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WINTER- 2024 EXAMINATION

Model Answer

Subject Name: Maths II

Subject Code 312301

Important Instructions to STUDENTS

1) The model answer given here are prepared from the answers from the previously uploaded model answers by Board.
2) These model answers are not uploaded by the MSBTE official site but MSBTE study resources website prepared it for students. This model answer has question paper also inbuilt in it, no need to download it separate.
3) Please rememeber that answers are not checked word to word but based on keywords which must be present in your answer
4) The model answer and the answer written by candidate may vary but the examiner may tryto assess the understanding level of the candidate
5) While assessing figures, examiner may give credit for principal components indicated in thefigure. The figures drawn bycandidate and model answer may vary. The examiner may give credit for anyequivalent figure drawn
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7) Students are advised to prepare all the syllabus from recommended book and use these model answers for the purpose of tests.

Q.NO	SUB Q N	ANSWER	Marking Scheme
1.		Solve any five.	10 Marks
	a]	Evaluate $\int\left(\frac{1}{1+x^2}+\cos x\right)dx$	
		$I=\int \frac{1}{1+x^2}+\cos x\cdot dx$	
		we know, $\int \frac{dx}{1+x^2}=\tan^{-1}(x)+C$	
		$\& \int \cos x\cdot dx = \sin x + C$	
		$\therefore \int \frac{1}{1+x^2}+\cos x \, dx$	
		$=\tan^{-1}x+\sin x+C.$	
		$\boxed{\int \frac{1}{1+x^2}+\cos x\, dx=\tan^{-1}x+\sin x+C.}$	

b] Evaluate $\int \sqrt{1+\cos 2x}\, dx$

$\rightarrow$ $\because 1+\cos\theta = 2\cos^2(\theta/2)$

Using relation,

$1+\cos 2x = 2\cos^2 x.$

$$I = \int \sqrt{1+\cos 2x}\, dx$$
$$= \int \sqrt{2\cos^2 x}\, dx$$
$$= \sqrt{2}\int \cos x \, dx$$
$$= \sqrt{2}\cdot \sin x + C.$$

$\therefore \boxed{\int \sqrt{1+\cos 2x}\cdot dx = \sqrt{2}\cdot \sin x + C.}$

c] Evaluate $\int_0^4 (4x - x^2)\cdot dx$

$\rightarrow$ $I = \int_0^4 4x - x^2 \cdot dx$

$$= \left[4\cdot\frac{x^2}{2} - \frac{x^3}{3}\right]_0^4$$
$$= \left[2x^2 - \frac{x^3}{3}\right]_0^4$$
$$= \left[2(4)^2 - \frac{4^3}{3}\right] - [0] = \frac{32}{3}$$

$\therefore \boxed{\int_0^4 (4x - x^2)\, dx = \frac{32}{3}}$

d] Find order & degree of diff. eq^n: $\frac{d^2y}{dx^2} = \sqrt{y - \frac{dy}{dx}}$

$\rightarrow$ $\frac{d^2y}{dx^2} = \sqrt{y - \frac{dy}{dx}}$; squaring both sides;

$$\left(\frac{d^2y}{dx^2}\right)^2 = y - \frac{dy}{dx}$$

$\because$ Higher order derivative is "2" & it's power is "2".

$\therefore$ $\boxed{\begin{array}{c}\text{Order} = 2 \\ \text{Degree} = 2\end{array}}$

Q.NO	Q N		Scheme

e] Show that, root of eq^n. $x^3-2x-5=0$, lies between 2 & 3.

→ Given eq^n. $x^3-2x-5=0$

$$f(x)=x^3-2x-5$$

Check:- $f(2)=2^3-2(2)-5$

$$f(2)=8-4-5$$

$$\boxed{f(2)=-1.}$$

& $f(3)=x^3-2x-5$

$$f(3)=3^3-2(3)-5$$

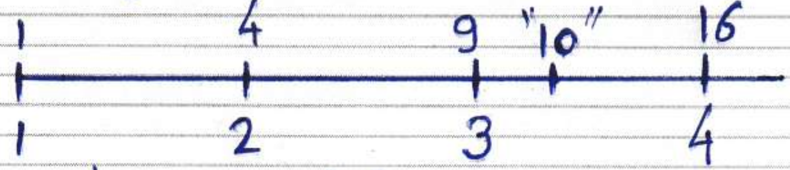
$$f(3)=27-6-5$$

$$\boxed{f(3)=16.}$$

∵ $f(2)$ is $-ve$ & $f(3)$ is $+ve$,
∴ Root lies between 2 & 3.

f] Find approximate square root of number "10" using Bakhshali Iteractive Method.

