12425

WINTER- 2024 EXAMINATION

Model Answer

Subject Name: Maths II

Subject Code 312301

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Q.NO	SUB Q N	ANSWER	Marking Scheme
1.		Solve any five.	10 Marks
	a)	Evaluate $\int\left(\frac{1}{1+x^2}+\cos x\right)dx$	
		$I=\int \frac{1}{1+x^2}+\cos x\cdot dx$	
		we know, $\int \frac{dx}{1+x^2}=\tan^{-1}(x)+C$	
		& $\int \cos x\cdot dx=\sin x+C$	
		$\therefore \int \frac{1}{1+x^2}+\cos x\,dx$	
		$=\tan^{-1}x+\sin x+C.$	
		$\boxed{\int \frac{1}{1+x^2}+\cos x\,dx=\tan^{-1}x+\sin x+C.}$	

Q.NO	SUB Q N	ANSWER	Marking Scheme

b] Evaluate $\int \sqrt{1+\cos 2x}\, dx$

→ $\because 1+\cos\theta = 2\cos^2(\theta/2)$

Using relation,

$1+\cos 2x = 2\cos^2 x.$

$I = \int \sqrt{1+\cos 2x}\, dx$

$= \int \sqrt{2\cos^2 x}\, dx$

$= \sqrt{2} \int \cos x \, dx$

$= \sqrt{2} \cdot \sin x + C.$

$\therefore \boxed{\int \sqrt{1+\cos 2x} \cdot dx = \sqrt{2} \cdot \sin x + C.}$

c] Evaluate $\int_0^4 (4x - x^2) \cdot dx$

→ $I = \int_0^4 4x - x^2 \cdot dx$

$= \left[4 \cdot \frac{x^2}{2} - \frac{x^3}{3}\right]_0^4$

$= \left[2x^2 - \frac{x^3}{3}\right]_0^4$

$= \left[2(4)^2 - \frac{4^3}{3}\right] - [0] = \frac{32}{3}$

$\therefore \boxed{\int_0^4 (4x - x^2)\, dx = \frac{32}{3}}$

d] Find order & degree of diff. eq^n: $\frac{d^2y}{dx^2} = \sqrt{y - \frac{dy}{dx}}$

→ $\frac{d^2y}{dx^2} = \sqrt{y - \frac{dy}{dx}}$; squaring both sides;

$\left(\frac{d^2y}{dx^2}\right)^2 = y - \frac{dy}{dx}$

$\because$ Higher order derivative is "2" & it's power is "2".

$\therefore$ Order $= 2$

Q.NO	SUB QN	ANSWER	Marking Scheme

e] Show that, root of eq^n. $x^3-2x-5=0$, lies between 2 & 3.

→ Given eq^n. $x^3-2x-5=0$

$$f(x)=x^3-2x-5$$

Check:- $f(2)=2^3-2(2)-5$

$$f(2)=8-4-5$$

$$\boxed{f(2)=-1.}$$

& $f(3)=x^3-2x-5$

$$f(3)=3^3-2(3)-5$$

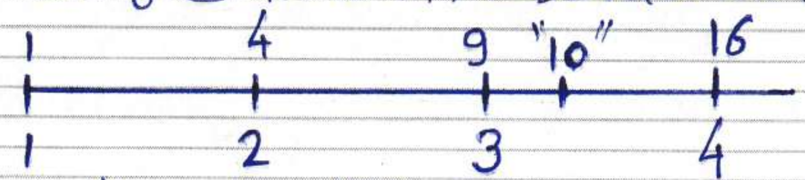
$$f(3)=27-6-5$$

$$\boxed{f(3)=16.}$$

$\because$ F(2) is −ve & f(3) is +ve,
∴ Root lies between 2 & 3.

f] Find approximate square root of number "10" using Bakhshali Iterative Method.

→



Nearest root is "9", whose sq. root is "3".

∴ $S=10$ & $N=3$.

∴ $d=(S-N^2)=(10-3^2)=1$.

