_____

SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics   **Model Answer**   Subject Code: | **22206** |

Important Instructions to Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more importance (Not applicable for subject English and Communication Skills).
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answer and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.		**Attempt any FIVE of the following:**	**10**
	a)	State whether the function $f(x) = \dfrac{a^x + a^{-x}}{2}$ is even or odd .	**02**
	Ans	$f(x) = \dfrac{a^x + a^{-x}}{2}$ $\therefore f(-x) = \dfrac{a^{-x} + a^{-(-x)}}{2}$ $= \dfrac{a^{-x} + a^{x}}{2}$ $= f(x)$ $\therefore$ function is even.	1 ½ ½
	b)	If $f(x) = x^2 + 6x + 10$ find $f(2) + f(-2)$	**02**
	Ans	$f(x) = x^2 + 6x + 10$ $\therefore f(2) = (2)^2 + 6(2) + 10 = 26$ $\therefore f(-2) = (-2)^2 + 6(-2) + 10 = 2$ $\therefore f(2) + f(-2) = 26 + 2 = 28$	½ ½ 1
	c)	If $y = \log(x^2 + 2x + 5)$, find $\dfrac{dy}{dx}$	**02**
	Ans	$y = \log(x^2 + 2x + 5)$ $\therefore \dfrac{dy}{dx} = \dfrac{1}{x^2 + 2x + 5}(2x + 2)$	02

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.		$\therefore \dfrac{dy}{dx} = \dfrac{2x+2}{x^2+2x+5}$	
		---	
	d)	Evaluate : $\int \dfrac{1}{\sin^2 x \cos^2 x}\, dx$	**02**
	Ans	$\int \dfrac{1}{\sin^2 x \cos^2 x}\, dx$	
		$= \int \dfrac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}\, dx$	½
		$= \int \dfrac{1}{\cos^2 x} + \dfrac{1}{\sin^2 x}\, dx$	½
		$= \int \left(\sec^2 x + \cos ec^2 x\right) dx$	½
		$= \tan x - \cot x + c$	½
		OR	
		$\int \dfrac{1}{\sin^2 x \cos^2 x}\, dx$	
		$= \int \cos ec^2 x . \sec^2 x\, dx$	½
		$= \int \left(1+\cot^2 x\right)\left(1+\tan^2 x\right) dx$	½
		$= \int \left(1+\tan^2 x + \cot^2 x + \tan^2 x \cot^2 x\right) dx$	
		$= \int \left(1+\tan^2 x + \cot^2 x + 1\right) dx$	
		$= \int \left(\sec^2 x + \cos ec^2 x\right) dx$	½
		$= \tan x - \cot x + c$	½
		---	
	e)	Find the area enclosed by the curve $y = 3x^2$, x-axis and the ordinates $x = 1$, $x = 3$	**02**
	Ans	Area $A = \int\limits_{a}^{b} y\, dx$	
		$= \int\limits_{1}^{3} 3x^2 dx$	½
		$= 3\left[\dfrac{x^3}{3}\right]_{1}^{3}$ $\qquad$ OR $= \left[x^3\right]_{1}^{3}$	½
		$= 3\left[\dfrac{3^3}{3} - \dfrac{1^2}{3}\right]$ $\qquad = \left[3^3 - 1^3\right]$	½
		$= 26$	½

_____

SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics   **Model Answer**   Subject Code: | 22206

