



SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics Model Answer

Subject Code: 17216

Important Instructions to examiners:

- 1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
- 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
- 3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills).
- 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
- 5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
- 6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
- 7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.		Attempt any <u>TEN</u> of the following:	20
	a)	Find x and y if $x(1-i) + y(2+i) + 6 = 0$	02
	Ans	$x(1-i) + y(2+i) + 6 = 0$ $x - ix + 2y + iy = -6 + i0$ $(x + 2y) + i(-x + y) = -6 + i0$ $x + 2y = -6 \text{ and } -x + y = 0$ $\therefore x = -2, y = -2$ <p>OR</p> $(x + 2y + 6) + i(-x + y) = 0$ $x + 2y + 6 = 0 \text{ and } -x + y = 0$ $\therefore x = -2, y = -2$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$
b)	Define composite function.	02	
Ans	<p>If y is function of u and u is function of x then y is composite function of x or it is called function of function $y = u(x)$</p> <p>OR</p> <p>If $f(x)$ and $g(x)$ are two functions then composite function is defined $f(g(x))$.</p> <p>OR</p> <p>If $f : x \rightarrow y, g : y \rightarrow z$ then $gof : x \rightarrow z$ called composite function</p>	<p>2</p> <p>2</p> <p>2</p>	
c)	If $f(x) = x^4 - 2x + 7$ find $f(0) + f(2)$	02	
Ans	$f(x) = x^4 - 2x + 7$		



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No.	Sub Q. N.	Answer	Marking Scheme
1.	c)	$f(0) = (0)^4 - 2(0) + 7 = 7$ $f(2) = (2)^4 - 2(2) + 7 = 19$ $f(0) + f(2) = 7 + 19$ $= 26$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
	d)	Express in the form of $a + ib$ if $z = \frac{1+i}{3-i}$	02
	Ans	$z = \frac{1+i}{3-i}$ $z = \frac{1+i}{3-i} \times \frac{3+i}{3+i}$ $= \frac{3+i+3i+i^2}{9-i^2}$ $= \frac{3+4i-1}{9+1}$ $= \frac{2+4i}{10}$ $= \frac{1}{5} + \frac{2}{5}i$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
e)	Evaluate $\lim_{x \rightarrow 4} \frac{2-\sqrt{x}}{4-x}$	02	
Ans	$\lim_{x \rightarrow 4} \frac{2-\sqrt{x}}{4-x}$ $= \lim_{x \rightarrow 4} \frac{2-\sqrt{x}}{4-x} \times \frac{2+\sqrt{x}}{2+\sqrt{x}}$ $= \lim_{x \rightarrow 4} \frac{4-x}{(4-x)(2+\sqrt{x})}$ $= \lim_{x \rightarrow 4} \frac{1}{2+\sqrt{x}}$ $= \frac{1}{2+\sqrt{4}}$ $= \frac{1}{4} = 0.25$ OR $\lim_{x \rightarrow 4} \frac{2-\sqrt{x}}{4-x}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	



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1.	e)	$= \lim_{x \rightarrow 4} \frac{2 - \sqrt{x}}{2^2 - (\sqrt{x})^2}$ $= \lim_{x \rightarrow 4} \frac{2 - \sqrt{x}}{(2 - \sqrt{x})(2 + \sqrt{x})}$ $= \lim_{x \rightarrow 4} \frac{1}{2 + \sqrt{x}}$ $= \frac{1}{2 + \sqrt{4}}$ $= \frac{1}{4} = 0.25$	<p>½</p> <p>½</p> <p>½</p> <p>½</p>
	f)	<p>Evaluate $\lim_{x \rightarrow 0} \frac{5 \sin x + 7x}{8x - 3 \tan x}$</p>	02
	Ans	$\lim_{x \rightarrow 0} \frac{5 \sin x + 7x}{8x - 3 \tan x}$ $= \lim_{x \rightarrow 0} \frac{\frac{5 \sin x}{x} + \frac{7x}{x}}{\frac{8x}{x} - \frac{3 \tan x}{x}}$ $= \frac{5 \lim_{x \rightarrow 0} \frac{\sin x}{x} + 7}{8 - 3 \lim_{x \rightarrow 0} \frac{\tan x}{x}}$ $= \frac{5 + 7}{8 - 3}$ $= \frac{12}{5} = 2.4$	<p>½</p> <p>1</p> <p>½</p>
	g)	<p>Evaluate $\lim_{x \rightarrow 0} \left(1 - \frac{7}{2x}\right)^x$</p>	02
	Ans	$\lim_{x \rightarrow 0} \left(1 - \frac{7}{2x}\right)^x$ $= \lim_{x \rightarrow 0} \left(\left(1 - \frac{7}{2x}\right)^{\frac{-2x}{7}} \right)^{\frac{-7}{2}}$ $= e^{\frac{-7}{2}}$	<p>1</p> <p>1</p>