$P=\frac{d}{2\times N}=\frac{1}{2\times 3}$ $\quad\boxed{P=\frac{1}{6}.}$

$A=N+P=3+\frac{1}{6}$ $\quad\boxed{A=\frac{19}{6}.}$

$$\sqrt{S}\cong\left[A-\frac{P^2}{2A}\right]$$

$$\sqrt{S}\cong\left[\frac{19}{6}-\frac{(1/6)^2}{2\times\frac{19}{6}}\right]\cong 3.1622$$

$$\therefore \boxed{\sqrt{10}\cong 3.1622.}$$

Q.NO	SUB Q N	ANSWER	Marking Scheme

g. A fair coin is tossed 8 times. Find probability of getting exactly "2" heads?

→ $n = 8, \quad p = \frac{1}{2}, \quad q = \frac{1}{2};$

To find $p(2)$ using Binomial distribution,

$$P(r) = nC_r \, p^r \cdot q^{n-r}$$

$$\therefore P(2) = 8C_2 \left(\tfrac{1}{2}\right)^2 \left(\tfrac{1}{2}\right)^{(8-2)}$$

$$\therefore \boxed{P(2) = \frac{7}{62}.}$$

2. Solve any three — 12 Marks

a] Evaluate $\int \frac{1}{\sqrt{1-x^2}\,(\sin^{-1}x)^2}\,dx$

→ Put $(\sin^{-1}x) = t$. diff. w.r.t. x, we get,

$$\frac{1}{\sqrt{1-x^2}} = \frac{dt}{dx} \qquad \therefore dx = \sqrt{1-x^2}\cdot dt.$$

$$I = \int \frac{1}{(\cancel{\sqrt{1-x^2}})\cdot t^2}\cdot(\cancel{\sqrt{1-x^2}})\,dt. = \int \frac{dt}{t^2}$$

$$I = \int t^{-2}\,dt$$

$$I = \frac{t^{(-2+1)}}{(-2+1)} + C$$

$$I = \frac{t^{(-1)}}{(-1)} + C$$

$$I = \frac{-1}{t} + C$$

$$I = \frac{-1}{\sin^{-1}x} + C$$

$$\therefore \boxed{\int \frac{1}{\sqrt{1-x^2}\,(\sin^{-1}x)^2}\,dx = \frac{-1}{(\sin^{-1}x)} + C.}$$

Q.NO	SUB Q N	ANSWER	Marking Scheme
	b)	Evaluate $\int \frac{\cos x}{(\sin x+1)(\sin x+2)} dx$	

→ Put, $\sin x = t$ (diff. w.r.t. x; we get)

$\cos x = \frac{dt}{dx} \quad \therefore dx = \frac{dt}{\cos x}$.

Now; $I = \int \frac{\cos x}{(t+1)(t+2)} \times \frac{dt}{\cos x}$

$I = \int \frac{1}{(t+1)(t+2)} dt.$

Let's resolve into Partial fraction,

$\frac{1}{(t+1)(t+2)} = \frac{A}{(t+1)} + \frac{B}{(t+2)}$

Multipling both sides by $[(t+1)(t+2)]$,

$\boxed{1 = A(t+2) + B(t+1).}$

Put $t+1=0$

$\therefore t = -1.$

$\therefore 1 = A(-1+2) + B(-1+1)$

$\therefore 1 = A(1) + B(0)$

$\therefore 1 = A. \qquad \therefore \boxed{A = 1.}$

Put, $t+2=0$

$\therefore t = -2.$

$\therefore 1 = A(-2+2) + B(-2+1)$

$\therefore 1 = 0 + (-1)B$

$\therefore -1 = B \qquad \therefore \boxed{B = -1.}$

Now,

$I = \int \frac{1}{(t+1)} + \frac{(-1)}{(t+2)} dt$

$= \log(t+1) - \log(t+2) + C$

$= \log\left[\frac{t+1}{t+2}\right] + C$

$\therefore \boxed{\int \frac{\cos x}{(\sin x+1)(\sin x+2)} dx = \log\left[\frac{\sin x+1}{\sin x+2}\right] + C}$

c] Evaluate $\int e^{x} \cdot \sin x \, dx$.

