## SUMMER – 2018 EXAMINATION

**Subject Name: Applied Mathematics** **Model Answer** Subject Code: 22224

**Important Instructions to Examiners:**

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more importance (Not applicable for subject English and Communication Skills).
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answer and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|---|---|---|---|
| **1.** | | **Attempt any FIVE of the following:** | **10** |
| | a) | State whether the function $f(x)=\dfrac{e^x+e^{-x}}{2}$ is odd or even. | **02** |
| | Ans | $f(x)=\dfrac{e^x+e^{-x}}{2}$ | |
| | | $\therefore f(-x)=\dfrac{e^{-x}+e^{-(-x)}}{2}$ | ½ |
| | | $=\dfrac{e^{-x}+e^{x}}{2}$ | ½ |
| | | $=f(x)$ | ½ |
| | | $\therefore$ function is even. | ½ |
| | | ------------------------------------------------------------ | |
| | b) | If $f(x)=\dfrac{x^2+1}{x^3-1}$ find $f\left(\dfrac{1}{2}\right)$ | **02** |
| | Ans | $f(x)=\dfrac{x^2+1}{x^3-1}$ | |
| | | $\therefore f\left(\dfrac{1}{2}\right)=\dfrac{\left(\dfrac{1}{2}\right)^2+1}{\left(\dfrac{1}{2}\right)^3-1}$ | 1 |
| | | $=\dfrac{-10}{7}$ OR $-1.429$ | 1 |
| | | ------------------------------------------------------------ | |

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| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|---|---|---|---|
| **1.** | c) | Find $\frac{dy}{dx}$ , if $y=\left(x^2+1\right)^5$ | **02** |
| | Ans | $y=\left(x^2+1\right)^5$ | |
| | | $\therefore \frac{dy}{dx}=5\left(x^2+1\right)^4.\frac{d}{dx}\left(x^2+1\right)$ | |
| | | $=5\left(x^2+1\right)^4.(2x)$ | 1 |
| | | $=10x\left(x^2+1\right)^4$ | 1 |
| | d) | Evaluate $\int(\tan x+\cot x)^2\ dx$ | **02** |
| | Ans | $\int(\tan x+\cot x)^2\ dx$ | |
| | | $=\int\left(\tan^2 x+2\tan x\cot x+\cot^2 x\right)dx$ | ½ |
| | | $=\int\left(\tan^2 x+2+\cot^2 x\right)dx$ | |
| | | $=\int\left[\left(\sec^2 x-1\right)+2+\left(co\sec^2 x-1\right)\right]dx$ | ½ |
| | | $=\int\left(\sec^2 x+co\sec^2 x\right)dx$ | |
| | | $=\tan x-\cot x+c$ | ½ +½ |
| | e) | Evaluate $\int \log x dx$ | **02** |
| | Ans | $\int \log x dx=\int \log x\cdot 1\ dx$ | ½ |
| | | $=\log x\int 1dx-\int\left(\int 1dx\ \frac{d}{dx}\log x\right)dx$ | ½ |
| | | $=\log x(x)-\int x\ \frac{1}{x}dx$ | ½ |
| | | $=x\log x-\int 1dx$ | |
| | | $=x\log x-x+c$ | ½ |
| | | $=x(\log x-1)+c$ | |
| | f) | Find the area between the lines $y=3x$ , $x$-axis and the ordinates $x=1$ and $x=5$ | **02** |
| | Ans | Area $A=\int_a^b ydx$ | |
| | | $=\int_1^5 3xdx$ | ½ |

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|---|---|---|---|
| **1.** | f) | $=3\int_1^5 xdx$ | |
| | | $=3\left[\frac{x^2}{2}\right]_1^5$ | ½ |
| | | $=3\left[\frac{5^2}{2}-\frac{1^2}{2}\right]$ | ½ |
| | | $=36$ | ½ |
| | g) | Show that there exist a root of the equation $x^2-2x-1=0$ in $(-1,0)$ and find approximate value of the root by using Bisection method. (Use two iterations) | **02** |
| | Ans | $x^2-2x-1=0$ | |
| | | $f(x)=x^2-2x-1$ | |
| | | $f(-1)=2$ | |
| | | $f(0)=-1$ | ½ |
| | | root is in $(-1,0)$ | |
| | | $\therefore x_1=\frac{-1+0}{2}=-0.5$ | ½ |
| | | $\therefore f(-0.5)=0.25$ | ½ |
| | | $\therefore$ root is in $(-0.5,0)$ | |
| | | $\therefore x_2=\frac{-0.5+0}{2}=-0.25$ | ½ |
| | | *OR* | |
| | | $x^2-2x-1=0$ | |
| | | $f(x)=x^2-2x-1$ | |
| | | $f(-1)=2$ | |
| | | $f(0)=-1$ | |
| | | root is in $(-1,0)$ | |

| $a$ | $b$ | $x=\frac{a+b}{2}$ | $f(x)$ | Marking Scheme |
|---|---|---|---|---|
| -1 | 0 | -0.5 | 0.25 | 1 |
| -0.5 | 0 | -0.25 | ----- | 1 |

