WINTER-23

~~SUMMER~~- 2023 EXAMINATION

Model Answer

Subject Name: Basic Mathematics.

Subject Code 311302

Important Instructions to STUDENTS

1) The model answer given here are prepared from the answers from the previously uploaded model answers by Board.
2) These model answers are not uploaded by the MSBTE official site but MSBTE study resources website prepared it for students. This model answer has question paper also inbuilt in it, no need to download it separate.
3) Please remembter that answers are not checked word to word but based on keywords which must be present in your answer.
4) The model answer and the answer written by candidate may vary but the examiner may tryto assess the understanding level of the candidate
5) While assessing figures, examiner may give credit for principal components indicated in thefigure. The figures drawn bycandidate and model answer may vary. The examiner may give credit for anyequivalent figure drawn
6) For programming language papers, credit may be given to any other program based on equivalentconcept
7) Students are advised to prepare all the syllabus from recommended book and use these model answers for the purpose of tests.

Q.NO	SUB Q N	ANSWER	Marking Scheme
1		Solve any five (10)	
	a	Find the value of x if $\log_5(x^2-5x+11)=1$	
	→	$\log_5(x^2-5x+11)=1$	
		$5^1 = x^2-5x+11$ — convert to exp form	1/2
		$\therefore 5 = x^2-5x+11$	
		$\therefore 0 = x^2-5x+11-5$	
		$\therefore 0 = x^2-5x+6$	1/2
		$\therefore x^2-5x+6=0$; -3, -2	1/2
		$(x-3)(x-2)=0$	
		$\boxed{\therefore x=3 \text{ or } x=2}$	1/2
	b	Find the value of $\sin(15^\circ)$ using compound angle.	
	→	$\sin 15^\circ = \sin(45^\circ-30^\circ)$	1/2
		$= \sin 45 \cdot \cos 30 - \cos 45 \cdot \sin 30$	1/2
		$= \left[\frac{1}{\sqrt2}\cdot\frac{\sqrt3}{2}\right]-\left[\frac{1}{\sqrt2}\cdot\frac{1}{2}\right]$	1/2
		$= \frac{\sqrt3}{2\sqrt2}-\frac{1}{2\sqrt2} =$	
		$\boxed{\sin 15^\circ = \frac{\sqrt3-1}{2\sqrt2}}$	1/2

Q.NO	SUB Q N	ANSWER	Marking Scheme

Q1 (c) Find Intercept of line $2x+3y=6$ on both axes.

→ $2x+3y=6$

$2x+3y-6=0$

$ax+by+c=0$ – std eqn — 1/2

$a=2,\ b=3,\ c=-6$ — 1/2

x- intercept $= -\frac{c}{a} = -\frac{-6}{2} = 3$ — 1/2

y- intercept $= -\frac{c}{b} = -\frac{-6}{3} = 2$ — 1/2

Q1. (d) State whether function is even or odd $f(x) = x^3+4x+\sin x$

→ $f(x) = x^3+4x+\sin x$

$f(-x) = (-x)^3+4(-x)+\sin(-x)$ — 1/2

$f(-x) = -x^3-4x-\sin x$ — 1/2

$\therefore f(x) \neq f(-x)$ — 1/2

Give function is '<u>ODD</u>' — 1/2

1 (e) At which point on the curve $y=3x-x^2$ slope of tangent is -5?

→ $y=3x-x^2$

diff w.r to x,

$\frac{dy}{dx} = 3(1)-2x = 3-2x$ — 1/2

But slope $=-5$, $dy/dx=-5$ — 1/2

$-5 = 3-2x$

$2x = 3+5 \quad \therefore 2x=8 \quad \therefore x=4$ — 1/2

& $y = 3(4)-4^2 = 12-16 = -4$

$\therefore$ Point is $(4,-4)$ — 1/2

1 (f) Divide 100 into two parts such that their product is maximum.