→



Nearest root is "9", whose sq. root is "3".

∴ $S=10$ & $N=3$.

∴ $\underline{d}=(S-N^2)=(10-3^2)=\underline{1}$.

$p=\frac{d}{2\times N}=\frac{1}{2\times 3}$ $\boxed{P=\frac{1}{6}.}$

$A=N+P=3+\frac{1}{6}$ $\boxed{A=\frac{19}{6}.}$

$$\sqrt{5}\cong\left[A-\frac{P^2}{2A}\right]$$

$$\sqrt{5}\cong\left[\frac{19}{6}-\frac{(1/6)^2}{2\times\frac{19}{6}}\right]\cong 3.1622$$

∴ $\boxed{\sqrt{10}\cong 3.1622.}$

Q.NO	SUB Q N	ANSWER	Marking Scheme

g. A fair coin is tossed 8 times Find probability of getting exactly "2" heads?

→ $n=8, \quad p=\frac{1}{2}, \quad q=\frac{1}{2};$

To find $P(2)$ using Binomial distribution,

$$P(r) = nC_r \, p^r \cdot q^{n-r}$$

$$\therefore P(2) = 8C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{(8-2)}$$

$$\therefore \boxed{P(2) = \frac{7}{62}.}$$

2. Solve any three — 12 Marks

a) Evaluate $\int \frac{1}{\sqrt{1-x^2}\,(\sin^{-1}x)^2}\,dx$

→ Put $(\sin^{-1}x) = t$. diff. w.r.t. x, we get,

$$\frac{1}{\sqrt{1-x^2}} = \frac{dt}{dx} \quad \therefore dx = \sqrt{1-x^2}\cdot dt.$$

$$I = \int \frac{1}{\cancel{(\sqrt{1-x^2})}\cdot t^2} \cdot \cancel{(\sqrt{1-x^2})}\, dt. = \int \frac{dt}{t^2}$$

$$I = \int t^{-2}\, dt$$

$$I = \frac{t^{(-2+1)}}{(-2+1)} + C$$

$$I = \frac{t^{(-1)}}{(-1)} + C$$

$$I = \frac{-1}{t} + C$$

$$I = \frac{-1}{\sin^{-1}x} + C$$

$$\therefore \boxed{\int \frac{1}{\sqrt{1-x^2}\,(\sin^{-1}x)^2}\,dx = \frac{-1}{(\sin^{-1}x)} + C.}$$

Q.NO	SUB Q N	ANSWER	Marking Scheme

b) Evaluate $\int \frac{\cos x}{(\sin x+1)(\sin x+2)}dx$

→ Put, $\sin x = t$ (diff. w.r.t. x; we get)

$\cos x = \frac{dt}{dx} \therefore dx = \frac{dt}{\cos x}$.

Now; $I = \int \frac{\cancel{\cos x}}{(t+1)(t+2)} \times \frac{dt}{\cancel{\cos x}}$

$I = \int \frac{1}{(t+1)(t+2)} dt.$

Let's resolve into Partial fraction,

$\frac{1}{(t+1)(t+2)} = \frac{A}{(t+1)} + \frac{B}{(t+2)}$

Multipling both sides by $[(t+1)(t+2)]$,

$\boxed{1 = A(t+2) + B(t+1).}$

Put $t+1=0$

$\therefore t = -1.$

$\therefore 1 = A(-1+2) + B(-1+1)$

$\therefore 1 = A(1) + B(0)$

$\therefore 1 = A.$ $\therefore \boxed{A = 1.}$

Put, $t+2=0$

$\therefore t = -2.$

$\therefore 1 = A(-2+2) + B(-2+1)$

$\therefore 1 = 0 + (-1)B$

$\therefore -1 = B$ $\therefore \boxed{B = -1.}$

Now,

$I = \int \frac{1}{(t+1)} + \frac{(-1)}{(t+2)} dt$

$= \log(t+1) - \log(t+2) + C$

$= \log\left[\frac{t+1}{t+2}\right] + C$

$\therefore \boxed{\int \frac{\cos x}{(\sin x+1)(\sin x+2)}dx = \log\left[\frac{\sin x+1}{\sin x+2}\right]+C}$

c] Evaluate $\int e^{x} \cdot \sin x \, dx$.