→ Check LIATE Rule; $\underset{\text{Exp.}}{\overset{\downarrow}{e^{x}}}$ $\underset{\text{Trig.}}{\overset{\downarrow}{\sin x}}$

∴ Change Sequence;

$I = \int \sin x \cdot e^{x} \, dx$; Using Int^{n}. by parts;

$I = \sin x \int e^{x} dx - \int \left[\int e^{x} dx \cdot \frac{d}{dx} \sin x\right] dx$

$I = \sin x \cdot e^{x} - \int \cos x \cdot e^{x} \, dx + C$

Again using Integration by parts;

$I = \sin x \cdot e^{x} - \left[\cos x \int e^{x} dx - \int \left[\int e^{x} dx \cdot \frac{d}{dx} \cos x\right] dx\right] + C$

$I = \sin x \cdot e^{x} - \left[\cos x \cdot e^{x} - \int e^{x} (-\sin x) \, dx\right] + C$

$I = \sin x \, e^{x} - \cos x \cdot e^{x} - \underline{\int e^{x} \cdot \sin x \, dx} + C$

$I = \sin x \cdot e^{x} - \cos x \cdot e^{x} - I + C$

$I + I = e^{x} (\sin x - \cos x) + C$

$2I = e^{x} (\sin x - \cos x) + C$

$$\boxed{I = \frac{1}{2}\left[e^{x} (\sin x - \cos x)\right] + C.}$$

d] Solve $\int \frac{1}{\sqrt{16 - 6x - x^{2}}} \cdot dx$

→ $I = \int \frac{1}{\sqrt{-(x^{2} + 6x - 16)}} \, dx$

Third term $= \left[\frac{1}{2x} \times 6x\right]^{2} = 9$

$I = \int \frac{1}{\sqrt{-(x^{2} + 6x + 9 - 16 - 9)}} \, dx$

$I = \int \frac{1}{\sqrt{(5)^{2} - (x+3)^{2}}} \, dx$

$$\therefore \boxed{\int \frac{1}{\sqrt{16 - 6x - x^{2}}} \cdot dx = \sin^{-1}\left(\frac{x+3}{5}\right) + C}$$

Q.NO	SUB Q N	ANSWER	Marking Scheme
3.		Solve any three	12 Marks
	a]	$\int_0^{\pi/2} \frac{dx}{5+4\cos x}$	

→ Put, $\tan\left(\frac{x}{2}\right) = t \therefore \cos x = \frac{(1-t^2)}{(1+t^2)}$ & $dx = \frac{2 \cdot dt}{(1+t^2)}$

∴ limit change;

x	0	$\pi/2$
t	0	∞

∴ Integral becomes;

$$I = \int_0^{\infty} \frac{2dt/(1+t^2)}{5+4\left(\frac{1-t^2}{1+t^2}\right)} dt.$$

$$I = \int_0^{\infty} \frac{2dt/(1+t^2)}{\frac{5}{1} + \frac{4-4t^2}{(1+t^2)}} dt$$

$$I = \int_0^{\infty} \frac{2dt/\cancel{(1+t^2)}}{\frac{5+5t^2+4-4t^2}{\cancel{(1+t^2)}}} dt$$

$$I = 2\int_0^{\infty} \frac{dt}{t^2+9}$$

$$I = 2\int_0^{\infty} \frac{dt}{(t)^2+(3)^2}$$

$$I = 2\left[\frac{1}{3} \cdot \tan^{-1}\left(\frac{t}{3}\right)\right]_0^{\infty}$$

$$I = \frac{2}{3}\left[\tan^{-1}(\infty) - \tan^{-1}(0)\right]$$

$$I = \frac{2}{3}\left[\frac{\pi}{2} - 0\right]$$

$$\therefore \boxed{\int_0^{\pi/2} \frac{dx}{5+4\cos x} = \frac{\pi}{3}}$$

Q.NO	SUB Q N	ANSWER	Marking Scheme

b) $$\int_0^4 \frac{\sqrt[3]{x+5}}{\sqrt{x+5}+\sqrt[3]{9-x}}\,dx$$

→ using Property, $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$$I = \int_0^4 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\,dx \quad \text{— eq}^n \text{ ①}$$

$$I = \int_0^4 \frac{\sqrt[3]{(0+4-x)+5}}{\sqrt[3]{(0+4-x)+5}+\sqrt[3]{9-(0+4-x)}}\,dx$$

$$I = \int_0^4 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+5}}\,dx \quad \text{— eq}^n \text{ ②}$$