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	f)	An unbaised coin is tossed 5 times .Find the probability of getting a head.	**02**
	Ans	$n = 5, p = \dfrac{1}{2}, q = \dfrac{1}{2}, r = 1$	½
		$p(r) = nc_r (p)^r (q)^{n-r}$	
		$p(1) = 5c_1 \left(\dfrac{1}{2}\right)^1 \left(\dfrac{1}{2}\right)^{5-1}$	½
		$= \dfrac{5}{32} \ or \ 0.156$	1
	g)	Evaluate: $\int x \cos x \, dx$	**02**
	Ans	$\int x \cos x \, dx = x \int \cos x \, dx - \int \left(\int \cos x . \dfrac{d}{dx} x \right) dx$	½
		$= x \sin x - \int (\sin x . 1) dx$	1
		$= x \sin x + \cos x + c$	½
2		**Attempt any THREE of the following:**	**12**
	(a)	If $e^x + e^y = e^{x+y}$, find $\dfrac{dy}{dx}$	**04**
	Ans	$e^x + e^y = e^{x+y}$	
		$e^x + e^y \dfrac{dy}{dx} = e^{x+y} \left[1 + \dfrac{dy}{dx} \right]$	1
		$e^y \dfrac{dy}{dx} - e^{x+y} \dfrac{dy}{dx} = e^{x+y} - e^x$	1
		$\dfrac{dy}{dx} (e^y - e^{x+y}) = e^{x+y} - e^x$	1
		$\dfrac{dy}{dx} = \dfrac{e^{x+y} - e^x}{e^y - e^{x+y}}$	1
	b)	If $x = a(\theta + \sin \theta), y = a(1 - \cos \theta)$, find $\dfrac{dy}{dx}$ at $\theta = \dfrac{\pi}{2}$	**04**
	Ans	$x = a(\theta + \sin \theta)$	
		$\dfrac{dx}{d\theta} = a(1 + \cos \theta)$	1
		$y = a(1 - \cos \theta)$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	b)	$\dfrac{dy}{d\theta} = a\big(-(-\sin\theta)\big) = a\sin\theta$	1
		$\dfrac{dy}{dx} = \dfrac{\dfrac{dy}{d\theta}}{\dfrac{dx}{d\theta}} = \dfrac{a\sin\theta}{a(1+\cos\theta)}$	
		$= \dfrac{\sin\theta}{(1+\cos\theta)}$ OR $= \dfrac{2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}}{2\cos^2\dfrac{\theta}{2}} = \tan\dfrac{\theta}{2}$	½
		at $\theta = \dfrac{\pi}{2}$, $\dfrac{dy}{dx} = \dfrac{\sin\dfrac{\pi}{2}}{\left(1+\cos\dfrac{\pi}{2}\right)}$ $= \tan\dfrac{\pi}{4}$	½
		$= \dfrac{1}{1+0} = 1$ $= 1$	1
		---	
	c)	Find the maximum and minimum values of $y = 2x^3 - 3x^2 - 36x + 10$	**04**
	Ans	Let $y = 2x^3 - 3x^2 - 36x + 10$	
		$\therefore \dfrac{dy}{dx} = 6x^2 - 6x - 36$	½
		$\therefore \dfrac{d^2y}{dx^2} = 12x - 6$	½
		Consider $\dfrac{dy}{dx} = 0$	
		$6x^2 - 6x - 36 = 0$	
		$x^2 - x - 6 = 0$	
		$\therefore x = -2$, $x = 3$	1
		at $x = -2$	
		$\dfrac{d^2y}{dx^2} = 12(-2) - 6 = -30 < 0$	½
		$\therefore y$ is maximum at $x = -2$	
		$y_{max} = 2(-2)^3 - 3(-2)^2 - 36(-2) + 10$	
		$= 54$	½
		at $x = 3,$ $\dfrac{d^2y}{dx^2} = 12(3) - 6 = 30 > 0$	½
		$\therefore y$ is minimum at $x = 3$	
		$y_{min} = 2(3)^3 - 3(3)^2 - 36(3) + 10$	
		$= -71$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	d)	A telegraph wire hangs in the form of a curve $y = a\log\left(\sec\left(\dfrac{x}{a}\right)\right)$ where 'a' is constant. Show that radius of curvature at any point is $a\sec\left(\dfrac{x}{a}\right)$	**04**
	Ans	$y = a\log\left(\sec\left(\dfrac{x}{a}\right)\right)$	
		$\dfrac{dy}{dx} = a\dfrac{1}{\sec\left(\dfrac{x}{a}\right)}\sec\left(\dfrac{x}{a}\right)\tan\left(\dfrac{x}{a}\right)\left(\dfrac{1}{a}\right)$	1
		$\dfrac{dy}{dx} = \tan\left(\dfrac{x}{a}\right)$	
		$\dfrac{d^2 y}{dx^2} = \sec^2\left(\dfrac{x}{a}\right)\left(\dfrac{1}{a}\right)$	1
		$\therefore$ Radius of curvature is $\rho = \dfrac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\dfrac{d^2 y}{dx^2}}$	
		$\therefore \rho = \dfrac{\left[1+\tan^2\left(\dfrac{x}{a}\right)\right]^{\frac{3}{2}}}{\sec^2\left(\dfrac{x}{a}\right)\left(\dfrac{1}{a}\right)}$	½
		$\therefore \rho = \dfrac{a\left[\sec^2\left(\dfrac{x}{a}\right)\right]^{\frac{3}{2}}}{\sec^2\left(\dfrac{x}{a}\right)}$	½
		$\therefore \rho = \dfrac{a\sec^3\left(\dfrac{x}{a}\right)}{\sec^2\left(\dfrac{x}{a}\right)}$	½
		$\therefore \rho = a\sec\left(\dfrac{x}{a}\right)$	½

