2}} + \sin^{-1} x\right]}$$

g. Find range & C.R. 2, 3, 1, 10, 6, 31, 17, 20, 24

→ 2, 3, 1, 10, 6, 31, 17, 20, 24.

Higher Number = H = 31. Lower Number = L = 1.

Range = H − L
Range = 31 − 1
$\boxed{\text{Range} = 30.}$

Coeff. of range $= \frac{H - L}{H + L}$

$$C.R. = \frac{31 - 1}{31 + 1} = \frac{30}{32}$$

$\boxed{C.R. = 0.9375}$

Q.NO	Q N	ANSWER	Marking Scheme

2. a)

If $A = \begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix}, B = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

verify that, $(AB)^T = B^T \cdot A^T$.

$\rightarrow$ $L.H.S. = (AB)^T$

$$L.H.S. = \left\{ \begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 3 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix} \right\}^T$$

$$L.H.S. = \left\{ \begin{bmatrix} \boxed{2 \ \ -3} \boxed{\begin{matrix} 3 \\ 1 \end{matrix}} & \boxed{2 \ \ -3} \boxed{\begin{matrix} -1 \\ 0 \end{matrix}} & \boxed{2 \ \ -3} \boxed{\begin{matrix} 2 \\ 1 \end{matrix}} \\ \boxed{1 \ \ 5} \boxed{\begin{matrix} 3 \\ 1 \end{matrix}} & \boxed{1 \ \ 5} \boxed{\begin{matrix} -1 \\ 0 \end{matrix}} & \boxed{1 \ \ 5} \boxed{\begin{matrix} 2 \\ 1 \end{matrix}} \end{bmatrix} \right\}^T$$

$$L.H.S. = \left\{ \begin{bmatrix} (6)+(-3) & (-2)+0 & (4)+(-3) \\ (3)+(5) & (-1)+0 & (2)+(3) \end{bmatrix} \right\}^T$$

$$L.H.S. = \left\{ \begin{bmatrix} 3 & -2 & 1 \\ 8 & -1 & 7 \end{bmatrix} \right\}^T$$

$$L.H.S. = \left\{ \begin{bmatrix} 3 & 8 \\ -2 & -1 \\ 1 & 7 \end{bmatrix} \right\}$$ Apply Transpose.

$R.H.S. = B^T \cdot A^T$

$$R.H.S. = \begin{bmatrix} 3 & 1 \\ -1 & 0 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix}$$

$$R.H.S. = \begin{bmatrix} \boxed{3 \ \ 1} \boxed{\begin{matrix} 2 \\ -3 \end{matrix}} & \boxed{3 \ \ 1} \boxed{\begin{matrix} 1 \\ 5 \end{matrix}} \\ \boxed{-1 \ \ 0} \boxed{\begin{matrix} 2 \\ -3 \end{matrix}} & \boxed{-1 \ \ 0} \boxed{\begin{matrix} 1 \\ 5 \end{matrix}} \\ \boxed{2 \ \ 1} \boxed{\begin{matrix} 2 \\ -3 \end{matrix}} & \boxed{2 \ \ 1} \boxed{\begin{matrix} 1 \\ 5 \end{matrix}} \end{bmatrix}$$

Q.NO	SUB Q N	ANSWER	Marking Scheme

$$R.H.S. = \begin{bmatrix} (6)+(-3) & (3)+(5) \\ (-2)+(0) & (-1)+(0) \\ (4)+(-3) & (2)+(5) \end{bmatrix}$$

$$R.H.S. = \begin{bmatrix} 3 & 8 \\ -2 & -1 \\ 1 & 7 \end{bmatrix}$$

$\therefore$ $\boxed{L.H.S. = R.H.S. \quad \text{Prooved.}}$

b] Resolve into Partial fraction $\frac{x}{x^2-x-2}$.