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|---|---|---|---|
| **2.** | | **Attempt any THREE of the following:** | **12** |
| | a) | Find $\frac{dy}{dx}$ if $\cos\left(x^2+y^2\right)=\log\left(xy\right)$ | **04** |
| | Ans | $\cos\left(x^2+y^2\right)=\log\left(xy\right)$ | |
| | | $\therefore -\sin\left(x^2+y^2\right)\left(2x+2y\frac{dy}{dx}\right)=\frac{1}{xy}\left(x\frac{dy}{dx}+y\right)$ | 2 |
| | | $\therefore -2x\sin\left(x^2+y^2\right)-2y\sin\left(x^2+y^2\right)\frac{dy}{dx}=\frac{1}{y}\frac{dy}{dx}+\frac{1}{x}$ | |
| | | $\therefore \frac{dy}{dx}\left(-2y\sin\left(x^2+y^2\right)-\frac{1}{y}\right)=\frac{1}{x}+2x\sin\left(x^2+y^2\right)$ | 1 |
| | | $\therefore \frac{dy}{dx}=\frac{\frac{1}{x}+2x\sin\left(x^2+y^2\right)}{-2y\sin\left(x^2+y^2\right)-\frac{1}{y}}$ | 1 |
| | | ------------------------------------------------------------------------ | |
| | b) | If $x=a\cos^3\theta, y=a\sin^3\theta$, find $\frac{dy}{dx}$ at $\theta=\frac{\pi}{4}$ | **04** |
| | Ans | $x=a\cos^3\theta$ $\quad$ $y=a\sin^3\theta$ | |
| | | $\therefore \frac{dx}{d\theta}=-3a\cos^2\theta\sin\theta$ $\quad$ $\frac{dy}{d\theta}=3a\sin^2\theta\left(\cos\theta\right)$ | 1 |
| | | $\therefore \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ | |
| | | $\frac{dy}{dx}=\frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta}$ | 1 |
| | | $\frac{dy}{dx}=-\tan\theta$ | 1 |
| | | at $\theta=\frac{\pi}{4}$ | |
| | | $\frac{dy}{dx}=-\tan\frac{\pi}{4}$ | |
| | | $\frac{dy}{dx}=-1$ | 1 |

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|---|---|---|---|
| **2.** | c) | Find the maximum and minimum value of $2x^3-3x^2-36x+10$ | **04** |
| | Ans | Let $y=2x^3-3x^2-36x+10$ | |
| | | $\therefore \frac{dy}{dx}=6x^2-6x-36$ | ½ |
| | | $\therefore \frac{d^2y}{dx^2}=12x-6$ | ½ |
| | | Consider $\frac{dy}{dx}=0$ | |
| | | $6x^2-6x-36=0$ | ½ |
| | | $x^2-x-6=0$ | |
| | | $\therefore x=-2\ ,\ x=3$ | ½ |
| | | at $x=-2$ | |
| | | $\frac{d^2y}{dx^2}=12(-2)-6=-30<0$ | ½ |
| | | $\therefore y$ is maximum at $x=-2$ | |
| | | $y_{\max}=2(-2)^3-3(-2)^2-36(-2)+10$ <br> $=54$ | ½ |
| | | $at\ \ x=3$ | |
| | | $\frac{d^2y}{dx^2}=12(3)-6=30>0$ | ½ |
| | | $\therefore y$ is minimum at $x=3$ | |
| | | $y_{\min}=2(3)^3-3(3)^2-36(3)+10$ <br> $=-71$ | ½ |
| | | ---------------------------------------- | |
| | d) | A beam is bent in the form of the curve $y=2\sin x-\sin 2x$ Find the radius of curvature of the beam at the point $x=\frac{\pi}{2}$ | **04** |
| | Ans | $y=2\sin x-\sin 2x$ | |
| | | $\therefore \frac{dy}{dx}=2\cos x-2\cos 2x$ | ½ |
| | | $\therefore \frac{d^2y}{dx^2}=-2\sin x+4\sin 2x$ | ½ |
| | | $\therefore$ at $x=\frac{\pi}{2}$ | |
| | | $\frac{dy}{dx}=2\cos\left(\frac{\pi}{2}\right)-2\cos 2\left(\frac{\pi}{2}\right)=2$ | ½ |