Adding eq^n. ① & ②

$$I+I = \int_0^4 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\,dx + \int_0^4 \frac{\sqrt[3]{9-x}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\,dx$$

$$2I = \int_0^4 \frac{\cancel{(\sqrt[3]{x+5}+\sqrt[3]{9-x})}}{\cancel{(\sqrt[3]{x+5}+\sqrt[3]{9-x})}}\,dx$$

$$2I = \int_0^4 1\,dx$$

$$2I = [x]_0^4$$

$$2I = [4-0]$$

$$I = \frac{4}{2} = 2$$

$$\therefore \boxed{\int_0^4 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\,dx = 2.}$$

Q.NO	SUB Q N	ANSWER	Marking Scheme

c] Solve $(2xy+y^2)dx + (x^2+2xy+\sin y)dy = 0$

→ Check eq^n. for EXACT;

$M = 2xy + y^2$ & $N = x^2 + 2xy + \sin y$

$\frac{\partial M}{\partial y} = 2x(1) + 2y$ | $\frac{\partial N}{\partial x} = 2x + 2y(1) + 0$

$\frac{\partial M}{\partial y} = 2x + 2y$ | $\frac{\partial N}{\partial x} = 2x + 2y$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Given eq^n. is exact.

sol^n:- $\int M dx + \int N dy = C$

$\downarrow$ (terms free from x)

$\int 2xy + y^2 dx + \int \sin y \, dy = C$

$2(y)\frac{(x^2)}{2} + y^2 x + (-\cos y) = C$

$\therefore \boxed{x^2 y + xy^2 - \cos y = C.}$

d] Using Bisection method find $x^3 - x - 1 = 0$

→ step(I) To find root lies between,

$f(x) = x^3 - x - 1$

$f(0) = 0 - 0 - 1$

$f(0) = -1.$

$f(1) = 1 - 1 - 1$

$f(1) = -1.$

$f(2) = 2^3 - 2 - 1$

$f(2) = 5.$

Root lies between 1 & 2

Step II

(P.T.O.

Q.NO	SUB Q.N	ANSWER	Marking Scheme

Step II Interations.

-VE (a)	+VE (b)	$x = \frac{a+b}{2}$	$f(x) = x^3 - x - 1$	sign
$a=1$	$b=2$	$x = \frac{1+2}{2}$ $x = 1.5$	$= (1.5)^3 - (1.5) - 1$ $= 0.875$	+VE
$a=1$	$b=1.5$	$x = \frac{1+1.5}{2}$ $x = 1.25$	$= (1.25)^3 - 1.25 - 1$ $= -0.296$	-VE
$a=1.25$	$b=1.5$	$x = \frac{1.25+1.5}{2}$ $x = 1.375$		

Approximate root of eq^n. is $x = 1.375$.

4 Solve any three — 12 Marks

a] Find root of $x^3 + 2x^2 - 8 = 0$ using Regula falsi method.

→ Step I] To find root lies between,

$f(x) = x^3 + 2x^2 - 8$

$f(0) = 0 + 0 - 8$

$f(0) = -8.$

$f(1) = 1^3 + 2(1)^2 - 8$

$f(1) = -5.$

$f(2) = 2^3 + 2(2)^2 - 8$

$f(2) = 8.$

∴ Root lies between 1 & 2.

(P.T.O

Q.NO	SUB Q N	ANSWER	Marking Scheme

Step II Interations

-VE (a)	+VE (b)	$x=\dfrac{a\cdot f(b)-b\cdot f(a)}{f(b)-F(a)}$	$f(x)=x^3+2x-8$	Sign
$a=1$, $f(a)=-5$	$b=2$, $f(b)=8$	$x=\dfrac{1(8)-2(-5)}{8-(-5)}$, $x=1\cdot384$	$=(1\cdot384)^3+2(1\cdot384)-8$ $=-1\cdot518$	-VE
$a=1\cdot384$, $f(a)=-1\cdot518$	$b=2$, $f(b)=8$	$x=\dfrac{(1\cdot384\times8)-(2\times-1\cdot518)}{8-(-1\cdot518)}$, $x=1\cdot482$	$=(1\cdot482)^3+2(1\cdot482)-8$ $=-0\cdot352$	-VE
$a=1\cdot482$, $f(a)=-0\cdot352$	$b=2$, $f(b)=8$	$x=\dfrac{(1\cdot482\times8)-(2\times-0\cdot352)}{8-(-0\cdot352)}$, $x=1\cdot503$		

Approximate root of eq^n is $x=1\cdot503$

b] Using Newton-Raphson Method find root of eq^n. $x^4-x-9=0$. Perform 3 Iterations.

