17301

13141 3 Hours / 100 Marks Seat No.

Instructions - (1) All Questions are Compulsory.

- (2) Illustrate your answers with neat sketches wherever necessary.
- (3) Figures to the right indicate full marks.
- (4) Assume suitable data, if necessary.
- (5) Use of Non-programmable Electronic Pocket Calculator is permissible.
- (6) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

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1. Attempt any <u>TEN</u> of the following:

- a) Find the point on the curve $y = 3x x^2$ at which slope is -5.
- b) Find the radius of curvature of the curve

$$y = \log(\sin x)$$
 at $x = \frac{\pi}{2}$

c) Integrate w.r.to x of $\sqrt{1 + \cos 2x}$

d) Evaluate
$$\int \frac{\cos(\log x) \, dx}{x}$$

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e) Evaluate
$$\int \frac{dx}{x(x+1)}$$

f) Evaluate
$$\int \tan^{-1} x \, dx$$

g) Evaluate
$$\int_{1}^{2} \frac{dx}{3x-2}$$

- h) Find the area contained by the curve $y=1+x^3+2\sin x$ from x=0 to $x=\pi$.
- i) Find the order and degree of the

D.E.
$$\frac{d^2 y}{dx^2} = \sqrt{y - \frac{dy}{dx}}$$

- j) Form a differential equation if $y = A \sin x + B \cos x$.
- k) From a pack of 52 cards one is drawn at random. Find the probability of getting a king.
- An unbiased coin is tossed 5 times. Find the probability of getting 3 heads.

2. Attempt any <u>FOUR</u> of the following:

- a) Find the equation of the tangent and normal to the curve $13x^3 + 2x^2y + y^3 = 1$ at (1, -2)
- b) A beam is bent in the form of the curve $y = 2 \sin x \sin 2x$. Find the radius of curvature of the beam at this point at

$$x=\frac{\pi}{2}$$
.

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c) Find the maximum and minimum values of $x^3 - 18x^2 + 96x$.

d) Evaluate
$$\int \frac{1 - \tan x}{1 + \tan x} dx$$

e) Evaluate
$$\int \sin^3 x \, dx$$

f) Evaluate
$$\int \frac{x+1}{x(x^2-4)} dx$$

3. Attempt any <u>FOUR</u> of the following:

a) Evaluate
$$\int_{0}^{4} \frac{dx}{\sqrt{4x - x^2}}$$

b) Evaluate
$$\int_{0}^{5} \frac{\sqrt{9-x} \, dx}{\sqrt{9-x} + \sqrt{x+4}}$$

c) Find the area of region included between the parabola
$$y = x^2 + 1$$
 and the line $y = 2x + 1$.

d) Solve the D.E. $x(1+y^2) dx + y(1+x^2) dy = 0$

e) Solve the D.E.
$$(4x + y)^2 \frac{dy}{dx} = 1$$

f) Solve the D.E. $x(x+y) dy - y^2 dx = 0$

4. Attempt any <u>FOUR</u> of the following:

a) Evaluate
$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1 + \sqrt{\cot x}} dx$$

b) Evaluate
$$\int_{0}^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$$

- c) Find the area of the circle $x^2 + y^2 = 16$ by integration.
- d) Solve the D.E. $\frac{dy}{dx} + y \tan x = \cos^2 x$
- e) Solve the D.E. $(e^x + 2xy^2 + y^3) dx + (a^y + 2x^2y + 3xy^2) dy = 0$

f) Verify that
$$y = \log x$$
 is solution of $\frac{x d^2 y}{dx^2} + \frac{dy}{dx} = 0$.

5. Attempt any <u>FOUR</u> of the following:

- a) A room has 3 electric lamps. From a collection of 15 electric bulbs of which only 10 are good, 3 are selected at random and put in the lamps. Find the probability that the room is lighted by at least one of the bulbs.
- b) If 20% of the bolts produce by a machine are defective. Find the probability that out of 4 bolts drawn
 - i) One is defective
 - ii) At most two are defective.

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c) A box contains 10 red, 5 white, 5 black balls. Two balls are drawn at random. Find the probability that they are not of the same colour.

d) Evaluate
$$\int \frac{dx}{4-5\cos x}$$

e) Evaluate
$$\int_{0}^{\frac{\pi}{4}} \log(1 + \tan x) \, dx$$

f) Solve the D.E.
$$\frac{dy}{dx} + \frac{y}{x} = y^2$$

6. Attempt any <u>FOUR</u> of the following:

a) If
$$P(A) = \frac{1}{5}$$
, $P(B') = \frac{3}{5}$ and $P\left(\frac{A}{B}\right) = \frac{3}{4}$

Find $P(A \cap B)$ and $P\left(\frac{B}{A}\right)$

b) Using Poisson distribution, find the probability that the ace of spades will be drawn from a pack of well shuffled cards atleast one in 104 consecutive trials.

- c) In a sample of 1000 cases, the mean of certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal find
 - i) How many students score between 12 and 15 ?
 - ii) How many students score above 18 ?

Given A (0.8) = 0.2881A (0.4) = 0.1554

A
$$(1.6) = 0.4452$$

- d) A manufacturer can sell x items at price of Rs. (330 x)each. The cost of producing x items in Rs. is $x^2 + 10x + 12$. How many items must be sold so that his profit is maximum.
- e) Find the equation of the tangents to the curve $y = x^2 2x 3$, where it cuts X-axis.
- f) Find the area of the region lying between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

3 Hours / 100 Marks