3		**Attempt any THREE of the following:**	**12**
	a)	Find the equation of tangent and normal to the curve $y = 2x - x^2$ at (2,0)	**04**

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	Ans	$y = 2x - x^2$	
		$\dfrac{dy}{dx} = 2 - 2x$	1
		at (2,0)	
		slope of tangent $m = \dfrac{dy}{dx} = 2 - 2(2) = -2$	½
		equation of tangent is,	
		$y - y_1 = m(x - x_1)$	
		$y - 0 = -2(x - 2)$	½
		$y = -2x + 4$	
		$2x + y - 4 = 0$	½
		slope of normal $m' = -\dfrac{1}{m} = \dfrac{1}{2}$	½
		equation of normal is,	
		$y - y_1 = m'(x - x_1)$	
		$y - 0 = \dfrac{1}{2}(x - 2)$	½
		$2y = x - 2$	
		$x - 2y - 2 = 0$	½
		--	
	b)	Differentiate $(\sin x)^{\tan x}$ w.r.t. x	**04**
	Ans	Let y $= (\sin x)^{\tan x}$	
		$\log y = \tan x \log(\sin x)$	½
		$\dfrac{1}{y}\dfrac{dy}{dx} = \tan x \dfrac{1}{\sin x}\cos x + \log(\sin x)\sec^2 x$	2
		$\dfrac{dy}{dx} = y\left(\tan x \cot x + \log(\sin x)\sec^2 x\right)$	1
		$\dfrac{dy}{dx} = (\sin x)^{\tan x}\left(1 + \log(\sin x)\sec^2 x\right)$	½
		--	
	c)	If $Y = \sqrt{\dfrac{1 - \cos 2x}{1 + \cos 2x}}$, find $\dfrac{dy}{dx}$	**04**
	Ans	$y = \sqrt{\dfrac{1 - \cos 2x}{1 + \cos 2x}}$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	c)	$y = \sqrt{\dfrac{2\sin^2 x}{2\cos^2 x}}$	1
		$y = \sqrt{\tan^2 x}$	1
		$y = \tan x$	1
		$\dfrac{dy}{dx} = \sec^2 x$	1
		--	
	d)	Evaluate: $\int \dfrac{\sin\sqrt{x}}{\sqrt{x}}\ dx$	**04**
	Ans	$\int \dfrac{\sin\sqrt{x}}{\sqrt{x}}\ dx$	
		Put $\sqrt{x} = t$	
		$\therefore \dfrac{1}{2\sqrt{x}} dx = dt$	1
		$\therefore \dfrac{1}{\sqrt{x}} dx = 2dt$	½
		$= \int \sin t\,(2dt)$	½
		$= -2\cos t + c$	1½
		$= -2\cos\sqrt{x} + c$	½
		--	
4		**Attempt any THREE of the following:**	**12**
	a)	Evaluate: $\int \dfrac{1}{\sqrt{1-x^2}\left(\sin^{-1} x\right)^2}\ dx$	**04**
	Ans	$\int \dfrac{1}{\sqrt{1-x^2}\left(\sin^{-1} x\right)^2}\ dx$	
		Put $\sin^{-1} x = t$	
		$\therefore \dfrac{1}{\sqrt{1-x^2}} dx = dt$	1
		$= \int \dfrac{1}{t^2}\ dt$	1
		$= \int t^{-2}\ dt$	½
		$= \dfrac{t^{-1}}{-1} + c$	
			1