→ Step I: To find root lies between

$f(x)=x^4-x-9 \quad \therefore f'(x)=4x^3-1.$

$f(0)=(0)-(0)-9$

$f(0)=-9$

$f(1)=1^4-1-9$

$f(1)=-9$

$f(2)=2^4-2-9$

$f(2)=5$

Q.NO	SUB Q N	ANSWER	Marking Scheme

let, initial root $x_0 = 2$

Iteration I) $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$$x_1 = 2 - \frac{2^4 - 2 - 9}{4(2)^3 - 1}$$

$$x_1 = 2 - \frac{5}{31} \quad \boxed{x_1 = 1.838}$$

Iteration II)

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 1.838 - \frac{1.838^4 - 1.838 - 9}{4(1.838)^3 - 1}$$

$$\boxed{x_2 = 1.813}$$

Iteration III)

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 1.813 - \frac{(1.813)^4 - 1.813 - 9}{4(1.813)^3 - 1}$$

$$\boxed{x_3 = 1.813}$$

c] Solve by Gauss-Seidal method,

$$5x - 2y + 3z = 18$$
$$x + 7y - 3z = 22$$
$$2x - y + 6z = 22.$$

→ Step I) Formation of equation,

from eq^n ① $x = \frac{1}{5}(18 + 2y - 3z)$

from eq^n ② $y = \frac{1}{7}(22 - x + 3z)$

from eq^n ③ $z = \frac{1}{6}(22 - 2x + y)$

Q.NO	SUB Q N	ANSWER	Marking Scheme

Step II) Iterations :-

Iteration I using $y=0$ & $z=0$

$x=\frac{1}{5}(18+0-0)=3.600$ $\boxed{x=3.600}$

using $x=3.6$ & $z=0$

$y=\frac{1}{7}[22-3.6+3(0)]$ $\boxed{y=2.628}$

using $x=3.6$ & $y=2.628$,

$z=\frac{1}{6}[22-2(3.6)+2.628]$ $\boxed{z=2.904.}$

Iteration II using $y=2.628$, $z=2.904$

$x=\frac{1}{5}[18+2(2.628)-3(2.904)]$ $\boxed{x=2.909}$

using $x=2.909$, $z=2.904$

$y=\frac{1}{7}[22-2.909+3(2.904)]$ $\boxed{y=3.972}$

using $x=2.909$, $y=3.972$

$z=\frac{1}{6}[22-2(2.909)+3.972]$ $\boxed{z=3.359}$

Iteration III using $y=3.972$, $z=3.359$

$x=\frac{1}{5}[18+2(3.972)-3(3.359)]$ $\boxed{x=3.1734}$

using $x=3.1734$, $z=3.359$,

$y=\frac{1}{7}[22-3.1734+3(3.359)]$ $\boxed{y=4.129}$

using $y=4.129$, $x=3.1734$,

$z=\frac{1}{6}[22-2(3.1734)+4.129]$ $\boxed{z=3.297.}$

Q.NO	SUB Q N	ANSWER	Marking Scheme

d] If 20% of bolts produced by a machine are defective, determine probability that out of 4 bolts drawn -
(i) One is defective
(ii) at the most two are defective.

→ Probability of defective bolts $P = \frac{20}{100} = 0.2$

$q = 1 - p \therefore q = 1 - 0.2 \quad q = 0.8$

$n = 4: \quad P(r) = {}^nC_r \cdot p^r \cdot q^{n-r}$

(i) One defective, $n = 4, r = 1$.

$$P(1) = {}^4C_1 \cdot (0.2)^1 \cdot (0.8)^{4-1}$$

$$\underline{P(1) = 0.4096.}$$

(ii) Probability of getting 2 defective in 4.

$$= P(0) + P(1) + P(2)$$

$$= {}^4C_0 (0.2)^0 (0.8)^{4-0} + {}^4C_1 (0.2)^1 (0.8)^{4-1}$$

$$+ {}^4C_2 (0.2)^2 (0.8)^{4-2}$$

$$= \underline{0.9728.}$$

e] If probability that an electric motor is defective is 0.01. What is probability that sample of 300 electric motors will contain exactly 5 defective motors?