→ Factoring denominator, we get,

$$\frac{x}{(x-2)(x+1)} = \frac{A}{(x-2)} + \frac{B}{(x+1)}$$

$$\therefore x = (x-2)(x+1)\left[\frac{A}{(x-2)} + \frac{B}{(x+1)}\right]$$

$$\boxed{x = A(x+1) + B(x-2)} \quad \text{— eq}^n \text{ (1)}$$

Put $x-2=0$ i.e. $x=2$ in eq^n (1)

$$2 = A(2+1) + 0$$
$$2 = 3A \qquad \therefore \boxed{A = \frac{2}{3}}$$

Put $x+1=0$ i.e. $x=-1$.

$$-1 = 0 + B(-1-2)$$
$$-1 = -3B \qquad \therefore \boxed{B = \frac{1}{3}}$$

$\therefore$ Sol^n =

$$\frac{x}{x^2-x-2} = \frac{(2/3)}{(x-2)} + \frac{(1/3)}{(x+1)}$$

$$\therefore \boxed{\frac{x}{x^2-x-2} = \frac{2}{3(x-2)} + \frac{1}{3(x+1)}}$$

Q.NO	SUB Q N	ANSWER	Marking Scheme

c) Simplyfy $\frac{1}{\log_5 10} + \frac{1}{\log_{20} 10}$

→ Using change of base theorem.

$$= \frac{1}{\frac{\log 10}{\log 5}} + \frac{1}{\frac{\log 10}{\log 20}}$$

$$= \frac{\log 5}{\log 10} + \frac{\log 20}{\log 10}$$

$$= \frac{\log 5 + \log 20}{\log 10}$$

$$= \frac{\log(100)}{\log(10)}$$

$$= \frac{\log(10)^2}{\log(10)} = \frac{2\log 10}{\log 10} = 2$$

$$\therefore \boxed{\frac{1}{\log_5 10} + \frac{1}{\log_{20} 10} = 2.}$$

d) If $\tan x = \frac{5}{6}$, $\tan y = \frac{1}{11}$. Show that, $x+y = \frac{\pi}{4}$.

→ $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$

$$\tan(x+y) = \frac{(5/6) + (1/11)}{1 - (5/6)(1/11)} = \frac{(61/66)}{(61/66)} = 1$$

$\therefore \tan(x+y) = 1$

$(x+y) = \tan^{-1}(1)$

$(x+y) = 45^\circ$ i.e. $\frac{\pi}{4}^c$

$\therefore \boxed{(x+y) = \frac{\pi}{4}}$ Prooved.

3 a] Prove that, $\dfrac{\cos A}{1-\tan A} + \dfrac{\sin A}{1-\cot A} = \sin A + \cos A$.

$\rightarrow$ $L.H.S. = \dfrac{\cos A}{1-\dfrac{\sin A}{\cos A}} + \dfrac{\sin A}{1-\dfrac{\cos A}{\sin A}}$

$L.H.S. = \dfrac{\cos A}{\left(\dfrac{\cos A - \sin A}{\cos A}\right)} + \dfrac{\sin A}{\left(\dfrac{\sin A - \cos A}{\sin A}\right)}$

$L.H.S. = \dfrac{\cos^2 A}{(\cos A - \sin A)} - \dfrac{\sin^2 A}{[\cos A - \sin A]}$

[−ve sign taken out from denominator of second term.]

$L.H.S. = \dfrac{\cos^2 A - \sin^2 A}{\cos A - \sin A}$

$L.H.S. = \dfrac{(\cos A + \sin A)\cancel{(\cos A - \sin A)}}{\cancel{(\cos A - \sin A)}}$

$L.H.S. = \cos A + \sin A$

i.e. $\boxed{L.H.S. = \sin A + \cos A = R.H.S.\ \text{Prooved.}}$

b] Without using calculator, Proove that,
$\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ = \frac{1}{16}$.