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|---|---|---|---|
| **2.** | d) | $\frac{d^2y}{dx^2} = -2\sin\left(\frac{\pi}{2}\right) + 4\sin 2\left(\frac{\pi}{2}\right) = -2$ | ½ |
| | | $\therefore$ Radius of curvature is $\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ | |
| | | $\therefore \rho = \frac{\left[1+(2)^2\right]^{\frac{3}{2}}}{-2}$ | 1 |
| | | $\therefore \rho = -5.590$ or $5.590$ | 1 |
| | | ------------------------------------------------------------------------------------------------ | |
| **3.** | | **Attempt any <u>THREE</u> of the following:** | **12** |
| | a) | Find the points on the curve $y = x^3 + 3x^2 - 9x + 7$ at which tangents drawn are parallel to $x-$axis. | **04** |
| | Ans | $y = x^3 + 3x^2 - 9x + 7$ | |
| | | $\frac{dy}{dx} = 3x^2 + 6x - 9$ | 1 |
| | | $\because$ tangent is parallel to $x$-axis | |
| | | $\therefore$ slope of tangent = slope of $x$-axis | |
| | | $\therefore \frac{dy}{dx} = 0$ | 1 |
| | | $\therefore 3x^2 + 6x - 9 = 0$ | |
| | | $\therefore x = 1$ ; $x = -3$ | |
| | | $\therefore y = 2$ ; $y = 34$ | 1 |
| | | $\therefore$ points are $(1,2)$ and $(-3,34)$ | 1 |
| | | ------------------------------------------------------------------------------------------------ | |
| | b) | Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ w.r.t. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ | **04** |
| | Ans | Let $u = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ and $v = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ | |
| | | Put $x = \tan\theta \Rightarrow \tan^{-1} x = \theta$ | |
| | | $u = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$ | ½ |
| | | $u = \tan^{-1}(\tan 2\theta)$ | ½ |
| | | $u = 2\theta$ | |
| | | $u = 2\tan^{-1} x$ | |

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|---|---|---|---|
| **3.** | b) | $\frac{du}{dx}=\frac{2}{1+x^2}$ | ½ |
| | | $v=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ | |
| | | $v=\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$ | ½ |
| | | $v=\sin^{-1}(\sin 2\theta)$ | ½ |
| | | $v=2\theta$ | |
| | | $v=2\tan^{-1}x$ | |
| | | $\frac{dv}{dx}=\frac{2}{1+x^2}$ | ½ |
| | | $\therefore\frac{du}{dv}=\frac{\frac{du}{dx}}{\frac{dv}{dx}}=\frac{\frac{2}{1+x^2}}{\frac{2}{1+x^2}}$ | ½ |
| | | $\therefore\frac{du}{dv}=1$ | ½ |
| | | *OR* | |
| | | Let $u=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ | |
| | | $\therefore\frac{du}{dx}=\frac{1}{1+\left(\frac{2x}{1-x^2}\right)^2}\times\left[\frac{(1-x^2)2-2x(-2x)}{(1-x^2)^2}\right]$ | 1 |
| | | $\therefore\frac{du}{dx}=\frac{(1-x^2)^2}{(1-x^2)^2+4x^2}\left[\frac{2+2x^2}{(1-x^2)^2}\right]$ | |
| | | $\therefore\frac{du}{dx}=\frac{2+2x^2}{(1-x^2)^2+4x^2}$ | |
| | | $\therefore\frac{du}{dx}=\frac{2(1+x^2)}{(1-x^2)^2+4x^2}$ | |
| | | $\therefore\frac{du}{dx}=\frac{2(1+x^2)}{(1+x^2)^2}=\frac{2}{1+x^2}$ | ½ |
| | | $\therefore v=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ | |

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|---|---|---|---|
| 3. | b) | $\therefore \frac{dv}{dx}=\frac{1}{\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}}\times\left[\frac{(1+x^2)2-2x(2x)}{(1+x^2)^2}\right]$ | 1 |
| | | $\therefore \frac{dv}{dx}=\frac{(1+x^2)}{\sqrt{(1+x^2)^2-4x^2}}\left[\frac{2-2x^2}{(1+x^2)^2}\right]$ | |
| | | $\therefore \frac{dv}{dx}=\frac{(2-2x^2)}{(1+x^2)\sqrt{(1+x^2)^2-4x^2}}$ | |
| | | $\therefore \frac{dv}{dx}=\frac{2(1-x^2)}{(1+x^2)(1-x^2)}$ | |
| | | $\therefore \frac{dv}{dx}=\frac{2}{(1+x^2)}$ | ½ |
| | | $\therefore \frac{du}{dv}=\frac{\frac{du}{dx}}{\frac{dv}{dx}}=\frac{\frac{2}{(1+x^2)}}{\frac{2}{(1+x^2)}}$ | ½ |
| | | $\therefore \frac{du}{dv}=1$ | ½ |
| | | ------------------------------------------------------------------------------------------------ | |
| | c) | Find $\frac{dy}{dx}$ if $y=(\log x)^x+x^{\cos^{-1}x}$ | **04** |
| | Ans | Let $u=(\log x)^x$ | ½ |
| | | $\log u=x\log(\log x)$ | 1 |
| | | $\frac{1}{u}\frac{du}{dx}=x\frac{1}{\log x}\frac{1}{x}+\log(\log x)$ | |
| | | $\therefore \frac{du}{dx}=u\left(\frac{1}{\log x}+\log(\log x)\right)$ | |
| | | $\therefore \frac{du}{dx}=(\log x)^x\left[\frac{1}{\log x}+\log(\log x)\right]$ | ½ |
| | | Let $v=x^{\cos^{-1}x}$ | |
| | | $\log v=\cos^{-1}x\log x$ | ½ |