→ Let one part be x,

Another part will be $(100-x)$

$\therefore$ Product $P = x \times (100-x)$

$P = 100x-x^2$ — 1/2

Diff w.r.t x $\frac{dP}{dx} = 100(1)-2x = 100-2x$ — 1/2

But at maxima, $dy/dx=0$ — 1/2

$\therefore 0=100-2x \quad 2x=100 \quad \therefore x=50$, Parts 50 & 50. — 1/2

Q.NO	SUB Q N	ANSWER	Marking Scheme

1 (g) If the mean is 34.5 and std. deviation is 5. Calculate coefficient of variance

→ coefficient of variance (C.V) $= \frac{\sigma}{\bar{x}} \times 100$ — 1

$$CV = \frac{5}{34.5} \times 100$$ — 1

$$\boxed{C.V = 14.492} \quad \boxed{14.492}$$

(2) Attempt any three of following **(12)**

2 (a) If $A = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}$

Find Matrix 'X' such that,
$2X + 3A - 4B = I$, wher I = identity matrix

→
$$2X + 3A - 4B = I$$
$$2X + 3\begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$2X + \begin{bmatrix} 9 & -3 \\ 6 & 12 \end{bmatrix} - \begin{bmatrix} 4 & 8 \\ -12 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$2X + \begin{bmatrix} 9-4 & -3-8 \\ 6+12 & 12-0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$2X + \begin{bmatrix} 5 & -11 \\ 18 & 12 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$2X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 5 & -11 \\ 18 & 12 \end{bmatrix}$$
$$2X = \begin{bmatrix} 1-5 & 0-(-11) \\ 0-18 & 1-12 \end{bmatrix}$$
$$2X = \begin{bmatrix} -4 & 11 \\ -18 & -11 \end{bmatrix}$$
$$\boxed{\therefore X = \frac{1}{2}\begin{bmatrix} -4 & 11 \\ -18 & -11 \end{bmatrix}}$$

2 (b) If $A = \begin{bmatrix} -2 & 0 & 2 \\ 3 & 4 & 5 \end{bmatrix}$ & $B = \begin{bmatrix} 2 & 1 \\ 3 & 5 \\ 0 & 2 \end{bmatrix}$

Whether AB is singular or non-singular?

→
$$AB = \begin{bmatrix} -2 & 0 & 2 \\ 3 & 4 & 5 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 5 \\ 0 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} (-2\times2)+(0\times3)+(2\times0) & (-2\times1)+(0\times5)+(2\times2) \\ (3\times2)+(4\times3)+(5\times0) & (3\times1)+(4\times5)+(5\times2) \end{bmatrix}$$

$$AB = \begin{bmatrix} -4 & 2 \\ 18 & 33 \end{bmatrix}$$

To find Determinant of AB

$$|AB| = \begin{vmatrix} -4 & 2 \\ 18 & 33 \end{vmatrix}$$

$$= [(-4 \times 33) - (18 \times 2)]$$

$$= -168$$

$\because |AB| \neq 0$ AB is non-singular Matrix

Resolve into partial Fractions

$$\frac{3x-2}{(x+2)(x^2+4)}$$

let,

$$\frac{3x-2}{(x+2)(x^2+4)} = \frac{A}{(x+2)} + \frac{Bx}{x^2+4} + \frac{c}{x^2+4}$$

OR

$$= \frac{A}{(x+2)} + \frac{(Bx+C)}{(x^2+4)}$$

$$\boxed{\therefore 3x-2 = A(x^2+4) + (Bx+C)(x+2)} \quad -(1)$$

Put $x+2=0 \therefore x=-2$ in eqn (1)

$$\therefore 3(-2)-2 = A[(-2)^2+4] + 0$$

$$-8 = A(8) \quad \boxed{\therefore A=-1}$$

Put $x=0$ & $A=-1$, in eqn (1)