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	a)	$= -\left(\sin^{-1} x\right)^{-1} + c$ --	½
	b) Ans	Evaluate : $\int \dfrac{1}{5 + 4\cos x} dx$ $\int \dfrac{1}{5 + 4\cos x} dx$ Put $\tan \dfrac{x}{2} = t \quad \cos x = \dfrac{1 - t^2}{1 + t^2}, dx = \dfrac{2dt}{1 + t^2}$ $\therefore \int \dfrac{dx}{5 + 4\cos x} = \int \dfrac{1}{5 + 4\left(\dfrac{1 - t^2}{1 + t^2}\right)} \cdot \dfrac{2dt}{1 + t^2}$ $= 2\int \dfrac{1}{t^2 + 9} dt$ $= 2\int \dfrac{1}{t^2 + 3^2} dt$ $= 2 \times \dfrac{1}{3} \tan^{-1}\left(\dfrac{t}{3}\right) + c$ $= \dfrac{2}{3} \tan^{-1}\left(\dfrac{\tan \dfrac{x}{2}}{3}\right) + c$	**04** 1 1 ½ 1 ½
		--	
	c) Ans	Evaluate: $\int \dfrac{x}{1 + \cos 2x} dx$ $\int \dfrac{x}{1 + \cos 2x} dx$ $= \int \dfrac{x}{2\cos^2 x} dx$ $= \dfrac{1}{2} \int x \sec^2 x \, dx$ $= \dfrac{1}{2}\left[x \int \sec^2 x \, dx - \int \left(\int \sec^2 x \, dx \cdot \dfrac{d}{dx} x \right) dx \right]$ $= \dfrac{1}{2}\left[x \tan x - \int \tan x \cdot 1 \, dx \right]$ $= \dfrac{1}{2}\left[x \tan x - \log(\sec x) \right] + c$ --	**04** ½ ½ 1 1 1