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|---|---|---|---|
| **3.** | c) | $\frac{1}{v}\frac{dv}{dx}=\cos^{-1}x\left(\frac{1}{x}\right)+\log x\left(\frac{-1}{\sqrt{1-x^2}}\right)$ | ½ |
| | | $\therefore \frac{dv}{dx}=x^{\cos^{-1}x}\left[\left(\cos^{-1}x\right)\left(\frac{1}{x}\right)-\log x\left(\frac{1}{\sqrt{1-x^2}}\right)\right]$ | ½ |
| | | $\therefore \frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx}$ | |
| | | $\therefore \frac{dy}{dx}=\left(\log x\right)^x\left[\frac{1}{\log x}+\log\left(\log x\right)\right]+x^{\cos^{-1}x}\left[\left(\cos^{-1}x\right)\left(\frac{1}{x}\right)-\log x\left(\frac{1}{\sqrt{1-x^2}}\right)\right]$ | ½ |
| | | ----------------------------------------------------------------------------------------------- | |
| | d) | Evaluate: $\int\frac{\sec x\cos ecx}{\log\tan x}\,dx$ | **04** |
| | Ans | $\int\frac{\sec x\cos ecx}{\log\tan x}\,dx$ | |
| | | Put $\log\tan x=t$ | 1 |
| | | $\therefore \frac{1}{\tan x}\sec^2 xdx=dt$ | |
| | | $\therefore \frac{\cos x}{\sin x}\frac{1}{\cos^2 x}dx=dt$ | |
| | | $\therefore \sec x\cos ecxdx=dt$ | 1 |
| | | $=\int\frac{1}{t}\,dt$ | ½ |
| | | $=\log t+c$ | 1 |
| | | $=\log\left(\log\left(\tan x\right)\right)+c$ | ½ |
| | | ----------------------------------------------------------------------------------------------- | |
| **4.** | | **Attempt any THREE of the following:** | **12** |
| | a) | Evaluate : $\int\frac{1}{2x^2+3x+1}dx$ | **04** |
| | Ans | $\int\frac{1}{2x^2+3x+1}dx$ | |
| | | $=\frac{1}{2}\int\frac{1}{x^2+\frac{3}{2}x+\frac{1}{2}}dx$ | ½ |

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|---|---|---|---|
| **4.** | a) | Third term $=\left(\frac{1}{2}\times\frac{3}{2}\right)^2=\frac{9}{16}$ | |
| | | $=\frac{1}{2}\int\frac{1}{x^2+\frac{3}{2}x+\frac{9}{16}-\frac{9}{16}+\frac{1}{2}}dx$ | 1 |
| | | $=\frac{1}{2}\int\frac{1}{\left(x+\frac{3}{4}\right)^2-\left(\frac{1}{4}\right)^2}dx$ | 1 |
| | | $=\frac{1}{2}\left[\frac{1}{2\left(\frac{1}{4}\right)}\log\left(\frac{x+\frac{3}{4}-\frac{1}{4}}{x+\frac{3}{4}+\frac{1}{4}}\right)\right]+c$ | 1½ |
| | | $=\log\left(\frac{2x+1}{2x+2}\right)+c$ | |
| | | *OR* | |
| | | $\int\frac{1}{2x^2+3x+1}dx=\int\frac{1}{(2x+1)(x+1)}dx$ | ½ |
| | | Let $\frac{1}{(2x+1)(x+1)}=\frac{A}{2x+1}+\frac{B}{x+1}$ | |
| | | $1=A(x+1)+B(2x+1)$ | |
| | | Put $x=\frac{-1}{2}$ | |
| | | $\therefore A=2$ | ½ |
| | | Put $x=-1$ | |
| | | $\therefore B=-1$ | ½ |
| | | $\frac{1}{(2x+1)(x+1)}=\frac{2}{2x+1}+\frac{-1}{x+1}$ | ½ |
| | | $\int\frac{1}{(2x+1)(x+1)}dx=\int\left(\frac{2}{2x+1}+\frac{-1}{x+1}\right)dx$ | |
| | | $=\frac{2\log(2x+1)}{2}-\log(x+1)+c$ | 1+1 |
| | | $=\log(2x+1)-\log(x+1)+c$ | |
| | | *OR* | |