$\rightarrow$ $L.H.S. = \cos 20 \cdot \cos 40 \cdot (1/2) \cdot \cos 80$

$L.H.S. = (1/2)\,[\cos 20 \cdot \cos 40 \times \cos 80]$

$L.H.S. = (1/2)\,[(1/2)\,[\cos(20+40) + \cos(20-40)] \cdot \cos 80$

$L.H.S. = \left(\frac{1}{4}\right)[\cos 60 \cdot \cos 80 + \cos 20 \cdot \cos 80]$

$L.H.S. = \left(\frac{1}{4}\right)\left[\frac{1}{2}\cos 80 + \frac{1}{2}[\cos 100 + \cos 60]\right]$

$L.H.S. = \left(\frac{1}{4}\right)\left[\frac{1}{2}(\cos 80 + \cos 100) + \frac{1}{2}\left(\frac{1}{2}\right)\right]$

$L.H.S. = \frac{1}{4}\left[\frac{1}{2}(\cos 80 + \cos 100) + \frac{1}{4}\right]$

Q.NO	SUB Q N	ANSWER	Marking Scheme

$$L.H.S. = \frac{1}{4}\left\{\frac{1}{2}\left[2\cos\left(\frac{80+100}{2}\right)\cdot\cos\left(\frac{80-100}{2}\right)\right]+\frac{1}{4}\right\}$$

$$L.H.S. = \frac{1}{4}\left\{\frac{1}{2}\left[2\cos 90 \cdot \cos 10\right] + \frac{1}{4}\right\}$$

$$L.H.S. = \frac{1}{4}\left\{\frac{1}{2}(0) + \frac{1}{4}\right\} = \frac{1}{4}\left(\frac{1}{4}\right)$$

$$L.H.S. = \frac{1}{16} = R.H.S.$$

$$\therefore \boxed{L.H.S. = R.H.S. \text{ Prooved.}}$$

c) Find eq^n of straight line passing through point of intersection of $x+y=4$ & $2x+y=4$ & parallel to x axis.

→ line parallel to x axis i.e. slope $= m = 0$.

$$\begin{array}{r} x + y = 4 \\ -\ 2x + y = 4 \\ \hline -x = 0 \end{array}$$

Substracting eq^n ① & ②

$\therefore \boxed{x = 0.}$

We know, $x + y = 4$

$0 + y = 4$ $\therefore \boxed{y = 4.}$

Now, Point $(0, 4)$ & slope $= m = 0$.
$\qquad\qquad x_1 \ y_1$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = 0(x - 0)$$

$$y - 4 = 0$$

$\therefore \boxed{\text{Eq}^n \text{ of line is } y-4=0 \text{ or } y=4.}$

Q.NO	SUB Q N	ANSWER	Marking Scheme
3	d)	Find $\frac{dy}{dx}$ if $x^3+y^3=30xy$	

$\rightarrow$ $x^3+y^3=30xy$ diff. w.r.t. x

$$3x^2+3y^2\frac{dy}{dx}=30\left[x\cdot\frac{dy}{dx}+y(1)\right]$$

$$3x^2+3y^2\frac{dy}{dx}=30x\frac{dy}{dx}+30y$$

$$3y^2\frac{dy}{dx}-30x\frac{dy}{dx}=30y-3x^2$$

$$\therefore \frac{dy}{dx}(3y^2-30x)=30y-3x^2$$

$$\therefore \frac{dy}{dx}=\frac{30y-3x^2}{3y^2-30x}$$

$$\therefore \frac{dy}{dx}=\frac{\cancel{3}[10y-x^2]}{\cancel{3}[y^2-10x]}$$

$$\therefore \frac{dy}{dx}=\frac{10y-x^2}{y^2-10x}.$$

$\therefore$ if $x^3+y^3=30xy$,

$$\boxed{\frac{dy}{dx}=\frac{10y-x^2}{y^2-10x}.}$$

Q.NO	SUB Q N	ANSWER	Marking Scheme

4 a] If $x = a\cos^3\theta$ & $y = a\sin^3\theta$ find $\frac{dy}{dx}$, $\theta = \frac{\pi}{3}$