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1.	g)	Note: If the student has considered 2^x and tried to solve give appropriate marks and if the student has considered $2x$ and attempted to solve give appropriate marks. -----	
	h)	If $y = \log(1+x^2)$ Find $\frac{dy}{dx}$	02
	Ans	$y = \log(1+x^2)$ $\frac{dy}{dx} = \frac{1}{1+x^2} \times 2x$ $= \frac{2x}{1+x^2}$	2
	i)	Find $\frac{dy}{dx}$ if $y = \frac{\sin x}{1+\cos x}$	02
Ans	$y = \frac{\sin x}{1+\cos x}$ $\frac{dy}{dx} = \frac{(1+\cos x)\cos x - \sin x(-\sin x)}{(1+\cos x)^2}$ $= \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$ $= \frac{\cos x + 1}{(1+\cos x)^2}$ $= \frac{1}{(1+\cos x)}$	1	
		<i>OR</i>	
		$y = \frac{\sin x}{1+\cos x}$ $y = \frac{2\sin \frac{x}{2} \cos \frac{x}{2}}{2\cos^2 \frac{x}{2}}$ $y = \tan \frac{x}{2}$ $\therefore \frac{dy}{dx} = \frac{1}{2} \sec^2 \left(\frac{x}{2} \right)$	½
			½
			1
	j)	Find $\frac{dy}{dx}$ if $x^3 + y^3 = 3axy$	02



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1.	j)	$x^3 + y^3 = 3axy$	
	Ans	$3x^2 + 3y^2 \frac{dy}{dx} = 3a \left(x \frac{dy}{dx} + y \right)$ $3x^2 + 3y^2 \frac{dy}{dx} = 3ax \frac{dy}{dx} + 3ay$ $3y^2 \frac{dy}{dx} - 3ax \frac{dy}{dx} = 3ay - 3x^2$ $(3y^2 - 3ax) \frac{dy}{dx} = 3ay - 3x^2$ $\frac{dy}{dx} = \frac{3ay - 3x^2}{3y^2 - 3ax} = \frac{ay - x^2}{y^2 - ax}$	1
	k)	Using Gauss seidal method find first iteration for system of equations: $8x + 2y + 3z = 30$, $x - 9y + 2z = 1$, $2x + 3y + 6z = 31$	02
Ans	$x = \frac{30 - 2y - 3z}{8}$ $y = \frac{1 - x - 2z}{-9}$ $z = \frac{31 - 2x - 3y}{6}$ <p>Initial approximations : $x_0 = y_0 = z_0 = 0$</p> $x = 3.75$ $y = 0.306$ $z = 3.764$	1	
1.	l)	Show that the root of the equation $xe^x - 3 = 0$ lies in the interval(1, 2)	02
	Ans	<p>Let $f(x) = xe^x - 3$</p> $f(1) = -0.282 < 0$ $f(2) = 11.778 > 0$ \therefore root lies between 1 and 2	1 1
2.		Attempt any <u>FOUR</u> of the following:	16
	a)	Find modulus and argument of $-3 + 3i$	04
Ans	$z = -3 + 3i$ $\therefore r = z = \sqrt{(-3)^2 + (3)^2} = \sqrt{18} = 3\sqrt{2}$	2	