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|---|---|---|---|
| **4.** | a) | $\int \frac{1}{2x^2+3x+1}dx$ | |
| | | Third term $=\frac{(M.T.)^2}{4(F.T.)}=\frac{9}{8}$ | ½ |
| | | $=\int \frac{1}{2x^2+3x+\frac{9}{8}-\frac{9}{8}+1}dx$ | 1 |
| | | $=\int \frac{1}{\left(\sqrt{2}x+\frac{3}{\sqrt{8}}\right)^2-\frac{1}{8}}dx$ | 1 |
| | | $=\int \frac{1}{\left(\sqrt{2}x+\frac{3}{\sqrt{8}}\right)^2-\left(\frac{1}{\sqrt{8}}\right)^2}dx$ | ½ |
| | | $=\frac{1}{\sqrt{2}}\left[\frac{1}{2\left(\frac{1}{\sqrt{8}}\right)}\log\left(\frac{\sqrt{2}x+\frac{3}{\sqrt{8}}-\frac{1}{\sqrt{8}}}{\sqrt{2}x+\frac{3}{\sqrt{8}}+\frac{1}{\sqrt{8}}}\right)\right]+c$ | 1 |
| | | $=\log\left(\frac{2x+1}{2x+2}\right)+c$ | |
| | | ------------------------------------------------------------------------------------------------------------------- | |
| | b) | Evaluate : $\int \frac{dx}{a^2\sin^2x+b^2\cos^2x}$ | **04** |
| | Ans | $\int \frac{dx}{a^2\sin^2x+b^2\cos^2x}$ | |
| | | $=\int \frac{dx/\cos^2x}{\frac{a^2\sin^2x+b^2\cos^2x}{\cos^2x}}$ | ½ |
| | | $=\int \frac{\sec^2 xdx}{a^2\tan^2x+b^2}$ Put $\tan x=t$ $\therefore \sec^2 xdx=dt$ | ½ |
| | | $=\int \frac{dt}{a^2t^2+b^2}$ | 1 |
| | | $=\int \frac{dt}{a^2\left(t^2+\frac{b^2}{a^2}\right)}$ | ½ |

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|---|---|---|---|
| **4.** | b) | $=\frac{1}{a^2}\int\frac{dt}{t^2+\left(\frac{b}{a}\right)^2}=\frac{1}{a^2}\cdot\frac{1}{\frac{b}{a}}\tan^{-1}\left(\frac{t}{\frac{b}{a}}\right)+c$ | 1 |
| | | $=\frac{1}{ab}\tan^{-1}\left(\frac{a\tan x}{b}\right)+c$ | ½ |
| | | *OR* | |
| | | $\int\frac{dx}{a^2\sin^2x+b^2\cos^2x}$ | |
| | | $=\int\frac{dx/\cos^2x}{\frac{a^2\sin^2x+b^2\cos^2x}{\cos^2x}}$ | ½ |
| | | $=\int\frac{\sec^2xdx}{a^2\tan^2x+b^2}$    Put $\tan x=t$, $\therefore\sec^2xdx=dt$ | ½ |
| | | $=\int\frac{dt}{a^2t^2+b^2}$ | 1 |
| | | $=\frac{1}{b}\tan^{-1}\left(\frac{at}{b}\right)\frac{1}{a}+c$ | 1½ |
| | | $=\frac{1}{ab}\tan^{-1}\left(\frac{a\tan x}{b}\right)+c$ | ½ |
| | | ------------------------------------------------------------ | |
| | c) | Evaluate : $\int x\cos ec^{-1}xdx$ | **04** |
| | Ans | $\int x\cos ec^{-1}xdx$ | |
| | | $=\cos ec^{-1}x\int xdx-\int\left(\int xdx\ \frac{d}{dx}\cos ec^{-1}x\right)dx$ | ½ |
| | | $=\cos ec^{-1}x\cdot\frac{x^2}{2}-\int\frac{x^2}{2}\cdot\left(\frac{-1}{x\sqrt{x^2-1}}\right)\cdot dx$ | 1 |
| | | $=\cos ec^{-1}x\cdot\frac{x^2}{2}+\frac{1}{2}\int\frac{x}{\sqrt{x^2-1}}\cdot dx$ | |
| | | $=\cos ec^{-1}x\cdot\frac{x^2}{2}+\frac{1}{4}\int\frac{2x}{\sqrt{x^2-1}}\cdot dx$ | 1 |
| | | $=\cos ec^{-1}x\cdot\frac{x^2}{2}+\frac{1}{4}\left(2\sqrt{x^2-1}\right)+c$ | 1 |
| | | $=\cos ec^{-1}x\cdot\frac{x^2}{2}+\frac{1}{2}\left(\sqrt{x^2-1}\right)+c$ | ½ |
| | | ------------------------------------------------------------ | |

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|---|---|---|---|
| **4.** | d) | Evaluate : $\int \frac{1}{x(2-\log x)(2\log x-1)}dx$ | **04** |
| | Ans | $\int \frac{1}{x(2-\log x)(2\log x-1)}dx$    Put $\log x = t$, $\therefore \frac{1}{x}dx = dt$ | ½ |
| | | $\int \frac{1}{(2-t)(2t-1)}dt$ | ½ |
| | | $\frac{1}{(2-t)(2t-1)} = \frac{A}{2-t} + \frac{B}{2t-1}$ | |
| | | $1 = A(2t-1) + B(2-t)$ | |
| | | $\therefore$ Put $t = 2$ , $A = \frac{1}{3}$ | ½ |
| | | Put $t = \frac{1}{2}$ , $B = \frac{2}{3}$ | ½ |
| | | $\therefore \frac{1}{(2-t)(2t-1)} = \frac{\frac{1}{3}}{2-t} + \frac{\frac{2}{3}}{2t-1}$ | |
| | | $\int \frac{1}{(2-t)(2t-1)}dt = \int \left( \frac{\frac{1}{3}}{2-t} + \frac{\frac{2}{3}}{2t-1} \right) dt$ | ½ |
| | | $= -\frac{1}{3}\log[2-t] + \frac{2}{6}\log[2t-1] + c$ | 1 |
| | | $= -\frac{1}{3}\log[2-\log x] + \frac{1}{3}\log[2\log x-1] + c$ | ½ |
| | | OR | |
| | | $\int \frac{1}{x(2-\log x)(2\log x-1)}dx$    Put $\log x = t$, $\therefore \frac{1}{x}dx = dt$ | ½ |
| | | $\int \frac{1}{(2-t)(2t-1)}dt$ | |
| | | $= \int \frac{1}{-2t^2+5t-2}dt$ | |
| | | $= \frac{-1}{2}\int \frac{1}{t^2 - \frac{5}{2}t + 1}dt$ | ½ |