→ Check LIATE Rule; $\underset{\text{Exp.}}{e^{x}} \quad \underset{\text{Trig.}}{\sin x}$

∴ Change Sequence;

$I = \int \sin x \cdot e^{x} \, dx$; Using Int^{n}. by parts;

$I = \sin x \int e^{x} dx - \int \left[\int e^{x} dx \cdot \frac{d}{dx} \sin x\right] dx$

$I = \sin x \cdot e^{x} - \int \cos x \cdot e^{x} \, dx + C$

Again using Integration by parts;

$I = \sin x \cdot e^{x} - \left[\cos x \int e^{x} dx - \int \left[\int e^{x} dx \cdot \frac{d}{dx} \cos x\right] dx\right] + C$

$I = \sin x \cdot e^{x} - \left[\cos x \cdot e^{x} - \int e^{x} (-\sin x) \, dx\right] + C$

$I = \sin x \, e^{x} - \cos x \cdot e^{x} - \underline{\int e^{x} \cdot \sin x \, dx} + C$

$I = \sin x \cdot e^{x} - \cos x \cdot e^{x} - I + C$

$I + I = e^{x} (\sin x - \cos x) + C$

$2I = e^{x} (\sin x - \cos x) + C$

$$\boxed{I = \frac{1}{2} \left[e^{x} (\sin x - \cos x)\right] + C.}$$

d] Solve $\int \frac{1}{\sqrt{16 - 6x - x^{2}}} \cdot dx$

→ $I = \int \frac{1}{\sqrt{-(x^{2} + 6x - 16)}} \, dx$

Third term $= \left[\frac{1}{2x} \times 6x\right]^{2} = 9$

$I = \int \frac{1}{\sqrt{-(x^{2} + 6x + 9 - 16 - 9)}} \, dx$

$I = \int \frac{1}{\sqrt{(5)^{2} - (x+3)^{2}}} \, dx$

$$\therefore \boxed{\int \frac{1}{\sqrt{16 - 6x - x^{2}}} \cdot dx = \sin^{-1}\left(\frac{x+3}{5}\right) + C}$$

Q.NO	SUB Q.N	ANSWER	Marking Scheme
3.		Solve any three	12 Marks

a] $\int_0^{\pi/2} \frac{dx}{5+4\cos x}$

→ Put, $\tan\left(\frac{x}{2}\right) = t \therefore \cos x = \frac{(1-t^2)}{(1+t^2)}$ & $dx = \frac{2\cdot dt}{(1+t^2)}$

∴ limit change;

x	0	$\pi/2$
t	0	∞

∴ Integral becomes;

$$I = \int_0^{\infty} \frac{2dt/(1+t^2)}{5+4\left(\frac{1-t^2}{1+t^2}\right)} dt.$$

$$I = \int_0^{\infty} \frac{2dt/(1+t^2)}{\frac{5}{1} + \frac{4-4t^2}{(1+t^2)}} dt$$

$$I = \int_0^{\infty} \frac{2dt/\cancel{(1+t^2)}}{\frac{5+5t^2+4-4t^2}{\cancel{(1+t^2)}}} dt$$

$$I = 2\int_0^{\infty} \frac{dt}{t^2+9}$$

$$I = 2\int_0^{\infty} \frac{dt}{(t)^2+(3)^2}$$

$$I = 2\left[\frac{1}{3}\cdot \tan^{-1}\left(\frac{t}{3}\right)\right]_0^{\infty}$$

$$I = \frac{2}{3}\left[\tan^{-1}(\infty) - \tan^{-1}(0)\right]$$

$$I = \frac{2}{3}\left[\frac{\pi}{2} - 0\right]$$

$$\therefore \boxed{\int_0^{\pi/2} \frac{dx}{5+4\cos x} = \frac{\pi}{3}}$$

Q.NO	SUB Q N	ANSWER	Marking Scheme

b) $\int_0^4 \frac{\sqrt[3]{x+5}}{\sqrt{x+5}+\sqrt[3]{9-x}}\,dx$

$\rightarrow$ using Property, $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$$I = \int_0^4 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\,dx \quad \text{— eq}^n \text{ ①}$$

$$I = \int_0^4 \frac{\sqrt[3]{(0+4-x)+5}}{\sqrt[3]{(0+4-x)+5}+\sqrt[3]{9-(0+4-x)}}\,dx$$

$$I = \int_0^4 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+5}}\,dx \quad \text{— eq}^n \text{ ②}$$