$$\therefore 3(0)-2 = -1(0+4) + (0+c)(0+2)$$

$$-2 = -4 + 2c$$

$$-2+4 = 2c$$

$$2 = 2c \quad \boxed{\therefore c=1}$$

Put $x=1$ (or any value), $A=-1$ & $c=1$,

$$\therefore 3(1)-2 = -1(1^2+4) + (B(1)+1)(1+2)$$

$$1 = -5 + (B+1)\cdot 3$$

$$1+5 = 3(B+1)$$

$$6 = 3(B+1) \quad \therefore 2 = B+1 \quad \boxed{\therefore B=1}$$

$$\boxed{\therefore \frac{3x-2}{(x+2)(x^2+4)} = \frac{-1}{(x+2)} + \frac{(1x+1)}{x^2+4}} \text{ — Ans.}$$

Q.NO	SUB Q N	ANSWER	Marking Scheme

2 (d) If A and B are obtuse angles and $\sin A = 5/13$, & $\cos B = -4/5$ find $\sin(A+B)$.

→ $\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

• To find $\cos A$ & $\sin B$.

Given $\sin A = 5/13$

$$\sin^2 A + \cos^2 A = 1$$

$$\cos^2 A = 1 - \sin^2 A$$

$$\cos A = \pm\sqrt{1-(5/13)^2}$$

$$= \pm 12/13$$

But $\angle A$ is in second Quadrant, where cos is $-$ve. $\boxed{\therefore \cos A = -12/13}$

• Given $\cos B = -4/5$

$$\therefore \sin^2 B + \cos^2 B = 1$$

$$\sin^2 B = 1 - \cos^2 B$$

$$\sin B = \pm\sqrt{1-\cos^2 B}$$

$$= \pm\sqrt{1-(-4/5)^2}$$

$$= \pm 3/5$$

But $\angle B$ is obtuse, means second Quadrant where sin is positive, hence we choose +ve value,

$$\therefore \sin B = +3/5$$

$$\therefore \sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$$

$$= \left(\frac{5}{13}\right) \times \left(\frac{-4}{5}\right) + \left(\frac{-12}{13}\right)\left(\frac{3}{5}\right)$$

$$\boxed{\sin(A+B) = -56/65}$$

(3) Attempt any three (12)

3 (a) prove that $\frac{\sin 3A - \sin A}{\cos 3A + \cos A} = \tan A$

→ $$L.H.S = \frac{\sin 3A - \sin A}{\cos 3A + \cos A}$$

$$= \frac{2 \cdot \cos\left(\frac{3A+A}{2}\right) \cdot \sin\left(\frac{3A-A}{2}\right)}{2 \cdot \cos\left(\frac{3A+A}{2}\right) \cdot \cos\left(\frac{3A-A}{2}\right)}$$

Q.NO	SUB Q N	ANSWER	Marking Scheme

3 (a)

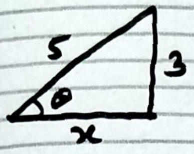
$$= \frac{2\cdot \cos\left(\frac{4A}{2}\right)\cdot \sin\left(\frac{2A}{2}\right)}{2\cdot \cos\left(\frac{4A}{2}\right)\cdot \cos\left(\frac{2A}{2}\right)}$$

$$= \frac{\cancel{2}\cdot \cancel{\cos 2A}\cdot \sin A}{\cancel{2}\cdot \cancel{\cos 2A}\cdot \cos A} = \frac{\sin A}{\cos A} = \tan A$$

$= RHS$ (proved)

3 (b) **Prove that, $\sin^{-1}\left(\frac{3}{5}\right) - \sin^{-1}\left(\frac{8}{17}\right) = \cos^{-1}\left(\frac{84}{85}\right)$**

→ • To convert $\sin^{-1}\left(\frac{3}{5}\right)$ into $\tan^{-1}$.