→ $x = a\cos^3\theta$

diff. w.r.t. θ

$$\frac{dx}{d\theta} = 3a\cos^2\theta \cdot \frac{d}{d\theta}\cos\theta$$

$$\frac{dx}{d\theta} = 3a\cos^2\theta(-\sin\theta)$$

$$\frac{dx}{d\theta} = -3a\sin\theta\cos^2\theta$$

$y = a\sin^3\theta$

diff. w.r.t. θ

$$\frac{dy}{d\theta} = 3a\sin^2\theta\frac{d}{d\theta}\sin\theta$$

$$\frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$$

$$\frac{dy}{d\theta} = 3a\sin^2\theta \cdot \cos\theta.$$

Now, $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$

$$\therefore \frac{dy}{dx} = \frac{\cancel{3}\cdot\cancel{a}\cdot \sin^{\cancel{2}}\theta\,\cancel{\cos\theta}}{-\cancel{3}\cdot\cancel{a}\cdot\cancel{\sin\theta}\cos^{\cancel{2}}\theta} = -\tan\theta$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{3}} = -\tan\left(\frac{\pi}{3}\right) = -\tan 60 = -\sqrt{3}.$$

$$\therefore \boxed{\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{3}} = -\sqrt{3}.}$$

b] Find maximum & minimum value of function

$$y = 2x^3 - 21x^2 + 36x - 20$$

→ diff. w.r.t. x; $\frac{dy}{dx} = 6x^2 - 42x + 36$

Again diff. w.r.t. x; $\frac{d^2y}{dx^2} = 12x - 42.$

To find $x = a$ & $x = b$ where maxima & minima occurs. At maxima & minima $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = 6x^2 - 42x + 36$$

$$0 = 6x^2 - 42x + 36$$

Divide each term by "6", we get,

$$0 = x^2 - 7x + 6$$

$$\therefore 0 = (x-6)(x-1)$$

$\left\{ x^2 - 7x + 6 \quad -6\ \ -1 \right.$

Q.NO	Q N		Scheme

$(x-6)(x-1) = 0$

$\therefore x-6 = 0$ or $x-1 = 0$

$x = 6$ or $x = 1.$

To find where maxima occurs,

$\left.\frac{d^2y}{dx^2}\right|_{x=6} = 12x - 42$

$\left.\frac{d^2y}{dx^2}\right|_{x=6} = (12\times 6) - 42$

$\left.\frac{d^2y}{dx^2}\right|_{x=6} = 30 \quad (+ve)$

Hence f^n. is minimum at $x = 6$,

& f^n. is maximum at $x = 1$.

$\therefore y_{max} = y|_{x=1} = 2(1)^3 - 21(1)^2 + 36(1) - 20$

$\boxed{y_{max} = -3 \text{ (Maxima)}}$

$y_{min} = y|_{x=6} = 2(6)^3 - 21(6)^2 + 36(6) - 20$

$\boxed{y_{min} = -128 \text{ (Minima).}}$

c] Compute M.D. For 15, 22, 27, 11, 9, 21, 14, 9.

$\rightarrow \bar{x} = \frac{15+22+27+11+9+21+14+9}{8} \quad \therefore \bar{x} = 16$

x_i	$d_i = \lvert \bar{x} - x_i \rvert =$
15	$= \lvert 16 - 15 \rvert = 1$
22	$= \lvert 16 - 22 \rvert = 6$
27	$= \lvert 16 - 27 \rvert = 11$
11	$= \lvert 16 - 11 \rvert = 5$
9	$= \lvert 16 - 9 \rvert = 7$
21	$= \lvert 16 - 21 \rvert = 5$
14	$= \lvert 16 - 14 \rvert = 2$
9	$= \lvert 16 - 9 \rvert = 7$
	$\Sigma d_i = 44$

$M.D. = \frac{\Sigma d_i}{N}$

$M.D. = \frac{44}{8}$

Mean Deviation

$\boxed{M.D. = 5.5.}$

Q.NO	SUB Q N	ANSWER	Marking Scheme

4. d) Calculate mean & S.D. of following frequency dist^n.