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|---|---|---|---|
| **4.** | d) | $=\frac{-1}{2}\int\frac{1}{t^2-\frac{5}{2}t+\frac{25}{16}-\frac{25}{16}+1}dt$ | ½ |
| | | $=\frac{-1}{2}\int\frac{1}{\left(t-\frac{5}{4}\right)^2-\left(\frac{3}{4}\right)^2}dt$ | 1 |
| | | $=\frac{-1}{2}\frac{1}{2\frac{3}{4}}\log\left\|\frac{t-\frac{5}{4}-\frac{3}{4}}{t-\frac{5}{4}+\frac{3}{4}}\right\|+c$ | 1 |
| | | $=\frac{-1}{3}\log\left\|\frac{t-2}{t-\frac{1}{2}}\right\|+c$ | |
| | | $=\frac{-1}{3}\log\left\|\frac{\log x-2}{\log x-\frac{1}{2}}\right\|+c$ | ½ |
| | | ------------------------------------------------------------------ | |
| | e) | Evaluate: $\int_1^4\frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+4}}\,dx$ | **04** |
| | Ans | $I=\int_1^4\frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+4}}\,dx$ --------------------(1) | |
| | | $I=\int_1^4\frac{\sqrt[3]{9-(5-x)}}{\sqrt[3]{9-(5-x)}+\sqrt[3]{(5-x)+4}}\,dx$ | 1 |
| | | $\therefore I=\int_1^4\frac{\sqrt[3]{x+4}}{\sqrt[3]{x+4}+\sqrt[3]{9-x}}\,dx$--------------------(2) | |
| | | add (1) and (2), $I+I=\int_1^4\frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+4}}\,dx+\int_1^4\frac{\sqrt[3]{x+4}}{\sqrt[3]{x+4}+\sqrt[3]{9-x}}\,dx$ | |
| | | $\therefore 2I=\int_1^4\frac{\sqrt[3]{9-x}+\sqrt[3]{x+4}}{\sqrt[3]{9-x}+\sqrt[3]{x+4}}\,dx$ | 1 |
| | | $\therefore 2I=\int_1^4 1\,dx$ | |
| | | $\therefore 2I=(x)_1^4$ | 1 |
| | | $\therefore I=\frac{3}{2}$ | 1 |

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|---|---|---|---|
| **5.** | | **Attempt any <u>TWO</u> of the following:** | **12** |
| | a) | Find the volume of the solid generated by revolving the ellipse $\frac{x^2}{9}+\frac{y^2}{4}$ about $x-$axis | **06** |
| | Ans | Consider $\frac{x^2}{9}+\frac{y^2}{4}=1$ | |
| | | $\therefore y^2=\frac{4}{9}\left(9-x^2\right)$ | 1 |
| | | Volume of solid $V=\pi\int_{-a}^{a} y^2dx$ | |
| | | $V=\pi\int_{-3}^{3}\frac{4}{9}\left(9-x^2\right)dx$ | 1 |
| | | $\therefore V=2\pi\int_{0}^{3}\frac{4}{9}\left(9-x^2\right)dx$ | 1 |
| | | $\therefore V=\frac{8\pi}{9}\left[9x-\frac{x^3}{3}\right]_0^3$ | 1 |
| | | $\therefore V=\frac{8\pi}{9}\left[\left(9(3)-\frac{3^3}{3}\right)-\left(9(0)-\frac{0^3}{3}\right)\right]$ | 1 |
| | | $V=16\pi$ | 1 |
| | | (Note :If student has considered/assumed other value than 1and attempted to solve the problem , give appropriate marks.) | |
| | | ------------------------------------------------------------------------------------------------ | |
| | b) | **Attempt the following:** | **06** |
| | (i) | Form the diffrential equation by eliminating the arbitrary constants if $y= a\cos\left(\log x\right)+b\sin\left(\log x\right)$ | **03** |
| | Ans | $y= a\cos\left(\log x\right)+b\sin\left(\log x\right)$ | |
| | | $\therefore \frac{dy}{dx}=-a\sin\left(\log x\right)\frac{1}{x}+b\cos\left(\log x\right)\frac{1}{x}$ | 1 |
| | | $\therefore x\frac{dy}{dx}=-a\sin\left(\log x\right)+b\cos\left(\log x\right)$ | |
| | | $\therefore x\frac{d^2y}{dx^2}+\frac{dy}{dx}=-a\cos\left(\log x\right)\frac{1}{x}-b\sin\left(\log x\right)\frac{1}{x}$ | 1 |
| | | $\therefore x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}=-\left(a\cos\left(\log x\right)+b\sin\left(\log x\right)\right)$ | ½ |