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2.	a)	$\theta = \pi - \tan^{-1} \left \frac{3}{-3} \right $	1
		$= \pi - \tan^{-1}(1)$	
		$= \pi - \frac{\pi}{4}$	1
		$= \frac{3\pi}{4}$	1
		OR	
		$\theta = \tan^{-1} \left(\frac{3}{-3} \right)$	1
		$= \tan^{-1}(-1)$	
		$= -\frac{\pi}{4}$	1
	b)	Using De-Moivre's Theorem , simplify,	
		$\frac{(\cos \theta - i \sin \theta)^5 (\cos 3\theta + i \sin 3\theta)^{-4}}{(\cos 3\theta + i \sin 3\theta)^{-2} (\cos 5\theta - i \sin 5\theta)^3}$	04
	Ans	$\frac{(\cos \theta - i \sin \theta)^5 (\cos 3\theta + i \sin 3\theta)^{-4}}{(\cos 3\theta + i \sin 3\theta)^{-2} (\cos 5\theta - i \sin 5\theta)^3}$	
		$= \frac{(\cos \theta + i \sin \theta)^{-5} (\cos \theta + i \sin \theta)^{-12}}{(\cos \theta + i \sin \theta)^{-6} (\cos \theta + i \sin \theta)^{-15}}$	2
		$= (\cos \theta + i \sin \theta)^{-5-12+6+15}$	1
		$= (\cos \theta + i \sin \theta)^4$	½
		$= \cos 4\theta + i \sin 4\theta$	½
		Note:If the student has considered l or i and attempted to solve give appropriate marks.	
	c)	Find all required roots of $(-1)^{\frac{1}{5}}$ using De-Moivre's Theorem	04
	Ans	Let $x = (-1)^{\frac{1}{5}} \therefore x^5 = -1$	
		Let $z = -1 = -1 + 0i$	
		$\therefore r = \sqrt{(-1)^2 + 0^2} = 1$	½
		$\therefore \theta = \pi - \tan^{-1} \left \frac{0}{-1} \right = \pi$	½
		In general polar form	



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2.	c)	$z = \cos(2\pi k + \theta) + i \sin(2\pi k + \theta)$ $\therefore -1 = \cos(2\pi k + \pi) + i \sin(2\pi k + \pi)$ $\therefore (-1)^{\frac{1}{5}} = \left[\cos(2\pi k + \pi) + i \sin(2\pi k + \pi) \right]^{\frac{1}{5}}$ $= \cos\left(\frac{2\pi k + \pi}{5}\right) + i \sin\left(\frac{2\pi k + \pi}{5}\right)$ <p>For $k = 0$, $z_1 = \cos\left(\frac{\pi}{5}\right) + i \sin\left(\frac{\pi}{5}\right)$</p> <p>For $k = 1$, $z_2 = \cos\left(\frac{3\pi}{5}\right) + i \sin\left(\frac{3\pi}{5}\right)$</p> <p>For $k = 2$, $z_3 = \cos(\pi) + i \sin(\pi)$</p> <p>For $k = 3$, $z_4 = \cos\left(\frac{7\pi}{5}\right) + i \sin\left(\frac{7\pi}{5}\right)$</p> <p>For $k = 4$, $z_5 = \cos\left(\frac{9\pi}{5}\right) + i \sin\left(\frac{9\pi}{5}\right)$</p>	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>
	d)	<p>If $\cos(A + iB) = x + iy$ show that $\frac{x^2}{\cos^2 A} - \frac{y^2}{\sin^2 A} = 1$ and</p> $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$	04
	Ans	$\cos(A + iB) = x + iy$ $\cos A \cos(iB) - \sin A \sin(iB) = x + iy$ $\cos A \cosh B - i \sin A \sinh B = x + iy$ <p>$\therefore x = \cos A \cosh B$ and $y = -\sin A \sinh B$</p> <p>i) $\frac{x^2}{\cos^2 A} - \frac{y^2}{\sin^2 A} = \frac{\cos^2 A \cosh^2 B}{\cos^2 A} - \frac{\sin^2 A \sinh^2 B}{\sin^2 A}$</p> $= \cosh^2 B - \sinh^2 B$ $= 1$	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2} + \frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>
		<p>ii) $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = \frac{\cos^2 A \cosh^2 B}{\cosh^2 B} + \frac{\sin^2 A \sinh^2 B}{\sinh^2 B}$</p> $= \cos^2 A + \sin^2 A$ $= 1$	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>
		Note: If the student has considered l or i and attempted to solve give appropriate marks.	