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	d)	Evaluate : $\int \dfrac{\sec^2 x}{(1+\tan x)(2+\tan x)}$	**04**
	Ans	$\int \dfrac{\sec^2 x}{(1+\tan x)(2+\tan x)}dx$ $\qquad$ $\begin{vmatrix} Put\ \tan x = t \\ \therefore \sec^2 x dx = dt \end{vmatrix}$	1
		$\int \dfrac{1}{(1+t)(2+t)}dt$	
		$\dfrac{1}{(1+t)(2+t)} = \dfrac{A}{1+t} + \dfrac{B}{2+t}$	½
		$1 = A(2+t) + B(1+t)$	
		$\therefore$ Put $t = -1$, $A = 1$	½
		Put $t = -2$, $B = -1$	½
		$\therefore \dfrac{1}{(1+t)(2+t)} = \dfrac{1}{1+t} - \dfrac{1}{2+t}$	
		$\int \dfrac{1}{(1+t)(2+t)}dt = \int \left(\dfrac{1}{1+t} - \dfrac{1}{2+t} \right) dt$	
		$= \log[1+t] - \log[2+t] + c$	1
		$= \log[1+\tan x] - \log[2+\tan x] + c$	½
		OR	
		$\int \dfrac{\sec^2 x}{(1+\tan x)(2+\tan x)}dx$ $\qquad$ $\begin{vmatrix} Put\ \tan x = t \\ \therefore \sec^2 x dx = dt \end{vmatrix}$	1
		$\int \dfrac{1}{(1+t)(2+t)}dt$	
		$= \int \dfrac{1}{t^2+3t+2}dt$	½
		$= \int \dfrac{1}{t^2+3t+\dfrac{9}{4}-\dfrac{9}{4}+2}dt$	½
		$= \int \dfrac{1}{\left(t+\dfrac{3}{2}\right)^2 - \left(\dfrac{1}{2}\right)^2}dt$	½
		$= \dfrac{1}{2\dfrac{1}{2}}\log\left|\dfrac{t+\dfrac{3}{2}-\dfrac{1}{2}}{t+\dfrac{3}{2}+\dfrac{1}{2}}\right| + c$	1
		$= \log\left|\dfrac{t+1}{t+2}\right| + c$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.		$= \log\left\|\dfrac{\tan x+1}{\tan x+2}\right\| + c$	½
		--	
	e)	Evaluate : $\displaystyle\int_0^{\pi/2} \dfrac{\sqrt[3]{\sin x}}{\sqrt[3]{\cos x}+\sqrt[3]{\sin x}}\,dx$	**04**
	Ans	$I = \displaystyle\int_0^{\pi/2} \dfrac{\sqrt[3]{\sin x}}{\sqrt[3]{\cos x}+\sqrt[3]{\sin x}}\,dx - - - - - - -(1)$	
		$= \displaystyle\int_0^{\pi/2} \dfrac{\sqrt[3]{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt[3]{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt[3]{\sin\left(\frac{\pi}{2}-x\right)}}\,dx$	1
		$I = \displaystyle\int_0^{\pi/2} \dfrac{\sqrt[3]{\cos x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}\,dx - - - - - - -(2)$	½
		Add (1) and (2)	½
		$\therefore 2I = \displaystyle\int_0^{\pi/2} \dfrac{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}\,dx$	½
		$= \displaystyle\int_0^{\pi/2} 1\cdot dx$	
		$= \big[x\big]_0^{\pi/2}$	1
		$2I = \dfrac{\pi}{2}$	
		$\therefore I = \dfrac{\pi}{4}$	1
		OR	
		$I = \displaystyle\int_0^{\pi/2} \dfrac{\sqrt[3]{\sin x}}{\sqrt[3]{\cos x}+\sqrt[3]{\sin x}}\,dx$ $\left\| \begin{array}{l} \text{Replace } x \to \dfrac{\pi}{2}-x \\[4pt] \therefore \sin x \to \cos x \\[2pt] \&\cos x \to \sin x \end{array} \right.$	1
		$\therefore I = \displaystyle\int_0^{\pi/2} \dfrac{\sqrt[3]{\cos x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}\,dx$	½
		$\therefore 2I = \displaystyle\int_0^{\pi/2} \dfrac{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}\,dx$	½
		$= \displaystyle\int_0^{\pi/2} 1\cdot dx$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	e)	$=\left[x\right]_0^{\pi/2}$ $$2I = \frac{\pi}{2}$$ $$\therefore I = \frac{\pi}{4}$$	1 1
5		**Attempt any TWO of the following:**	**12**
	a)	Find the area of the region bounded by the parabola $y = 4x - x^2$ and the x-axis.	**06**
	Ans	$y = 4x - x^2$ put $y = 0$, $4x - x^2 = 0$ $x = 0, x = 4$	1
		Area $=\int_a^b y\,dx$	
		$=\int_0^4 \left(4x - x^2\right) dx$	1
		$=4\left[\frac{x^2}{2}\right]_0^4 - \left[\frac{x^3}{3}\right]_0^4$ $\quad$ OR $\quad =\left[4\frac{x^2}{2} - \frac{x^3}{3}\right]_0^4$	2
		$=4\left[\frac{4^2}{2} - \frac{0^2}{2}\right] - \left[\frac{4^3}{3} - \frac{0^3}{3}\right]$ $\quad$ OR $\quad =\left[\left(2(4)^2 - \frac{4^3}{3}\right) - 0\right]$	1
		$=\frac{32}{3} = 10.667$	1
	b)	Attempt the following:	**06**
	(i)	Form the D.E. by eliminating the arbitrary constants if $y = A\cos 3x + B\sin 3x$	**03**
	Ans	$y = A\cos 3x + B\sin 3x$	
		$\therefore \frac{dy}{dx} = -3A\sin 3x + 3B\cos 3x$	1
		$\therefore \frac{d^2 y}{dx^2} = -9A\cos 3x - 9B\sin 3x$	1
		$\therefore \frac{d^2 y}{dx^2} = -9\left(A\cos 3x + B\sin 3x\right)$	½
		$\frac{d^2 y}{dx^2} = -9y$	½
		$\frac{d^2 y}{dx^2} + 9y = 0$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	b)(ii)	Solve : $x(1+y^2)dx + y(1+x^2)dy = 0$	**03**
	Ans	$x(1+y^2)dx + y(1+x^2)dy = 0$	
		$\therefore \dfrac{x}{1+x^2}dx = -\dfrac{y}{1+y^2}dy$	1
		$\therefore \int \dfrac{x}{1+x^2}dx = -\int \dfrac{y}{1+y^2}dx$	1
		$\therefore \dfrac{1}{2}\log(1+x^2) = -\dfrac{1}{2}\log(1+y^2) + c$	1
		$\therefore \log(1+x^2) = -\log(1+y^2) + C$	
		--	
	(c)	A particle starting with velocity 6m/sec has an acceleration $(1-t^2)$ m/sec^2, when does it first come to rest? How far has it then travelled?	**06**
	Ans	Acceleration $= \dfrac{dv}{dt} = 1 - t^2$	
		$\therefore dv = (1-t^2)dt$	
		$\therefore \int dv = \int (1-t^2)dt$	
		$\therefore v = t - \dfrac{t^3}{3} + c$	1
		given $v = 6$ and initially $t = 0$	
		$\therefore c = 6$	½
		$\therefore v = t - \dfrac{t^3}{3} + 6$	
		The particle comes to rest when $v = 0$	
		$\therefore t - \dfrac{t^3}{3} + 6 = 0$	½
		$\therefore t^3 - 3t - 18 = 0$	
		$\therefore t = 3$	1
		$\therefore v = \dfrac{dx}{dt}$	
		$\therefore \dfrac{dx}{dt} = t - \dfrac{t^3}{3} + 6$	½
		$\therefore dx = \left(t - \dfrac{t^3}{3} + 6\right)dt$	
		$\therefore \int dx = \int \left(t - \dfrac{t^3}{3} + 6\right)dt$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	c)	$\therefore x = \dfrac{t^2}{2} - \dfrac{t^4}{12} + 6t + c_1$	1
		$\therefore$ initially $x = 0$, $t = 0$	
		$c_1 = 0$	½
		$\therefore x = \dfrac{t^2}{2} - \dfrac{t^4}{12} + 6t$	
		put $t = 3$	
		$\therefore x = \dfrac{(3)^2}{2} - \dfrac{(3)^4}{12} + 6(3)$	
		$\therefore x = 15.75$	1
		--	
6		**Attempt any TWO of the following:**	**12**
	a)	Attempt the following:	**06**
	i)	A person fires 10 shots at target. The probability that any shot will hit the target 3/5. Find the probability that the target is hit exactly 5 times.	**03**
	Ans	$n = 10, p = \dfrac{3}{5}$	
		$q = 1 - p = 1 - \dfrac{3}{5} = \dfrac{2}{5}$	
		$r = 5$	
		$p(r) = {}^n c_r \, (p)^r \, (q)^{n-r}$	
		$p(5) = {}^{10} c_5 \left(\dfrac{3}{5}\right)^5 \left(\dfrac{2}{5}\right)^{10-5}$	2
		$= 0.2007$	1
		--	
	ii)	If 20% of the bolt produce by a machine are defective. Find the Probability that out of 4 bolts drawn ,