Adding eq^n. ① & ②

$$I+I = \int_0^4 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\,dx + \int_0^4 \frac{\sqrt[3]{9-x}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\,dx$$

$$2I = \int_0^4 \frac{\cancel{(\sqrt[3]{x+5}+\sqrt[3]{9-x})}}{\cancel{(\sqrt[3]{x+5}+\sqrt[3]{9-x})}}\,dx$$

$$2I = \int_0^4 1\,dx$$

$$2I = [x]_0^4$$

$$2I = [4-0]$$

$$I = \frac{4}{2} = 2$$

$$\therefore \boxed{\int_0^4 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\,dx = 2.}$$

Q.NO	SUB Q N	ANSWER	Marking Scheme

c] Solve $(2xy+y^2)dx+(x^2+2xy+\sin y)dy=0$

→ Check eq^n. for EXACT ;

$M=2xy+y^2$ || & $N=x^2+2xy+\sin y$

$\frac{\partial M}{\partial y}=2x(1)+2y$ || $\frac{\partial N}{\partial x}=2x+2y(1)+0$

$\frac{\partial M}{\partial y}=2x+2y$ || $\frac{\partial N}{\partial x}=2x+2y$

$\therefore \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ Given eq^n. is exact.

sol^n:- $\int M dx + \int N dy = C$

$\downarrow$
(terms free from x)

$\int 2xy+y^2\,dx+\int \sin y\,dy=C$

$2(y)\frac{(x^2)}{2}+y^2x+(-\cos y)=C$

$\therefore \boxed{x^2y+xy^2-\cos y=C.}$

d] Using Bisection method find $x^3-x-1=0$

→ step(I) To find root lies between,

$f(x)=x^3-x-1$

$f(0)=0-0-1$

$f(0)=-1.$

$f(1)=1-1-1$

$f(1)=-1.$

$f(2)=2^3-2-1$

$f(2)=5.$

Root lies between 1 & 2

Step II

(P.T.O.

Q.NO	SUB Q.N	ANSWER	Marking Scheme

Step II Interations.

-VE (a)	+VE (b)	$x=\frac{a+b}{2}$	$f(x)=x^3-x-1$	sign
$a=1$	$b=2$	$x=\frac{1+2}{2}$, $x=1\cdot5$	$=(1\cdot5)^3-(1\cdot5)-1$ $=0\cdot875$	+VE
$a=1$	$b=1\cdot5$	$x=\frac{1+1\cdot5}{2}$, $x=1\cdot25$	$=(1\cdot25)^3-1\cdot25-1$ $=-0\cdot296$	-VE
$a=1\cdot25$	$b=1\cdot5$	$x=\frac{1\cdot25+1\cdot5}{2}$, $x=1\cdot375$		

Approximate root of eq^n. is $x=1\cdot375$.

4 Solve any three — 12 Marks

a] Find root of $x^3+2x^2-8=0$ using Regula falsi method.

→ Step I] To find root lies between;

$f(x)=x^3+2x^2-8$

$f(0)=0+0-8$

$f(0)=-8.$

$f(1)=1^3+2(1)^2-8$

$f(1)=-5.$

$f(2)=2^3+2(2)^2-8$

$f(2)=8.$

∴ Root lies between 1 & 2.

(P.T.O.

Step II Interations

-VE (a)	+VE (b)	$x = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - F(a)}$	$f(x) = x^3 + 2x - 8$	Sign
$a = 1$, $f(a) = -5$	$b = 2$, $f(b) = 8$	$x = \frac{1(8) - 2(-5)}{8 - (-5)}$, $x = 1.384$	$= (1.384)^3 + 2(1.384) - 8$, $= -1.518$	-VE
$a = 1.384$, $f(a) = -1.518$	$b = 2$, $f(b) = 8$	$x = \frac{(1.384 \times 8) - (2 \times -1.518)}{8 - (-1.518)}$, $x = 1.482$.	$= (1.482)^3 + 2(1.482) - 8$, $= -0.352$	-VE
$a = 1.482$, $f(a) = -0.352$	$b = 2$, $f(b) = 8$	$x = \frac{(1.482 \times 8) - (2 \times -0.352)}{8 - (-0.352)}$, $x = 1.503$		

Approximate root of eq^n is $x = 1.503$

b] Using Newton-Raphson Method find root of eq^n. $x^4 - x - 9 = 0$. Perform 3 Iterations.