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2.	e)	If $f(x) = y = \frac{ax+1}{5x-a}$ show that $f(y) = x$	04
	Ans	$f(x) = \frac{ax+1}{5x-a}$ $\therefore f(y) = \frac{ay+1}{5y-a}$ $= \frac{a\left(\frac{ax+1}{5x-a}\right)+1}{5\left(\frac{ax+1}{5x-a}\right)-a}$ $= \frac{\left(\frac{a^2x+a+5x-a}{5x-a}\right)}{\left(\frac{5ax+5-5ax+a^2}{5x-a}\right)}$ $= \frac{a^2x+5x}{5+a^2}$ $= \frac{x(a^2+5)}{5+a^2}$ $= x$ $\therefore f(y) = x$	<p>½</p> <p>½</p> <p>1</p> <p>1</p> <p>1</p>
	f)	If $f(x) = \log\left(\frac{x-1}{x}\right)$ show that $f(y^2) = f(y) + f(-y)$	04
	Ans	$LHS = f(y^2)$ $= \log\left(\frac{y^2-1}{y^2}\right)$ $= \log(y^2-1) - \log(y^2)$ $= \log(y^2-1) - 2\log y \quad \dots\dots\dots(1)$ $RHS = f(y) + f(-y)$ $= \log\left(\frac{y-1}{y}\right) + \log\left(\frac{-y-1}{-y}\right)$ $= \log\left(\frac{y-1}{y}\right) + \log\left(\frac{y+1}{y}\right)$ $= \log(y-1) - \log y + \log(y+1) - \log y$ $= \log(y-1) + \log(y+1) - 2\log y$	<p>½</p> <p>½</p> <p>½</p> <p>½</p> <p>½</p>



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2.	f)	$= \log(y^2 - 1) - 2 \log y \dots\dots\dots(2)$	$\frac{1}{2}$
		From (1) and (2)	
		$\therefore f(y^2) = f(y) + f(-y)$	$\frac{1}{2}$
		OR	
		$f(y^2) = \log\left(\frac{y^2 - 1}{y^2}\right)$	$\frac{1}{2}$
		$= \log(y^2 - 1) - \log(y^2)$	$\frac{1}{2}$
		$= \log((y - 1)(y + 1)) - 2 \log y$	$\frac{1}{2}$
		$= \log(y - 1) + \log(y + 1) - 2 \log y$	$\frac{1}{2}$
		$= [\log(y - 1) - \log y] + [\log(y + 1) - \log y]$	$\frac{1}{2}$
		$= \log\left(\frac{y - 1}{y}\right) + \log\left(\frac{y + 1}{y}\right)$	$\frac{1}{2}$
		$= \log\left(\frac{y - 1}{y}\right) + \log\left(\frac{-1 - y}{-y}\right)$	$\frac{1}{2}$
		$= f(y) + f(-y)$	$\frac{1}{2}$
		$\therefore f(y^2) = f(y) + f(-y)$	
		OR	
		$f(y) + f(-y) = \log\left(\frac{y - 1}{y}\right) + \log\left(\frac{-y - 1}{-y}\right)$	1
		$= \log\left(\frac{y - 1}{y}\right) + \log\left(\frac{y + 1}{y}\right)$	$\frac{1}{2}$
		$= \log\left[\left(\frac{y - 1}{y}\right) \times \left(\frac{y + 1}{y}\right)\right]$	1
		$= \log\left(\frac{y^2 - 1}{y^2}\right)$	1
		$= f(y^2)$	$\frac{1}{2}$
3.		Attempt any <u>FOUR</u> of the following :	16
	a)	If $f(x) = \frac{x-1}{x+1}, x \neq -1$, show that $f\left(\frac{x-1}{x+1}\right) = -\frac{1}{x}$	04
	Ans	$f(x) = \frac{x-1}{x+1}$	