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|---|---|---|---|
| **5.** | b)(i) | $\therefore x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}=-y$ | ½ |
| | | $\therefore x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+y=0$ | |
| | | ------------------------------------------------------------ | |
| | b)(ii) | Solve the differential equation: $\dfrac{dy}{dx}+y\tan x=\cos^2 x$ | **03** |
| | Ans | $\dfrac{dy}{dx}+y\tan x=\cos^2 x$ Comparing with $\dfrac{dy}{dx}+Py=Q$ | ½ |
| | | $\therefore P=\tan x$ and $Q=\cos^2 x$ | |
| | | $IF=e^{\int\tan x dx}=e^{\log(\sec x)}=\sec x$ | ½ |
| | | $\therefore y\cdot IF=\int Q\cdot IF dx+c$ | |
| | | $y\cdot\sec x=\int\cos^2 x\sec x dx+c$ | 1 |
| | | $y\cdot\sec x=\int\cos x dx+c$ | |
| | | $y\cdot\sec x=\sin x+c$ | 1 |
| | | ------------------------------------------------------------ | |
| | c) | In a single closed electrical circuit the current ' I ' at time t is given by $E-RI-L\dfrac{dI}{dt}=0$. Find the current I at time t, given that t=0 , I=0 and L,R,E are constants. | **06** |
| | Ans | $E-RI-L\dfrac{dI}{dt}=0$ | |
| | | $\therefore \dfrac{dI}{dt}+\dfrac{R}{L}I=\dfrac{E}{L}$ Comparing with $\dfrac{dy}{dx}+Py=Q$ | ½ |
| | | $\therefore P=\dfrac{R}{L}$ and $Q=\dfrac{E}{L}$ | |
| | | $IF=e^{\int\frac{R}{L}dt}=e^{\frac{R}{L}t}$ | 1 |
| | | $\therefore I\cdot IF=\int Q\cdot IF dt+c$ | |
| | | $I\cdot e^{\frac{R}{L}t}=\int\dfrac{E}{L}e^{\frac{R}{L}t}dt+c$ | 1 |
| | | $I\cdot e^{\frac{R}{L}t}=\dfrac{E}{L}\dfrac{e^{\frac{R}{L}t}}{\frac{R}{L}}+c$ | 1 |
| | | $I\cdot e^{\frac{R}{L}t}=\dfrac{E}{R}e^{\frac{R}{L}t}+c$ | |
| | | When $t=0$ , $\mathrm{I}=0$ | |

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|---|---|---|---|
| **5.** | c) | $\therefore c=-\frac{E}{R}$ | 1 |
| | | $I\cdot e^{\frac{R}{L}t}=\frac{E}{R}e^{\frac{R}{L}t}-\frac{E}{R}$ | ½ |
| | | $I=\frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)$ | 1 |
| | | ------------------------------------------------------------ | |
| **6.** | | **Attempt any <u>TWO</u> of the following:** | **12** |
| | a) | **Attempt the following:** | **06** |
| | (i) | Solve the following system of by equtions by Jacobi's -Iteration method. $\left(\text{Two iterations}\right)$ <br> $5x+2y+z=12$ , $x+4y+2z=15$ , $x+2y+5z=20$ | **03** |
| | Ans | $x=\frac{1}{5}(12-2y-z)$ <br> $y=\frac{1}{4}(15-x-2z)$ <br> $z=\frac{1}{5}(20-x-2y)$ | 1 |
| | | Starting with $x_0=y_0=z_0=0$ <br> $x_1=2.4$ <br> $y_1=3.75$ <br> $z_1=4$ | 1 |
| | | $x_2=0.1$ <br> $y_2=1.15$ <br> $z_2=2.02$ | 1 |
| | | ------------------------------------------------------------ | |
| | a(ii) | Solve the following system of equation by using Gauss-Seidel method. <br> $\left(\text{Two iterations}\right)$ <br> $15x+2y+z=18$ , $2x+20y-3z=19$ , $3x-6y+25z=22$ | **03** |
| | Ans | $x=\frac{1}{15}(18-2y-z)$ <br> $y=\frac{1}{20}(19-2x+3z)$ <br> $z=\frac{1}{25}(22-3x+6y)$ | 1 |

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**Subject Name: Applied Mathematics** **Model Answer** Subject Code: 22224