→ $P = 0.01, \; n = 300, \; m = 0.01 \times 300 = 3$

$$P(\text{Exactly } 5) = P(5) = \frac{e^{-m} \cdot m^r}{r!}$$

$$P(5) = \frac{e^{-3} (3)^5}{5!}$$

$$\boxed{P(5) = 0.10082.}$$

5. Solve any two [2 Marks]

a) (i) Evaluate $\int \frac{dx}{3-2\sin^2 x}$

→ Divide Numerator & Denominator $\cos^2 x$

$$\therefore I = \int \frac{dx/\cos^2 x}{\frac{3-2\sin^2 x}{\cos^2 x}}$$

$$I = \int \frac{dx/\cos^2 x}{\frac{3}{\cos^2 x} - \frac{2\sin^2 x}{\cos^2 x}} = \int \frac{\sec^2 x \, dx}{3\sec^2 x - 2\tan^2 x}$$

$$I = \int \frac{\sec^2 x \, dx}{3(1+\tan^2 x) - 2\tan^2 x} = \int \frac{\sec^2 x \, dx}{3+\tan^2 x}$$

Put, $\tan x = t$. Diff. w.r.t. x,

$$\sec^2 x \, dx = dt$$

$$\therefore I = \int \frac{dt}{(\sqrt{3})^2 + t^2} = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{t}{\sqrt{3}}\right) + C$$

$$\boxed{\therefore \int \frac{dx}{3-2\sin^2 x} = \frac{1}{\sqrt{3}} \cdot \tan^{-1}\left(\frac{\tan x}{\sqrt{3}}\right) + C}$$

(ii) Evaluate $\int \frac{1-\tan x}{1+\tan x} dx$

$$\rightarrow I = \int \frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}} dx = \int \frac{\frac{\cos x - \sin x}{\cos x}}{\frac{\cos x + \sin x}{\cos x}} dx$$

$$I = \int \frac{\cos x - \sin x}{\cos x + \sin x} dx = \int f(x) = \cos x + \sin x$$

$$f'(x) = -\sin x + \cos x = \cos x - \sin x$$

$$\int \frac{f'(x)}{f(x)} dx = \log(f(x)) + C$$

$$\boxed{I = \log(\cos x + \sin x) + C.}$$

$$\int_0^1 \frac{dx}{x^2-x+1}$$

→ Third term $= \left[\frac{1}{2x} \times (-x)\right]^2 = \frac{1}{4}$

$$I = \int_0^1 \frac{dx}{x^2 - x + \frac{1}{4} + 1 - \frac{1}{4}}$$

$$I = \int_0^1 \frac{dx}{(x-\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}$$

$$I = \left[\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{x-\frac{1}{2}}{\sqrt{3}/2}\right)\right]_0^1$$

$$I = \frac{2}{\sqrt{3}}\left[\tan^{-1}\left(\frac{2(1-1/2)}{\sqrt{3}}\right) - \tan^{-1}\left(\frac{2(0-\frac{1}{2})}{\sqrt{3}}\right)\right]$$

$$I = \frac{2}{\sqrt{3}}\left\{\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) - \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)\right\}$$

$$I = \frac{2}{\sqrt{3}}\left(\frac{\pi}{6} + \frac{\pi}{6}\right) = \frac{4\pi}{6\sqrt{3}} = \frac{2\pi}{3\sqrt{3}}$$

$$\boxed{\int_0^1 \frac{dx}{x^2-x+1} = \frac{2\pi}{3\sqrt{3}}.}$$

(ii) Evaluate $\int_0^{\pi/2} \sin^3 x \cdot \cos x \cdot dx$

→ Put, $\sin x = t$; diff. w.r.t. x. $\cos x\, dx = dt$

$$I = \int_0^{\pi/2} t^3\, dt = \int_0^{\pi/2} t^3 \cdot \cancel{\cos x} \cdot \frac{dt}{\cancel{\cos x}}$$

Limit Change,

x	0	$\pi/2$
t	$\sin 0 = 0$	$\sin(\pi/2) = 1.$

$$I \int_0^1 t^3\, dt = \left[\frac{t^4}{4}\right]_0^1 = \frac{1}{4}$$

$$\boxed{\int_0^{\pi/2} \sin^3 x \cdot \cos x\, dx = \frac{1}{4}}$$

Q.NO	SUB Q N	ANSWER	Marking Scheme

5 c (i) Evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$

→ Let, $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ — ①

using property,

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin[0+\frac{\pi}{2}-x]}}{\sqrt{\sin[0+\frac{\pi}{2}-x]}+\sqrt{\cos[0+\frac{\pi}{2}-x]}}\,dx$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx \quad — ②$$