(1) one is defective

(2) at the most two are defective. | **03** |
| | Ans | Given $p = 20\% = \dfrac{20}{100} = 0.2, n = 4$ and $q = 1 - p = 0.8$

$p(r) = {}^n C_r \, p^r q^{n-r}$ | |

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	a)(ii)	(1) $p(\text{one is defective})$ $= p(1) = 4C_1 (0.2)^1 (0.8)^{4-1}$ $= 0.4096$ (2) $p(\text{at the most two are defective.})$ $= p(0) + p(1) + p(2)$ $= 4C_0 (0.2)^0 (0.8)^{4-0} + 4C_1 (0.2)^1 (0.8)^{4-1} + 4C_2 (0.2)^2 (0.8)^{4-2}$ $= 0.9728$ --	1 ½ 1 ½
	b) Ans	A company manufacture electric motors. The probability that an elecric motor is defective is 0.01. What is the probability that a sample of 300 electric motors will contain exactly 5 defective motors? (Given: $e^{-3} = 0.0498$) $p = 0.01, n = 300 , r = 5$ $\therefore m = np = 0.01 \times 300 = 3$ $p(r) = \dfrac{e^{-m} \cdot (m)^r}{r!}$ $p(5) = \dfrac{e^{-3} \cdot (3)^5}{5!}$ $p(5) = \dfrac{(0.0498) \cdot (3)^5}{5!}$ $= 0.1008$ --	**06** 2 2 1 1
	c) Ans	In a sample of 1000 cases the mean of certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal, find (1) how many students score above 18? (2) how many students score between 12 and 15? $[\text{Given: } A(0.4) = 0.1554, A(0.8) = 0.2881, A(1.6) = 0.4452]$ Given $\bar{x} = 14 \quad \sigma = 2.5 \quad N = 1000$ (1) $z = \dfrac{x - \bar{x}}{\sigma} = \dfrac{18 - 14}{2.5} = 1.6$ $\therefore p(\text{score above 18}) = A(\text{greater than } 1.6)$ $= 0.5 - A(1.6)$ $= 0.5 - 0.4452 = 0.0548$ $\therefore \text{No. of students} = N \cdot p$ $= 1000 \times 0.0548 = 54.8 \ i.e., 55$	**06** 1 1 1

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.		(2) $z = \dfrac{x - \bar{x}}{\sigma} = \dfrac{12 - 14}{2.5} = -0.8$	1
		$z = \dfrac{x - \bar{x}}{\sigma} = \dfrac{15 - 14}{2.5} = 0.4$	
		$\therefore p\left(\text{score between 12 and 15}\right) = A\left(-0.8\right) + A\left(0.4\right)$	
		$= 0.2881 + 0.1554$	
		$= 0.4435$	1
		$\therefore \text{No. of students} = N \cdot p = 1000 \times 0.4435$	
		$= 443.5 \ \textit{i.e., } 444$	1

Important Note

In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.