Class interval	0-10	10-20	20-30	30-40	40-50
Frequency	14	23	27	21	15

→

C.I.	class Marks x_i	frequency f_i	f_ix_i	x_i^2	$f_ix_i^2$
0-10	5	14	70	25	350
10-20	15	23	345	225	5175
20-30	25	27	675	625	16875
30-40	35	21	735	1225	25725
40-50	45	15	675	2025	30375
Total	—	100	2500	—	78500

$$\text{Mean} = \bar{x} = \frac{\Sigma f_ix_i}{\Sigma f_i} = \frac{2500}{100} \qquad \boxed{\bar{x} = 25}$$

$$S.D. = \sigma = \sqrt{\frac{\Sigma f_ix_i^2}{N} - (\bar{x}^2)} = \sqrt{\frac{78500}{100} - (25)^2}$$

$$\boxed{S.D. = \sigma = 12.65}$$

e) Find variance & C.V. for following dist^n.

→

Class interval	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	4	6	10	18	9	3

C.I.	x_i	f_i	x_if_i
10-20	15	4	60
20-30	25	6	150
30-40	35	10	350
40-50	45	18	810
50-60	55	9	495
60-70	65	3	195
		$\Sigma f_i = 50$	$\Sigma f_ix_i = 2060$

$$\therefore \text{Mean} = \bar{x} = \frac{\Sigma f_ix_i}{\Sigma f_i}$$

$$\bar{x} = \frac{2060}{50} \qquad \boxed{\bar{x} = 41.2}$$

Q.NO	SUB Q N	ANSWER	Marking Scheme

class	x_i	f_i	$d_i = \lvert\bar{x} - x_i\rvert$	$f_i d_i^2$
10-20	15	4	$= \lvert 41.2 - 15 \rvert = 26.2$	2745.76
20-30	25	6	$= \lvert 41.2 - 25 \rvert = 16.2$	1574.64
30-40	35	10	$= \lvert 41.2 - 35 \rvert = 6.2$	384.4
40-50	45	18	$= \lvert 41.2 - 45 \rvert = 3.8$	259.92
50-60	55	9	$= \lvert 41.2 - 55 \rvert = 13.8$	1713.96
60-70	65	3	$= \lvert 41.2 - 65 \rvert = 23.8$	1699.32
				$\Sigma f_i d_i^2 = 8378$

$$\text{Variance} = \frac{\Sigma f_i d_i^2}{\Sigma f_i} = \frac{8378}{50}$$

$$\boxed{\text{Variance} = 167.56}$$

$$\text{Coeff. of Variance} = \frac{\sigma}{\bar{x}} \times 100 = \frac{\sqrt{167.56}}{41.2} \times 100$$

$$\boxed{C.V. = 31.41\%}$$

5. a] Solve by matrixe inversion method,

$3x + y + 2z = 3$

$2x - 3y - z = -3$

$x + 2y + z = 4.$

→ Writting equations in matrixe form.

$$\begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$$

$$A \cdot X = B \quad — (1)$$

$\therefore X = A^{-1} \cdot B$ To find A^{-1};

$$|A| = \begin{vmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

$|A| = 3(-3+2) - 1(2+1) + 2(4+3)$

$|A| = 8$

$\because |A| \neq 0$

$\therefore A^{-1}$ exists. (P.T.O.)