→ Step I. To find root lies between

$f(x) = x^4 - x - 9 \quad \therefore f'(x) = 4x^3 - 1.$

$f(0) = (0) - (0) - 9$

$f(0) = -9$

$f(1) = 1^4 - 1 - 9$

$f(1) = -9$

$f(2) = 2^4 - 2 - 9$

$f(2) = 5$

Root lies between 1 & 2.

Q.NO	SUB Q N	ANSWER	Marking Scheme

let, initial root $x_0 = 2$

Iteration I) $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$$x_1 = 2 - \frac{2^4 - 2 - 9}{4(2)^3 - 1}$$

$$x_1 = 2 - \frac{5}{31} \quad \boxed{x_1 = 1.838}$$

Iteration II)

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 1.838 - \frac{1.838^4 - 1.838 - 9}{4(1.838)^3 - 1}$$

$$\boxed{x_2 = 1.813}$$

Itertion III)

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 1.813 - \frac{(1.813)^4 - 1.813 - 9}{4(1.813)^3 - 1}$$

$$\boxed{x_3 = 1.813}$$

c] Solve by Gauss-Seidal method,

$$5x - 2y + 3z = 18$$
$$x + 7y - 3z = 22$$
$$2x - y + 6z = 22.$$

→ Step I) Formation of equation,

from eqⁿ ① $x = \frac{1}{5}(18 + 2y - 3z)$

from eqⁿ ② $y = \frac{1}{7}(22 - x + 3z)$

from eqⁿ ③ $z = \frac{1}{6}(22 - 2x + y)$

Q.NO	SUB Q N	ANSWER	Marking Scheme

step II) Iterations :-

Iteration I using $y=0$ & $z=0$

$x = \frac{1}{5}(18+0-0) = 3.600$ $\boxed{x = 3.600}$

using $x = 3.6$ & $z = 0$

$y = \frac{1}{7}[22 - 3.6 + 3(0)]$ $\boxed{y = 2.628}$

using $x = 3.6$ & $y = 2.628$,

$z = \frac{1}{6}[22 - 2(3.6) + 2.628]$ $\boxed{z = 2.904.}$

Iteration II using $y = 2.628$, $z = 2.904$

$x = \frac{1}{5}[18 + 2(2.628) - 3(2.904)]$ $\boxed{x = 2.909}$

using $x = 2.909$, $z = 2.904$

$y = \frac{1}{7}[22 - 2.909 + 3(2.904)]$ $\boxed{y = 3.972}$

using $x = 2.909$, $y = 3.972$

$z = \frac{1}{6}[22 - 2(2.909) + 3.972]$ $\boxed{z = 3.359}$

Iteration III using $y = 3.972$, $z = 3.359$

$x = \frac{1}{5}[18 + 2(3.972) - 3(3.359)]$ $\boxed{x = 3.1734}$

using $x = 3.1734$, $z = 3.359$,

$y = \frac{1}{7}[22 - 3.1734 + 3(3.359)]$ $\boxed{y = 4.129}$

using $y = 4.129$, $x = 3.1734$,

$z = \frac{1}{6}[22 - 2(3.1734) + 4.129]$ $\boxed{z = 3.297.}$

Q.NO	SUB Q N	ANSWER	Marking Scheme

d] If 20% of bolts produced by a machine are defective, determine probability that out of 4 bolts drawn –
(i) One is defective
(ii) at the most two are defective.

→ Probability of defective bolts $P = \frac{20}{100} = 0.2$

$q = 1 - p \quad \therefore q = 1 - 0.2 \quad q = 0.8$

$n = 4; \quad P(r) = {}^{n}C_{r} \cdot p^{r} \cdot q^{n-r}$

(i) One defective, $n = 4$, $r = 1$.

$P(1) = {}^{4}C_{1} \cdot (0.2)^{1} \cdot (0.8)^{4-1}$

$P(1) = 0.4096.$

(ii) Probability of getting 2 defective in 4.

$= P(0) + P(1) + P(2)$

$= {}^{4}C_{0}(0.2)^{0}(0.8)^{4-0} + {}^{4}C_{1}(0.2)^{1}(0.8)^{4-1}$

$+ {}^{4}C_{2}(0.2)^{2}(0.8)^{4-2}$

$= 0.9728.$

e] If probability that an electric motor is defective is 0.01. What is probability that sample of 300 electric motors will contain exactly 5 defective motors?