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|---|---|---|---|
| **6.** | a)(ii) | Starting with $y_0 = z_0 = 0$<br>$x_1 = 1.2$<br>$y_1 = 0.83$<br>$z_1 = 0.935$ | 1 |
| | | $x_2 = 1.027$<br>$y_2 = 0.988$<br>$z_2 = 0.994$ | 1 |
| | b) | Solve the following system of equations by using Gauss-elimination method<br>$6x - y - z = 19$ , $3x + 4y + z = 26$ , $x + 2y + 6z = 22$ | **06** |
| | Ans | $6x - y - z = 19$<br>$3x + 4y + z = 26$<br>$x + 2y + 6z = 22$ | |
| | | $6x - y - z = 19$ and $18x + 24y + 6z = 156$<br>$3x + 4y + z = 26$ and $x + 2y + 6z = 22$<br>$+$ ______ $-$ ______<br>$9x + 3y = 45$ and $17x + 22y = 134$ | 1+1 |
| | | $\therefore\ 3x + y = 15$<br>$66x + 22y = 330$<br>$17x + 22y = 134$<br>$-$ ______<br>$49x = 196$ | 1 |
| | | $\therefore x = 4$ | 1 |
| | | $y = 3$ | 1 |
| | | $z = 2$ | 1 |
| | | ***Note:*** *In the above solution, first z is eliminated and then y is eliminated to find the value of x first. If in case the problem is solved by elimination of another unknown i. e., either first x or y, appropriate marks to be given as per above scheme of marking.* | |

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**Subject Name: Applied Mathematics** **Model Answer** Subject Code: 22224

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|---|---|---|---|
| **6.** | c) | Find the approximate root of the equation $x^4 - x - 10 = 0$, by Newton-Raphson method (Carry out four iterations) | |
| | Ans | Let $f(x) = x^4 - x - 10$ | |
| | | $f(1) = -10 < 0$ | |
| | | $f(2) = 4 > 0$ | 1 |
| | | $f'(x) = 4x^3 - 1$ | 1 |
| | | Initial root $x_0 = 2$ | |
| | | $\therefore f'(2) = 31$ | |
| | | $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{f(2)}{f'(2)} = 1.871$ | 1 |
| | | $x_2 = 1.871 - \frac{f(1.871)}{f'(1.871)} = 1.856$ | 1 |
| | | $x_3 = 1.856 - \frac{f(1.856)}{f'(1.856)} = 1.856$ | 1 |
| | | $x_4 = 1.856 - \frac{f(1.856)}{f'(1.856)} = 1.856$ | 1 |
| | | *OR* | |
| | | Let $f(x) = x^4 - x - 10$ | |
| | | $f(1) = -10 < 0$ | |
| | | $f(2) = 4 > 0$ | 1 |
| | | $f'(x) = 4x^3 - 1$ | 1 |
| | | Initial root $x_0 = 2$ | |
| | | $\therefore f'(2) = 31$ | |
| | | $x_i = \frac{xf'(x) - [f(x)]}{f'(x)} = \frac{x(4x^3 - 1) - [x^4 - x - 10]}{4x^3 - 1}$ | |
| | | $= \frac{3x^4 + 10}{4x^3 - 1}$ | 2 |
| | | $x_1 = 1.871$ | ½ |
| | | $x_2 = 1.856$ | ½ |
| | | $x_3 = 1.856$ | ½ |
| | | $x_4 = 1.856$ | ½ |
| | | *OR* | |

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**Subject Name: Applied Mathematics** **Model Answer** Subject Code: 22224

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|---|---|---|---|
| **6.** | c) | Let $f(x)=x^4-x-10$ | |
| | | $f(-1)=-8<0$ | |
| | | $f(-2)=8>0$ | 1 |
| | | $f'(x)=4x^3-1$ | 1 |
| | | Initial root $x_0=-2$ | |
| | | $\therefore f'(-2)=-33$ | |
| | | $x_1=x_0-\dfrac{f(x_0)}{f'(x_0)}=-2-\dfrac{f(-2)}{f'(-2)}=-1.758$ | 1 |
| | | $x_2=-1.758-\dfrac{f(-1.758)}{f'(-1.758)}=-1.700$ | 1 |
| | | $x_3=-1.700-\dfrac{f(-1.700)}{f'(-1.700)}=-1.697$ | 1 |
| | | $x_4=-1.697-\dfrac{f(-1.697)}{f'(-1.697)}=-1.697$ | 1 |
| | | *OR* | |
| | | Let $f(x)=x^4-x-10$ | |
| | | $f(-1)=-8<0$ | |
| | | $f(-2)=8>0$ | 1 |
| | | $f'(x)=4x^3-1$ | 1 |
| | | Initial root $x_0=-2$ | |
| | | $\therefore f'(-2)=-33$ | |
| | | $x_i=\dfrac{xf'(x)-[f(x)]}{f'(x)}=\dfrac{x(4x^3-1)-[x^4-x-10]}{4x^3-1}$ | |
| | | $=\dfrac{3x^4+10}{4x^3-1}$ | 2 |
| | | $x_1=-1.758$ | ½ |
| | | $x_2=-1.700$ | ½ |
| | | $x_3=-1.697$ | ½ |
| | | $x_4=-1.697$ | ½ |
| | | ------------------------------------------------------------ | |

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| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|---|---|---|---|
| | | ***Important Note*** <br> *In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.* <br> ------------------------------------------------------------ <br> ------------------------------------------------------------ | |