Finding Co-factors of each elements of $|A|$,

$C_{11} = \begin{vmatrix} -3 & -1 \\ 2 & 1 \end{vmatrix}$

$C_{11} = -3+2$

$\underline{C_{11} = -1.}$

$C_{12} = -\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix}$

$C_{12} = -(2+1)$

$\underline{C_{12} = -3}$

$C_{13} = +\begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix}$

$C_{13} = 4+3$

$\underline{C_{13} = 7}$

$C_{21} = -\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}$

$C_{21} = -(-1-4)$

$\underline{C_{21} = 3.}$

$C_{22} = +\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix}$

$C_{22} = 3-2$

$\underline{C_{22} = 1.}$

$C_{23} = -\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}$

$C_{23} = -(6-1)$

$\underline{C_{23} = -5.}$

$C_{31} = +\begin{vmatrix} 1 & 2 \\ -3 & -1 \end{vmatrix}$

$C_{31} = -1+6$

$\underline{C_{31} = 5.}$

$C_{32} = -\begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix}$

$C_{32} = -(-3-4)$

$\underline{C_{32} = 7}$

$C_{33} = +\begin{vmatrix} 3 & 1 \\ 2 & -3 \end{vmatrix}$

$C_{33} = -9-2$

$\underline{C_{33} = -11.}$

$\therefore P =$ Matrix of Co-factors

$$P = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} -1 & -3 & 7 \\ 3 & 1 & -5 \\ 5 & 7 & -11 \end{bmatrix}$$

Now, $\operatorname{Adj} A = P' = \begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}$

$$\therefore A^{-1} = \frac{1}{|A|} \cdot \operatorname{adj} A$$

$$\therefore A^{-1} = \frac{1}{8} \begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}$$

We know, $X = A^{-1} \cdot B$

$$\therefore X = \frac{1}{8} \begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix} \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} -3 & -9 & +20 \\ -9 & -3 & +28 \\ 21 & +15 & -44 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \quad \therefore \boxed{\begin{matrix} x = 1 \\ y = 2 \\ z = -1. \end{matrix}}$$

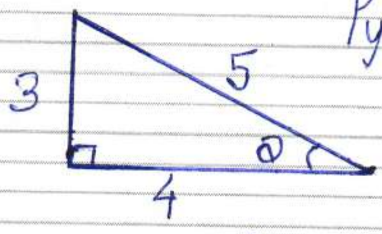
Scheme

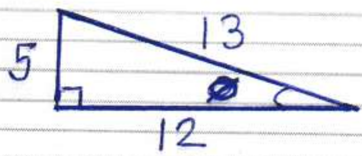
5. b (i) Prove that, $\cos^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\cos^{-1}\left(\frac{33}{65}\right)$

→ let, $\cos^{-1}\left(\frac{4}{5}\right)=\theta$ $\qquad \cos^{-1}\left(\frac{12}{13}\right)=\phi$

$\therefore \cos\theta=\left(\frac{4}{5}\right)$ $\qquad \therefore \cos\phi=\left(\frac{12}{13}\right)$

Pythagoras Triplets.





$\therefore \sin\theta=\left(\frac{3}{5}\right)$ $\qquad \sin\phi=\left(\frac{5}{13}\right)$

Now, $\cos(\theta+\phi)=\cos\theta\cdot\cos\phi-\sin\theta\cdot\sin\phi$

$\therefore \cos(\theta+\phi)=\left(\frac{4}{5}\right)\left(\frac{12}{13}\right)-\left(\frac{3}{5}\right)\left(\frac{5}{13}\right)$

$\cos(\theta+\phi)=\left(\frac{33}{65}\right)$

$\therefore (\theta+\phi)=\cos^{-1}\left(\frac{33}{65}\right)$

$\therefore \boxed{\cos^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\cos^{-1}\left(\frac{33}{65}\right) \text{ Prooved.}}$

(ii) If $\sin\theta=\left(\frac{15}{17}\right)$, θ in II^{nd} quad^{t} find $\tan\theta$.

→ $\sin^2\theta+\cos^2\theta=1$

$\therefore \cos^2\theta=1-\sin^2\theta$

$\therefore \cos^2\theta=1-\left(\frac{15}{17}\right)^2$

$\therefore \cos^2\theta=\frac{64}{289}$ $\therefore \cos\theta=\pm\frac{8}{17}$.