→ $P = 0.01$, $n = 300$, $m = 0.01 \times 300 = 3$

$P(\text{Exactly } 5) = P(5) = \frac{e^{-m} \cdot m^{r}}{r!}$

$P(5) = \frac{e^{-3}(3)^{5}}{5!}$

$\boxed{P(5) = 0.10082.}$

Q.NO	SUB Q N	ANSWER	Marking Scheme
5.		Solve any two	12 Marks
	a) (i)	Evaluate $\int \frac{dx}{3-2\sin^2 x}$	
	→	Divide Numerator & Denominator $\cos^2 x$	
		$\therefore I = \int \frac{dx/\cos^2 x}{\frac{3-2\sin^2 x}{\cos^2 x}}$	
		$I = \int \frac{dx/\cos^2 x}{\frac{3}{\cos^2 x} - \frac{2\sin^2 x}{\cos^2 x}} = \int \frac{\sec^2 x\,dx}{3\sec^2 x - 2\tan^2 x}$	
		$I = \int \frac{\sec^2 x\,dx}{3(1+\tan^2 x) - 2\tan^2 x} = \int \frac{\sec^2 x\,dx}{3+\tan^2 x}$	
		Put, $\tan x = t$. Diff. w.r.t. x,	
		$\sec^2 x\,dx = dt$	
		$\therefore I = \int \frac{dt}{(\sqrt{3})^2 + t^2} = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{t}{\sqrt{3}}\right) + C$	
		$\boxed{\therefore \int \frac{dx}{3-2\sin^2 x} = \frac{1}{\sqrt{3}} \cdot \tan^{-1}\left(\frac{\tan x}{\sqrt{3}}\right) + C}$	
	(ii)	Evaluate $\int \frac{1-\tan x}{1+\tan x}\,dx$	
	→	$I = \int \frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}}\,dx = \int \frac{\frac{\cos x - \sin x}{\cos x}}{\frac{\cos x + \sin x}{\cos x}}\,dx$	
		$I = \int \frac{\cos x - \sin x}{\cos x + \sin x}\,dx = \int f(x) = \cos x + \sin x$	
		$f'(x) = -\sin x + \cos x = \cos x - \sin x$	
		$\int \frac{f'(x)}{f(x)}\,dx = \log(f(x)) + C$	

5 b(i) $\int_0^1 \frac{dx}{x^2-x+1}$

$\rightarrow$ Third term $= \left[\frac{1}{2x} \times (-x)\right]^2 = \frac{1}{4}$

$$I = \int_0^1 \frac{dx}{x^2 - x + \frac{1}{4} + 1 - \frac{1}{4}}$$

$$I = \int_0^1 \frac{dx}{(x-\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}$$

$$I = \left[\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{x-\frac{1}{2}}{\sqrt{3}/2}\right)\right]_0^1$$

$$I = \frac{2}{\sqrt{3}}\left[\tan^{-1}\left(\frac{2(1-1/2)}{\sqrt{3}}\right) - \tan^{-1}\left(\frac{2(0-\frac{1}{2})}{\sqrt{3}}\right)\right]$$

$$I = \frac{2}{\sqrt{3}}\left\{\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) - \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)\right\}$$

$$I = \frac{2}{\sqrt{3}}\left(\frac{\pi}{6} + \frac{\pi}{6}\right) = \frac{4\pi}{6\sqrt{3}} = \frac{2\pi}{3\sqrt{3}}$$

$$\boxed{\int_0^1 \frac{dx}{x^2-x+1} = \frac{2\pi}{3\sqrt{3}}.}$$

(ii) Evaluate $\int_0^{\pi/2} \sin^3 x \cdot \cos x \cdot dx$

$\rightarrow$ Put, $\sin x = t$; diff. w.r.t. x. $\cos x\, dx = dt$

$$I = \int_0^{\pi/2} t^3\, dt = \int_0^{\pi/2} t^3 \cdot \cancel{\cos x} \cdot \frac{dt}{\cancel{\cos x}}$$

Limit Change,

x	0	$\pi/2$
t	$\sin 0 = 0$	$\sin(\pi/2) = 1.$

$$I \int_0^1 t^3\, dt = \left[\frac{t^4}{4}\right]_0^1 = \frac{1}{4}$$

$$\boxed{\int_0^{\pi/2} \sin^3 x \cdot \cos x\, dx = \frac{1}{4}}$$

Q.NO	SUB Q N	ANSWER	Marking Scheme
5	c (i)	Evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$	