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3.	a)	$f\left(\frac{x-1}{x+1}\right) = \frac{\left(\frac{x-1}{x+1}\right) - 1}{\left(\frac{x-1}{x+1}\right) + 1}$ $= \frac{x-1-x-1}{x-1+x+1}$ $= \frac{-2}{2x}$ $= \frac{-1}{x}$	1 1 1 1
	b)	For what values of x , $f(x) = f(2x+1)$ if $f(x) = x^2 - 3x + 4$	04
	Ans	$f(x) = f(2x+1)$ $x^2 - 3x + 4 = (2x+1)^2 - 3(2x+1) + 4$ $x^2 - 3x + 4 = 4x^2 + 4x + 1 - 6x - 3 + 4$ $3x^2 + x - 2 = 0$ $x = -1 \quad \text{or} \quad x = \frac{2}{3}$	1 1 1 1
c)	Evaluate $\lim_{x \rightarrow 3} \left(\frac{1}{x-3} - \frac{1}{x^2 - 5x + 6} \right)$	04	
Ans	$\lim_{x \rightarrow 3} \left(\frac{1}{x-3} - \frac{1}{x^2 - 5x + 6} \right)$ $= \lim_{x \rightarrow 3} \left(\frac{1}{x-3} - \frac{1}{(x-2)(x-3)} \right)$ $= \lim_{x \rightarrow 3} \frac{1}{x-3} \left(1 - \frac{1}{x-2} \right)$ $= \lim_{x \rightarrow 3} \frac{1}{x-3} \left(\frac{x-2-1}{x-2} \right)$ $= \lim_{x \rightarrow 3} \frac{x-3}{(x-3)(x-2)}$ $= \lim_{x \rightarrow 3} \frac{1}{x-2}$ $= 1$	1 1 1 1	



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3.	d)	Evaluate $\lim_{x \rightarrow \infty} x \left[\sqrt{x^2 + 1} - \sqrt{x^2 - 1} \right]$	04
	Ans	$\lim_{x \rightarrow \infty} x \left[\sqrt{x^2 + 1} - \sqrt{x^2 - 1} \right]$ $= \lim_{x \rightarrow \infty} x \left[\sqrt{x^2 + 1} - \sqrt{x^2 - 1} \right] \times \frac{\left[\sqrt{x^2 + 1} + \sqrt{x^2 - 1} \right]}{\left[\sqrt{x^2 + 1} + \sqrt{x^2 - 1} \right]}$ $= \lim_{x \rightarrow \infty} \frac{x \left[\left(\sqrt{x^2 + 1} \right)^2 - \left(\sqrt{x^2 - 1} \right)^2 \right]}{\left[\sqrt{x^2 + 1} + \sqrt{x^2 - 1} \right]}$ $= \lim_{x \rightarrow \infty} \frac{x \left(x^2 + 1 - x^2 + 1 \right)}{\left[\sqrt{x^2 + 1} + \sqrt{x^2 - 1} \right]}$ $= \lim_{x \rightarrow \infty} \frac{2x}{\left[\sqrt{x^2 + 1} + \sqrt{x^2 - 1} \right]}$ $= \lim_{x \rightarrow \infty} \frac{2x}{x \left[\sqrt{1 + \frac{1}{x^2}} + \sqrt{1 - \frac{1}{x^2}} \right]}$ $= \lim_{x \rightarrow \infty} \frac{2}{\left[\sqrt{1 + \frac{1}{x^2}} + \sqrt{1 - \frac{1}{x^2}} \right]}$ $= \frac{2}{\sqrt{1+0} + \sqrt{1-0}}$ $= \frac{2}{2} = 1$ <p>OR</p> $\lim_{x \rightarrow \infty} x \left[\sqrt{x^2 + 1} - \sqrt{x^2 - 1} \right]$ <p>Put $x = \frac{1}{t}$ as $x \rightarrow \infty, t \rightarrow 0$</p> $\lim_{t \rightarrow 0} \frac{1}{t} \left[\sqrt{\left(\frac{1}{t}\right)^2 + 1} - \sqrt{\left(\frac{1}{t}\right)^2 - 1} \right]$ $= \lim_{t \rightarrow 0} \frac{1}{t} \frac{1}{t} \left[\sqrt{1+t^2} - \sqrt{1-t^2} \right]$ $= \lim_{t \rightarrow 0} \frac{1}{t^2} \left[\sqrt{1+t^2} - \sqrt{1-t^2} \right] \times \frac{\left[\sqrt{1+t^2} + \sqrt{1-t^2} \right]}{\left[\sqrt{1+t^2} + \sqrt{1-t^2} \right]}$	<p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>