Adding eq^n ① & ②

$$I + I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx + \int_0^{\pi/2} \frac{\sqrt{\cos x}\,dx}{\sqrt{\cos x}+\sqrt{\sin x}}$$

$$2I = \int_0^{\pi/2} \frac{\cancel{(\sqrt{\sin x}+\sqrt{\cos x})}}{\cancel{(\sqrt{\sin x}+\sqrt{\cos x})}}\,dx$$

$$2I = \int_0^{\pi/2} 1\,dx$$

$$2I = [x]_0^{\pi/2}$$

$$2I = \frac{\pi}{2} - 0$$

$$I = \frac{\pi}{4}$$

$$\therefore \boxed{\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx = \frac{\pi}{4}}$$

Q.NO	SUB Q N	ANSWER	Marking Scheme

C (ii) Evaluate $\int_2^5 \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\,dx$

→ Let, $I = \int_2^5 \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\,dx$ — (1)

using property,

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$I = \int_2^5 \frac{\sqrt{2+5-x}}{\sqrt{7-(2+5-x)}+\sqrt{2+5-x}}\,dx$$

$$\therefore I = \int_2^5 \frac{\sqrt{7-x}}{\sqrt{x}+\sqrt{7-x}}\,dx \quad \text{— (2)}$$

Adding eq^n (1) & (2)

$$I + I = \int_2^5 \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\,dx + \int_2^5 \frac{\sqrt{7-x}}{\sqrt{x}+\sqrt{7-x}}\,dx$$

$$2I = \int_2^5 \frac{\cancel{(\sqrt{x}+\sqrt{7-x})}}{\cancel{(\sqrt{7-x}+\sqrt{x})}}\,dx$$

$$2I = \int_2^5 1\,dx$$

$$2I = [x]_2^5$$

$$2I = [5-2]$$

$$2I = 3 \qquad I = \frac{3}{2}.$$

$$\therefore \boxed{\int_2^5 \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\,dx = \frac{3}{2}}$$

Q.NO	SUB Q N	ANSWER	Marking Scheme

6. a (i) Form D.E. if $y = ax^2 + b$

→ $y = ax^2 + b$ Diff. w.r.t. x

$$\frac{dy}{dx} = a(2x) + 0$$

$$\therefore a = \frac{1}{2x} \cdot \frac{dy}{dx}$$ Again diff. w.r.t. x

$$\frac{d^2y}{dx^2} = 2a\,(1)$$

$$\frac{d^2y}{dx^2} = \cancel{2}\left(\frac{1}{\cancel{2}x}\right)\frac{dy}{dx}$$

$$\therefore \frac{d^2y}{dx^2} - \frac{1}{x} \cdot \frac{dy}{dx} = 0$$

multiplying by "x"

$$\boxed{x \cdot \frac{d^2y}{dx^2} - \frac{dy}{dx} = 0}$$

(ii) Solve: $\sec^2 x \cdot \tan y \, dx + \sec^2 y \cdot \tan x \, dy = 0$

→ $\sec^2 x \cdot \tan y \, dx = -\sec^2 y \cdot \tan x \, dy$

$$\frac{\sec^2 x}{\tan x} dx = -\frac{\sec^2 y}{\tan y} dy$$

Integrating both sides,

$$\int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 y}{\tan y} dy$$

$$\log(\tan x) = -\log(\tan y) + \log C$$

$$\log[\tan x] + \log[\tan y] = \log C$$

$$\log(\tan x \cdot \tan y) = \log C$$

$$\boxed{\tan x \cdot \tan y = C}$$

(i) Solve D.E. $x \cdot \frac{dy}{dx} + y = x^3$

→ Divide each term by x,

$$\frac{\cancel{x} \cdot \frac{dy}{dx}}{\cancel{x}} + \frac{y}{x} = \frac{x^3}{x}$$

$$\frac{dy}{dx} + \left(\frac{1}{x}\right) y = x^2$$

Eq^n. in form $\frac{dy}{dx} + Py = Q$

∴ Integrating Factors $I.F. = e^{\int P dx}$

$$I.F. = e^{\int \frac{1}{x} dx}$$

$$I.F. = e^{\log x}$$

∴ $\boxed{I.F. = x.}$

Solution of equation,

$$y \times I.F. = \int Q \times I.F.\, dx + C$$

$$y \cdot x = \int x^2 \times x\, dx + C$$

$$x \cdot y = \int x^3\, dx + C$$

$$\boxed{x \cdot y = \frac{x^4}{4} + C}$$

(ii) Show that, $(3x^2 + 6xy^2)dx + (6x^2y + 4y^2)dy = 0$ is an exact D.E.