→ Let, $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ — ①

using property,

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin[0+\frac{\pi}{2}-x]}}{\sqrt{\sin[0+\frac{\pi}{2}-x]}+\sqrt{\cos[0+\frac{\pi}{2}-x]}}\,dx$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx \quad \text{— ②}$$

Adding eq^n ① & ②

$$I+I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx + \int_0^{\pi/2} \frac{\sqrt{\cos x}\,dx}{\sqrt{\cos x}+\sqrt{\sin x}}$$

$$2I = \int_0^{\pi/2} \frac{\cancel{(\sqrt{\sin x}+\sqrt{\cos x})}}{\cancel{(\sqrt{\sin x}+\sqrt{\cos x})}}\,dx$$

$$2I = \int_0^{\pi/2} 1\,dx$$

$$2I = [x]_0^{\pi/2}$$

$$2I = \frac{\pi}{2} - 0$$

$$I = \frac{\pi}{4}$$

$$\therefore \boxed{\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx = \frac{\pi}{4}}$$

Q.NO	SUB Q N	ANSWER	Marking Scheme
	C (ii)	Evaluate $\int_2^5 \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\,dx$	

→ Let, $I = \int_2^5 \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\,dx$ — (1)

using property,

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$I = \int_2^5 \frac{\sqrt{2+5-x}}{\sqrt{7-(2+5-x)}+\sqrt{2+5-x}}\,dx$$

$$\therefore I = \int_2^5 \frac{\sqrt{7-x}}{\sqrt{x}+\sqrt{7-x}}\,dx \quad \text{— (2)}$$

Adding eq^n (1) & (2)

$$I + I = \int_2^5 \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\,dx + \int_2^5 \frac{\sqrt{7-x}}{\sqrt{x}+\sqrt{7-x}}\,dx$$

$$2I = \int_2^5 \frac{\cancel{(\sqrt{x}+\sqrt{7-x})}}{\cancel{(\sqrt{7-x}+\sqrt{x})}}\,dx$$

$$2I = \int_2^5 1\,dx$$

$$2I = [x]_2^5$$

$$2I = [5-2]$$

$$2I = 3 \qquad I = \frac{3}{2}.$$

$$\therefore \boxed{\int_2^5 \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\,dx = \frac{3}{2}}$$

Q.NO	SUB Q N	ANSWER	Marking Scheme
6.	a (i)	Form D.E. if $y = ax^2 + b$	

→ $y = ax^2 + b$ Diff. w.r.t. x

$$\frac{dy}{dx} = a(2x) + 0$$

$$\therefore a = \frac{1}{2x} \cdot \frac{dy}{dx}$$ Again diff. w.r.t. x

$$\frac{d^2y}{dx^2} = 2a\,(1)$$

$$\frac{d^2y}{dx^2} = \cancel{2}\left(\frac{1}{\cancel{2}x}\right)\frac{dy}{dx}$$

$$\therefore \frac{d^2y}{dx^2} - \frac{1}{x} \cdot \frac{dy}{dx} = 0$$

multiplying by "x"

$$\boxed{x \cdot \frac{d^2y}{dx^2} - \frac{dy}{dx} = 0}$$

(ii) Solve: $\sec^2 x \cdot \tan y \, dx + \sec^2 y \cdot \tan x \, dy = 0$

→ $\sec^2 x \cdot \tan y \, dx = -\sec^2 y \cdot \tan x \, dy$

$$\frac{\sec^2 x}{\tan x} dx = -\frac{\sec^2 y}{\tan y} dy$$

Integrating both sides,

$$\int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 y}{\tan y} dy$$

$$\log(\tan x) = -\log(\tan y) + \log C$$

$$\log[\tan x] + \log[\tan y] = \log C$$

$$\log(\tan x \cdot \tan y) = \log C$$

$$\boxed{\tan x \cdot \tan y = C}$$

Q.NO	Q.N		Scheme

6 b

(i) Solve D.E. $x \cdot \frac{dy}{dx} + y = x^3$

→ Divide each term by x,

$$\frac{x \cdot \frac{dy}{dx}}{x} + \frac{y}{x} = \frac{x^3}{x}$$

$$\frac{dy}{dx} + \left(\frac{1}{x}\right) y = x^2$$

Eqn. in form $\frac{dy}{dx} + Py = Q$

∴ Integrating Factors $I.F. = e^{\int P\,dx}$

$$I.F. = e^{\int \frac{1}{x} dx}$$

$$I.F. = e^{\log x}$$

∴ $\boxed{I.F. = x.}$

Solution of equation,

$$y \times I.F. = \int Q \times I.F.\, dx + C$$

$$y \cdot x = \int x^2 \times x\, dx + C$$

$$x \cdot y = \int x^3\, dx + C$$

$$\boxed{x \cdot y = \frac{x^4}{4} + C}$$

(ii) Show that, $(3x^2 + 6xy^2)dx + (6x^2y + 4y^2)dy = 0$ is an exact D.E.