let $\sin^{-1}\left(\frac{3}{5}\right) = \theta$

$\therefore \sin\theta = \frac{3}{5}$

$\therefore 5^2 = x^2 + 3^2$

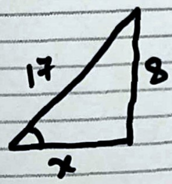
$\therefore 25 - 9 = x^2$

$16 = x^2$ $\boxed{\therefore x = 4}$

$\therefore \tan\theta = \frac{3}{4}$ $\therefore \theta = \tan^{-1}\left(\frac{3}{4}\right)$

$\therefore \sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{3}{4}\right)$

• To convert $\sin^{-1}(8/17)$ into $\tan^{-1}$.



let $\theta = \sin^{-1}(8/17)$ | $17^2 = x^2 + 8^2$

$\therefore \sin\theta = 8/17$ | $17^2 - 8^2 = x^2$

$225 = x^2$

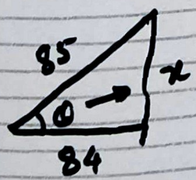
$x = 15$

$\therefore \tan\theta = 8/15$

$\therefore \theta = \tan^{-1}(8/15)$

$\therefore \sin^{-1}\left(\frac{8}{17}\right) = \tan^{-1}(8/15)$

• To convert $\cos^{-1}(84/85)$



let $\theta = \cos^{-1}(84/85)$ | $85^2 = x^2 + 84^2$

$\therefore \cos\theta = 84/85$ | $85^2 - 84^2 = x^2$

$169 = x^2$

$x = 13$

$\therefore \tan\theta = \frac{13}{84}$ $\therefore \theta = \tan^{-1}(13/84)$

$\therefore \cos^{-1}(84/85) = \tan^{-1}(13/84)$

$\therefore LHS = \sin^{-1}(3/5) - \sin^{-1}(8/17)$

$= \tan^{-1}(3/4) - \tan^{-1}(8/15)$

$= \tan^{-1}\left[\frac{3/4 - 8/15}{1 + (3/4 \cdot 8/15)}\right] = \tan^{-1}(13/84)$

$= RHS.$

Q.NO	SUB Q N	ANSWER	Marking Scheme

3. (c) Find the equation of straight line passing through the point of intersection of lines $4x+3y=8$ & $x+y=1$, and Parallel to the line $5x-7y=3$.

→ • Point of intersection of lines

$4x+3y=8$

$x+y=1 \quad \times -3$

$$\begin{array}{r} 4x+3y=8 \\ -3x-3y=-3 \\ \hline x \quad\quad =5 \end{array}$$

$\therefore x+y=1$

$5+y=1$

$y=1-5=-4$

$\therefore$ Point of intersection $(5,-4)$ 1

• slope of given line,

$5x-7y=3$

$5x-7y-3=0 \qquad \therefore m_1 = -\frac{a}{b} = \frac{-5}{-7} = \frac{5}{7}$

$ax+by+c=0$

$\therefore \quad m_2 = m_1$ (Parallel lines have same slope)

$\therefore m_2 = 5/7$ 1

• Eqn of required line,

Point $(5,-4)$ $\quad m_2 = 5/7$

Using slope-Point Form,

$(y-y_1) = m(x-x_1)$ 1

$(y-(-4)) = \frac{5}{7}(x-5)$

$7(y+4) = 5(x-5) \Rightarrow 7y+28 = 5x-25$

$-5x+7y+28+25=0 \Rightarrow -5x+7y+53=0$

$\boxed{\therefore 5x-7y-53=0}$ is eqn of req. line 1

3 (d) Find dy/dx if $x^3+xy^2 = y^3+yx^2$.