→ $M = (3x^2 + 6xy^2)$ & $N = (6x^2y + 4y^2)$

$$\frac{\partial M}{\partial y} = 0 + 6x(2y) \quad \& \quad \frac{\partial N}{\partial x} = 6y(2x) + 0$$

$$\frac{\partial M}{\partial y} = 12xy \quad \& \quad \frac{\partial N}{\partial x} = 12xy$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

∴ $\boxed{\text{Given eq}^n\text{. is exact.}}$

Q.NO	SUB Q N	ANSWER	Marking Scheme
6	C.	In a sample of 1000 cases, mean of certain test is 14 & S.D. is 2.5. Assuming distribution to be normal find –	

(i) How many students score betⁿ 12 & 15?

(ii) How many students score above 18?

Given –

$A(0.8) = 0.2881$

$A(0.4) = 0.1554$

$A(1.6) = 0.4452.$

→ $N = 1000$, $\bar{x} = 14$, $\sigma = 2.5.$

Case I] No. of students scoring bet^n 12 & 15.

$a_1 = 12$

$z_1 = \dfrac{a_1 - \bar{x}}{\sigma}$

$z_1 = \dfrac{12 - 14}{2.5}$

$z_1 = -0.8$

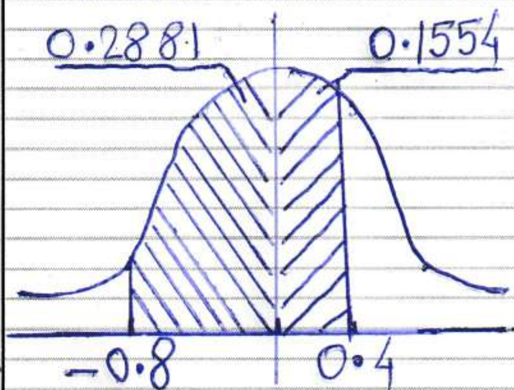
$a_2 = 15$

$z_2 = \dfrac{a_2 - \bar{x}}{\sigma}$

$Z_2 = \dfrac{15 - 14}{2.5}$

$Z_2 = +0.5$

$\therefore P(\text{bet}^n\ 12 \& 15) = A(-0.8 \leq x \leq 0.4)$



$= 0.2881 + 0.1554$

$= 0.4435$

$\therefore$ No. of students

$= 1000 \times 0.4435$

$= 443.5$

$\cong 444$ students

Case II) Score above 18 i.e. $a = 18$

$Z = \dfrac{a - \bar{x}}{\sigma}$

$Z = \dfrac{18 - 14}{2.5}$ $\therefore Z = +1.6$

Q.NO	SUB Q N	ANSWER	Marking Scheme

$P(\text{Above } 18) = P(Z \text{ above } 1.6)$

$= A(\text{right to } 1.6)$

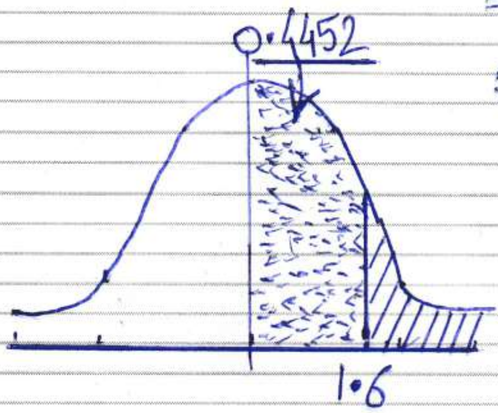
$= 0.5 - 0.4452$

$= 0.0548$

No. of students $= 0.0548 \times 1000$

$= 54.8$

$\simeq$ 55 students



Case I
Students scoring betn 12 & 15 are = 444

Case II
Students scoring above 18 are = 55.