$\because$ θ lies in 2^{nd} quadrant $\boxed{\cos\theta=-\frac{8}{17}}$

We know,

$\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{(15/17)}{(-8/17)}$

$\therefore \boxed{\tan\theta=-\frac{15}{8}}$

Q.NO	SUB Q N	ANSWER	Marking Scheme
5	c (i)	Find length of perpendicular from the point (5, 4) on the straight line $2x + y = 34$.	

→ $2x + y - 34 = 0$

where, $a = 2$, $b = 1$. & $c = -34$.

$$\therefore d_{pl} = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

$$\therefore d_{pl} = \left| \frac{(2 \times 5) + (1 \times 4) - 34}{\sqrt{(2)^2 + (1)^2}} \right|$$

$$\therefore d_{pl} = \left| \frac{10 + 4 - 34}{\sqrt{5}} \right|$$

$$\therefore \boxed{d_{pl} = 4\sqrt{5} \text{ units}}$$

(ii) Find the equation of a straight line that passes through (3, 4) & perpendicular to line $3x + 2y + 5 = 0$

→ $3x + 2y + 5 = 0$

where $a = 3$, $b = 2$

$\therefore$ Slope of Line $= m_1 = -\frac{a}{b} = -\frac{3}{2}$.

Required line ⊥er to given line.

$\therefore$ slope of required line $= m_2 = \frac{2}{3}$

$(\because m_1 \cdot m_2 = -1)$.

Line passes through (3, 4) with slope $(m_2) = \frac{2}{3}$.

[P.T.O.]

Q.NO	SUB Q N	ANSWER	Marking Scheme

$x_1 = 3$, $y_1 = 4$ & $m = \frac{2}{3}$.

Equation of line in slope point form is = $y - y_1 = m(x - x_1)$

$\therefore y - 4 = \frac{2}{3}(x - 3)$

$\therefore 3y - 12 = 2x - 6$

$\therefore 2x - 3y - 6 + 12 = 0$

$\therefore 2x - 3y + 6 = 0$

$\therefore \boxed{2x - 3y + 6 = 0}$ is eq^n. of line.

Q.NO	SUB Q N	ANSWER	Marking Scheme

6. a) Find equation of tangent & normal to the curve $4x^2+9y^2=40$ at point $(1,2)$.

→ Eq^n. of curve is $4x^2+9y^2=40$ — eq^n ①

diff. w.r.t. x,

$\therefore 8x+18y\cdot\frac{dy}{dx}=0$

$\therefore 18y\cdot\frac{dy}{dx}=-8x$

$\therefore \frac{dy}{dx}=\frac{-8x}{18y}$

$\therefore \frac{dy}{dx}=\frac{-4x}{9y}$

Now, Slope of tangent at $(1,2)$ is,

$$\left.\frac{dy}{dx}\right|_{(1,2)}=\frac{-4(1)}{9(2)}=\frac{-2}{9}.$$

$$\therefore \left.\frac{dy}{dx}\right|_{(1,2)}=\frac{-2}{9}.$$

$x_1=1,\ y_1=2,\ m=\frac{-2}{9},$

Equation of tangent in slope point form

$y-y_1=m(x-x_1)$

$\therefore y-2=\frac{-2}{9}(x-1)$

$\therefore 9y-18=-2x+2$

$\therefore 2x+9y=2+18$

$\therefore 2x+9y=20$ — eq^n. ②

Tangent $\perp$er normal at $(1,2)$

Q.NO	SUB Q N	ANSWER	Marking Scheme

Tangent ⊥er normal at (1,2)

∴ Slope of normal at (1,2)

$$\therefore m' = \frac{-1}{\text{Slope of tangent at } (1,2)}$$

$$\therefore m' = \frac{-1}{(-2/9)}$$

$$\therefore m' = \frac{9}{2}.$$

Normal also passes through same point (1,2)

$$x_1 = 1,\ y_1 = 2\ \&\ m' = \frac{9}{2}.$$

Equation of normal in slope point form

$$y - y_1 = m'(x - x_1)$$

$$\therefore y - 2 = \frac{9}{2}(x - 1)$$

$$\therefore 2y - 4 = 9x - 9$$

$$\therefore -9x + 2y = -9 + 4$$

$$\therefore -9x + 2y = -5$$

$$\therefore \boxed{9x - 2y = 5.}$$ is eqⁿ. of line.