→ $x^3+xy^2 = y^3+yx^2$

Diff with respect to x,

$$3x^2+\left[x\cdot\frac{d}{dx}y^2+y^2\cdot\frac{d}{dx}x\right] = 3y^2\frac{dy}{dx} + \left[y\cdot\frac{d}{dx}x^2+x^2\cdot\frac{d}{dx}y\right]$$

Q.NO	SUB Q N	ANSWER	Marking Scheme

$$3x^2 + \left[x \cdot 2y \cdot \frac{dy}{dx} + y^2 \cdot (1)\right] = 3y^2 \cdot \frac{dy}{dx} + \left[y \cdot 2x + x^2 \cdot \frac{dy}{dx}\right]$$

$$3x^2 + 2xy \cdot \frac{dy}{dx} + y^2 = 3y^2 \cdot \frac{dy}{dx} + 2xy + x^2 \cdot \frac{dy}{dx}$$

$$2xy \cdot \frac{dy}{dx} - 3y^2 \cdot \frac{dy}{dx} - x^2 \cdot \frac{dy}{dx} = 2xy - 3x^2 - y^2$$

$$\frac{dy}{dx}\left(2xy - 3y^2 - x^2\right) = 2xy - 3x^2 - y^2$$

$$\frac{dy}{dx} = \frac{2xy - 3x^2 - y^2}{2xy - 3y^2 - x^2}$$

Q 4 Attempt any three (12)

4 (a) If $x = a(\theta + \sin\theta)$ & $y = a(1 - \cos\theta)$ find dy/dx at $\theta = \pi/2$

→

$x = a(\theta + \sin\theta)$	$y = a(1 - \cos\theta)$
diff w.r. to θ	diff w.r. to θ,
$\frac{dx}{d\theta} = a(1 + \cos\theta)$	$\frac{dy}{d\theta} = a(0 - (-\sin\theta))$
	$= a \cdot \sin\theta$

$$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\cancel{a} \cdot \sin\theta}{\cancel{a}(1 + \cos\theta)}$$

$$\therefore \frac{dy}{dx} = \frac{\sin\theta}{1 + \cos\theta}$$

$$\left.\frac{dy}{dx}\right|_{\theta = \frac{\pi}{2}} = \frac{\sin \pi/2}{1 + \cos \pi/2}$$

$$\boxed{\frac{dy}{dx} = 1}$$

Q.NO	SUB Q N	ANSWER	Marking Scheme

4 (b) If $y = x^{\sin x} + \tan x^{x}$ find dy/dx

→ $y = x^{\sin x} + \tan x^{x}$

let $u = x^{\sin x}$ &

Taking log of b.s.

$\log u = \log x^{\sin x}$

$\log u = \sin x \cdot \log x$

Diff w.r to x,

$\frac{1}{u} \cdot \frac{du}{dx} = \sin x \cdot \frac{1}{x} + \log x \cdot \cos x$

$\frac{du}{dx} = u\left(\frac{\sin x}{x} + \log x \cdot \cos x\right)$

$= x^{\sin x}\left(\frac{\sin x}{x} + \log x \cdot \cos x\right)$

$v = \tan x^{x}$

Taking log of b.s.

$\log v = \log \tan x^{x}$

$\log v = x \cdot \log(\tan x)$

Diff w.r to x

$\frac{1}{v} \cdot \frac{dv}{dx}$

$= x \cdot \frac{1}{\tan x} \cdot \sec^2 x$

$+ \log(\tan x) \cdot 1$

$\therefore \frac{dv}{dx} = v \cdot \left[\frac{x \cdot \sec^2 x}{\tan x} + \log(\tan x)\right]$

$\frac{dy}{dx} = \tan x^{x}\left[\frac{x \cdot \sec^2 x}{\tan x} + \log(\tan x)\right]$

$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$= x^{\sin x}\left[\frac{\sin x}{x} + \log x \cdot \cos x\right] +$

$\tan x^{x}\left[\frac{x \cdot \sec^2 x}{\tan x} + \log(\tan x)\right]$

4 (c) Find range & coefficient of range

class	10-19	20-29	30-39	40-49	50-59
freq	15	25	13	17	10

→ make class continuous,

class	~~Exact~~ class	freq
10-19	9.5-19.5	15
20-29	19.5-29.5	25
30-39	29.5-39.5	13
40-49	39.5-49.5	17
50-59	49.5-59.5	10

$H = 59.5$

$L = 9.5$

Range $= H - L$

$= 59.5 - 9.5$

$= 50$

$C.R = \frac{H-L}{H+L}$

$= \frac{50}{69} = 0.847$

Q.NO	SUB Q N	ANSWER	Marking Scheme

4 (d) Calculate mean deviation about mean of following data.