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3.	d)	$= \lim_{t \rightarrow 0} \frac{1}{t^2} \frac{[1+t^2 - 1+t^2]}{[\sqrt{1+t^2} + \sqrt{1-t^2}]}$ $= \lim_{t \rightarrow 0} \frac{1}{t^2} \frac{2t^2}{[\sqrt{1+t^2} + \sqrt{1-t^2}]}$ $= \lim_{t \rightarrow 0} \frac{2}{[\sqrt{1+t^2} + \sqrt{1-t^2}]}$ $= \frac{2}{[\sqrt{1+0} + \sqrt{1-0}]}$ $= \frac{2}{2} = 1$	<p>½</p> <p>½</p> <p>½</p> <p>½</p>
	f)	<p>Evaluate $\lim_{x \rightarrow 0} \frac{\cos 3x - \cos 5x}{x^2}$</p> <p>Ans $\lim_{x \rightarrow 0} \frac{\cos 3x - \cos 5x}{x^2}$</p> $= \lim_{x \rightarrow 0} \frac{-2 \sin\left(\frac{3x+5x}{2}\right) \sin\left(\frac{3x-5x}{2}\right)}{x^2}$ $= \lim_{x \rightarrow 0} \frac{2 \sin 4x \sin x}{x \cdot x}$ $= 2 \left(\lim_{x \rightarrow 0} \frac{\sin 4x}{4x} \right) 4 \left(\lim_{x \rightarrow 0} \frac{\sin x}{x} \right)$ $= 2(1)4(1)$ $= 8$ <p>OR</p> $\lim_{x \rightarrow 0} \frac{\cos 3x - \cos 5x}{x^2}$ $= \lim_{x \rightarrow 0} \frac{-2 \sin\left(\frac{3x+5x}{2}\right) \sin\left(\frac{3x-5x}{2}\right)}{x^2}$ $= \lim_{x \rightarrow 0} \frac{-2 \sin 4x \sin(-x)}{x \cdot x}$ $= -2 \left(\lim_{x \rightarrow 0} \frac{\sin 4x}{4x} \right) \times 4 \times \left(\lim_{x \rightarrow 0} \frac{\sin(-x)}{(-x)} \right) (-1)$ $= -2(1)4(1)(-1)$ $= 8$	<p>04</p> <p>1</p> <p>½</p> <p>1</p> <p>1</p> <p>½</p> <p>1</p> <p>½</p> <p>1</p> <p>½</p>



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Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	f)	Evaluate $\lim_{x \rightarrow 0} \frac{10^x - 2^x - 5^x + 1}{x \tan x}$	04
	Ans	$\lim_{x \rightarrow 0} \frac{10^x - 2^x - 5^x + 1}{x \tan x}$ $= \lim_{x \rightarrow 0} \frac{5^x 2^x - 5^x - 2^x + 1}{x \tan x}$ $= \lim_{x \rightarrow 0} \frac{5^x (2^x - 1) - (2^x - 1)}{x \tan x}$ $= \lim_{x \rightarrow 0} \frac{(5^x - 1)(2^x - 1)}{x \tan x}$ $= \lim_{x \rightarrow 0} \frac{(5^x - 1)(2^x - 1)}{\frac{x^2}{x^2}}$ $= \frac{\lim_{x \rightarrow 0} \left(\frac{5^x - 1}{x} \right) \lim_{x \rightarrow 0} \left(\frac{2^x - 1}{x} \right)}{\lim_{x \rightarrow 0} \frac{\tan x}{x}}$ $= (\log 5)(\log 2)$	<p>½</p> <p>½</p> <p>1</p> <p>1</p> <p>1</p>
4.		Attempt any FOUR of the following:	16
	a)	By using first principle find the derivative of $y = \log x$	04
	Ans	$\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{\log(x+h) - \log x}{h}$ $\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{1}{h} \log \left(\frac{x+h}{x} \right)$ $\frac{dy}{dx} = \lim_{h \rightarrow 0} \log \left(1 + \frac{h}{x} \right)^{\frac{1}{h}}$ $\frac{dy}{dx} = \left[\lim_{h \rightarrow 0} \log \left(1 + \frac{h}{x} \right)^{\frac{x}{h}} \right]^{\frac{1}{x}}$ $\frac{dy}{dx} = \log e^{\frac{1}{x}}$ $\frac{dy}{dx} = \frac{1}{x} \log e = \frac{1}{x}$	<p>1</p> <p>½</p> <p>½</p> <p>½</p> <p>1</p> <p>½</p>