6. b]

(i) Find radius of curvature of curve $y = x^3$ at point (2, 8)

→ $y = x^3$ eq^n of Curve,

diff. w.r.t. x

$$\frac{dy}{dx} = 3x^2$$

Q.NO	SUB Q N	ANSWER	Marking Scheme

Now, $\left.\frac{dy}{dx}\right|_{(2,8)} = 3(2)^2 = \underline{\underline{12}}.$

Again diff. w.r.t. x

$$\frac{d^2y}{dx^2} = 6x$$

Hence, $\left.\frac{d^2y}{dx^2}\right|_{(2,8)} = 6(2) = \underline{\underline{12}}.$

Radius of curvature is given by,

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$$\rho_{(2,8)} = \frac{\left[1 + (12)^2\right]^{3/2}}{12}$$

$$\boxed{\rho_{(2,8)} = 145.50 \text{ units}}$$

6 b (ii) Find $\frac{dy}{dx}$, if $y = x^{\sin x}$

→ Taking log of both sides,

$$\log y = \log(x^{\sin x})$$

$$\therefore \log y = \sin x \cdot \log x$$

$$\left(\frac{1}{y}\right) \cdot \frac{dy}{dx} = \sin x\left(\frac{1}{x}\right) + \log x\,(\cos x)$$

$$\therefore \frac{dy}{dx} = y\left[\frac{\sin x}{x} + \log x \cdot \cos x\right]$$

$$\therefore \boxed{\frac{dy}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \log x \cdot \cos x\right]}$$

6 c) Scores of two batsmen A & B in ten innings during a certain season as under:-

A	32	28	47	63	71	39	10	60	96	14
B	19	31	48	53	67	90	10	62	40	80

Find which of two batsmen is more consistent in scoring why?

→ To compare coeff. of variance of A & B.

"A"

x_i	$d_i = \bar{x} - x_i$	d_i^2
32	−14	196
28	−18	324
47	1	1
63	17	289
71	25	625
39	−7	49
10	−36	1296
60	14	196
96	50	2500
14	−32	1024
460	—	6500

$\text{Mean} = \bar{x} = \frac{\Sigma x_i}{N}$

$\bar{x} = \frac{460}{0} = \underline{46}$

S.D. of A (σ_A)

$\sigma_A = \sqrt{\frac{\Sigma d_i^2}{N}}$

$\sigma_A = \sqrt{\frac{6500}{10}}$

$\boxed{\sigma_A = 25.5}$

$C.V._A = \frac{\sigma_A}{\bar{x}} \times 100$

$\boxed{C.V._A = 55.43\%}$

"B"

x_i	$d_i = \bar{x} - x_i$	d_i^2
19	−31	961
31	−19	361
48	−2	4
53	3	9
67	17	289
90	40	1600
10	−40	1600
62	12	144
40	−10	100
80	30	900
500	—	5968

$\text{Mean} = \bar{x} = \frac{\Sigma x_i}{N}$

$\bar{x} = \frac{500}{10} = \underline{50}$

S.D. of B (σ_B)

$\sigma_B = \sqrt{\frac{\Sigma d_i^2}{N}}$

$\sigma_B = \sqrt{\frac{5968}{10}}$

$\boxed{\sigma_B = 24.43}$

$C.V._B = \frac{\sigma_B}{\bar{x}} \times 100$

$\boxed{C.V._B = 48.86\%}$

$\because C.V._B < C.V._A$

Batsman "B" is less variable, i.e. more consistant.