→ $M = (3x^2 + 6xy^2)$ & $N = (6x^2y + 4y^2)$

$$\frac{dM}{dy} = 0 + 6x(2y) \quad \& \quad \frac{dN}{dx} = 6y(2x) + 0$$

$$\frac{dM}{dy} = 12xy \quad \& \quad \frac{dN}{dx} = 12xy$$

$$\therefore \frac{dM}{dy} = \frac{dN}{dx}$$

∴ $\boxed{\text{Given eq}^n \text{ is exact.}}$

Q.NO	SUB Q N	ANSWER	Marking Scheme
6	C.	In a sample of 1000 cases, mean of certain test is 14 & S.D. is 2.5. Assuming distribution to be normal find –	

(i) How many students score betn 12 & 15?

(ii) How many students score above 18?

Given –

$A(0.8) = 0.2881$

$A(0.4) = 0.1554$

$A(1.6) = 0.4452.$

→ $N = 1000$, $\bar{x} = 14$, $\sigma = 2.5.$

Case I] No. of students scoring betn 12 & 15.

$a_1 = 12$

$z_1 = \dfrac{a_1 - \bar{x}}{\sigma}$

$z_1 = \dfrac{12 - 14}{2.5}$

$z_1 = -0.8$

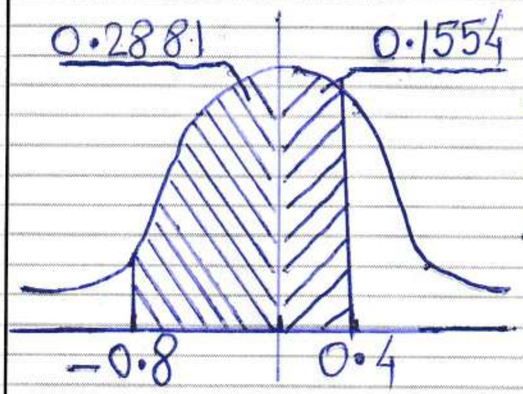
$a_2 = 15$

$z_2 = \dfrac{a_2 - \bar{x}}{\sigma}$

$Z_2 = \dfrac{15 - 14}{2.5}$

$Z_2 = +0.5$

$\therefore P(\text{bet}^n\ 12 \,\&\, 15) = A(-0.8 \leq x \leq 0.4)$



$= 0.2881 + 0.1554$

$= 0.4435$

$\therefore$ No. of students

$= 1000 \times 0.4435$

$= 443.5$

$\cong 444$ students

Case II) Score above 18 i.e. $a = 18$

$Z = \dfrac{a - \bar{x}}{\sigma}$

$Z = \dfrac{18 - 14}{2.5}$ $\therefore Z = +1.6$

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Website dedicated to Diploma Engineering study Resources

Q.NO	SUB Q N	ANSWER	Marking Scheme

$P(\text{Above } 18) = P(Z \text{ above } 1.6)$

$= A(\text{right to } 1.6)$

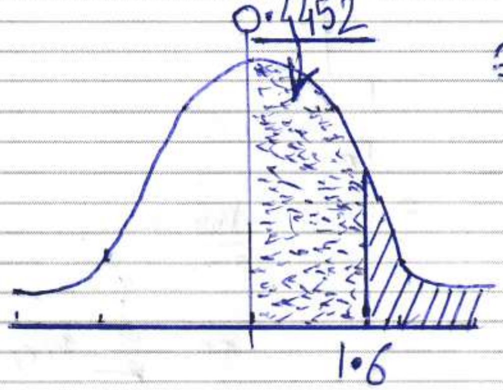
$= 0.5 - 0.4452$

$= 0.0548$

No. of students $= 0.0548 \times 1000$

$= 54.8$

$\cong$ **55 students**



Case I
Students scoring betn 12 & 15 are = 444

Case II
Students scoring above 18 are = 55.