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Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	b)	<p>If u and v are differentiable functions of x and $y = \frac{u}{v}$, where $v \neq 0$ then prove that</p> $\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$ <p>Ans Let $\delta u, \delta v, \delta y$ are small increments in u, v, y respectively corresponding to increment δx in x.</p> $\therefore y + \delta y = \frac{u + \delta u}{v + \delta v}$ $\delta y = \frac{uv + v\delta u - u(v + \delta v)}{v(v + \delta v)}$ $\delta y = \frac{v\delta u - u\delta v}{v^2 + v\delta v}$ $\frac{\delta y}{\delta x} = \frac{v\delta u - u\delta v}{v^2 + v\delta v}$ $\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \frac{v \lim_{\delta x \rightarrow 0} \frac{\delta u}{\delta x} - u \lim_{\delta x \rightarrow 0} \frac{\delta v}{\delta x}}{v^2}$ $\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \quad (\because \text{as } \delta x \rightarrow 0, \delta v \rightarrow 0)$	<p>04</p> <p>½</p> <p>1</p> <p>½</p> <p>1</p> <p>1</p>
	c)	<p>If $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ then show that $\frac{dy}{dx} = 1 - y^2$</p> <p>Ans $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$</p> <p>LHS = $\frac{dy}{dx}$</p> $= \frac{(e^x + e^{-x}) \frac{d(e^x - e^{-x})}{dx} - (e^x - e^{-x}) \frac{d(e^x + e^{-x})}{dx}}{(e^x + e^{-x})^2}$ $= \frac{(e^x + e^{-x})(e^x + e^{-x}) - (e^x - e^{-x})(e^x - e^{-x})}{(e^x + e^{-x})^2}$ $= \frac{e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x}}{(e^x + e^{-x})^2}$ $= \frac{4}{(e^x + e^{-x})^2} \quad \dots\dots(1)$	<p>04</p> <p>1</p> <p>1</p>



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4.	c)	RHS = $1 - y^2$	
		$= 1 - \left(\frac{e^x - e^{-x}}{e^x + e^{-x}} \right)^2$	½
		$= 1 - \frac{e^{2x} - 2 + e^{-2x}}{e^{2x} + 2 + e^{-2x}}$	
		$= \frac{e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x}}{e^{2x} + 2 + e^{-2x}}$	½
		$= \frac{4}{(e^x + e^{-x})^2} \dots\dots(2)$	½
		From (1) and (2)	
		$\therefore \frac{dy}{dx} = 1 - y^2$	½
		OR	
		$y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$	
		$\log y = \log \left(\frac{e^x - e^{-x}}{e^x + e^{-x}} \right)$	½
$\log y = \log(e^x - e^{-x}) - \log(e^x + e^{-x})$	½		
$\frac{1}{y} \frac{dy}{dx} = \frac{1}{e^x - e^{-x}} (e^x + e^{-x}) - \frac{1}{e^x + e^{-x}} (e^x - e^{-x})$	1		
$\frac{dy}{dx} = y \left[\frac{e^x + e^{-x}}{e^x - e^{-x}} - \frac{e^x - e^{-x}}{e^x + e^{-x}} \right]$	½		
$\frac{dy}{dx} = y \left[\frac{1}{y} - y \right]$	1		
$\frac{dy}{dx} = 1 - y^2$	½		
d)		If $y = \tan^{-1} \left[\frac{\sin 2x}{1 - \cos 2x} \right]$ Find $\frac{dy}{dx}$	04
Ans		$y = \tan^{-1} \left[\frac{\sin 2x}{1 - \cos 2x} \right]$	
		$y = \tan^{-1} \left[\frac{2 \sin x \cos x}{2 \sin^2 x} \right]$	1
		$y = \tan^{-1} \left[\frac{\cos x}{\sin x} \right]$	½