17, 15, 18, 23, 25, 22, 11, 5.

→ step I) To calculate mean

$$\text{Mean}(\bar{x}) = \frac{\text{Sum of all obsesvation}}{\text{No. of. observation}}$$

$$= \frac{17+15+18+23+25+22+11+5}{8}$$

$$\boxed{\bar{x} = 17}$$

step II) To find deviations from mean

x_i	$D_i = \lvert\bar{x} - x_i\rvert$
17	$= \lvert 17-17\rvert = 0$
15	$= \lvert 17-15\rvert = 2$
18	$= \lvert 17-18\rvert = 1$
23	$= \lvert 17-23\rvert = 6$
25	$= \lvert 17-25\rvert = 8$
22	$= \lvert 17-22\rvert = 5$
11	$= \lvert 17-11\rvert = 6$
5	$= \lvert 17-5\rvert = 12$

$\Sigma d_i = 40$

$$M.D = \frac{\Sigma d_i}{N} = \frac{40}{8}$$

$$\boxed{MD = 5}$$

Q4 (e) The following data pertains to two workers doing same Job in factory,

Details	Worker A	Worker B
Mean time	40	42
Std. deviation	8	6

which worker is more consistent?

→ coefficient of variance,

$$(CV)_A = \frac{\sigma}{\bar{x}} \times 100 = \frac{8}{40} \times 100 = 20\%.$$

$$(C.V.)_B = \frac{\sigma}{\bar{x}} \times 100 = \frac{6}{42} \times 100 = 14.28\%.$$

$\because (CV_B) < (CV)_A$

Worker B is more consistent.

Q.NO	SUB Q N	ANSWER	Marking Scheme

Q5 Attempt any TWO of following

5 (a) solve following system of eqn by matrix inversion method.

$x+y+z=3$, $3x-2y+3z=4$, $5x+5y+z=11$

• Write in Matrix Form,

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & -2 & 3 \\ 5 & 5 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 11 \end{bmatrix}$$

• To find A^{-1}

$$|A| = 1(-2-15)-1(3-15)+1(15+10)$$

$$= -17+12+25 = 20 \quad A^{-1} \text{ exists.}$$

Matrix of cofactors (P)

$$P = \begin{bmatrix} +\begin{vmatrix} -2 & 3 \\ 5 & 1 \end{vmatrix} & -\begin{vmatrix} 3 & 3 \\ 5 & 1 \end{vmatrix} & +\begin{vmatrix} 3 & -2 \\ 5 & 5 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ 5 & 1 \end{vmatrix} & +\begin{vmatrix} 1 & 1 \\ 5 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 5 & 5 \end{vmatrix} \\ +\begin{vmatrix} 1 & 1 \\ -2 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 3 & 3 \end{vmatrix} & +\begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix} \end{bmatrix}$$

$$P = \begin{bmatrix} -17 & 12 & 25 \\ 4 & -4 & 0 \\ 5 & 0 & -5 \end{bmatrix}$$

$$\text{Adj } A = P^T = \begin{bmatrix} -17 & 4 & 5 \\ 12 & -4 & 0 \\ 25 & 0 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{Adj} A = \frac{1}{20} \begin{bmatrix} -17 & 4 & 5 \\ 12 & -4 & 0 \\ 25 & 0 & -5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1} \cdot D = \frac{1}{20} \begin{bmatrix} -17 & 4 & 5 \\ 12 & -4 & 0 \\ 25 & 0 & -5 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 11 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$x=1$, $y=1$ & $z=1$