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Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	d)	$y = \tan^{-1}[\cot x] = \tan^{-1}\left[\tan\left(\frac{\pi}{2} - x\right)\right]$ $y = \frac{\pi}{2} - x$ $\frac{dy}{dx} = -1$	$\frac{1}{2}$ 1 1
	e)	Find derivative of $(\sin^{-1} x)^{\cos x}$	04
	Ans	$y = (\sin^{-1} x)^{\cos x}$ $\log y = \log(\sin^{-1} x)^{\cos x}$ $\log y = \cos x \log(\sin^{-1} x)$ $\frac{1}{y} \frac{dy}{dx} = \cos x \frac{1}{\sin^{-1} x} \frac{1}{\sqrt{1-x^2}} + \log(\sin^{-1} x)(-\sin x)$ $\frac{dy}{dx} = y \left[\frac{\cos x}{\sin^{-1} x \sqrt{1-x^2}} - \sin x \log(\sin^{-1} x) \right]$	$\frac{1}{2}$ $\frac{1}{2}$ 2½ $\frac{1}{2}$
f)	Differentiate $5^{\sqrt{x}}$ w.r.t. $(\sqrt{x})^x$	04	
Ans	Let $u = 5^{\sqrt{x}}$ $\log u = \log 5^{\sqrt{x}}$ $\log u = \sqrt{x} \log 5$ $\frac{1}{u} \frac{du}{dx} = \frac{1}{2\sqrt{x}} \log 5$ $\frac{du}{dx} = \frac{u}{2\sqrt{x}} \log 5$ $\frac{du}{dx} = \frac{5^{\sqrt{x}}}{2\sqrt{x}} \log 5$ and $v = (\sqrt{x})^x$ $\log v = \log(\sqrt{x})^x$ $\log v = x \log(\sqrt{x})$ $\log v = x \log x^{\frac{1}{2}}$ $\log v = \frac{1}{2} x \log x$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	



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Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	f)	$\frac{1}{v} \frac{dv}{dx} = \frac{1}{2} \left[x \frac{1}{x} + \log x \right]$	½
		$\frac{dv}{dx} = \frac{v}{2} [1 + \log x]$	
		$\frac{dv}{dx} = \frac{(\sqrt{x})^x}{2} [1 + \log x]$	½
		$\frac{du}{dv} = \frac{du}{dx} \cdot \frac{dx}{dv}$	
		$\frac{du}{dv} = \frac{5^{\sqrt{x}} \log 5}{2\sqrt{x}}$	½
		$\frac{du}{dv} = \frac{5^{\sqrt{x}} \log 5}{\sqrt{x} (\sqrt{x})^x [1 + \log x]}$	
		OR	1½
		Let $u = 5^{\sqrt{x}}$	
		$\frac{du}{dx} = 5^{\sqrt{x}} \log 5 \frac{1}{2\sqrt{x}}$	
		$\frac{du}{dx} = \frac{5^{\sqrt{x}}}{2\sqrt{x}} \log 5$	
		and $v = (\sqrt{x})^x$	
		$\log v = \log (\sqrt{x})^x$	
$\log v = x \log (\sqrt{x})$			
$\frac{1}{v} \frac{dv}{dx} = x \frac{1}{\sqrt{x}} \frac{1}{2\sqrt{x}} + \log (\sqrt{x})$	½		
$\frac{dv}{dx} = v \left(\frac{1}{2} + \log (\sqrt{x}) \right)$			
$\frac{dv}{dx} = (\sqrt{x})^x \left(\frac{1}{2} + \frac{1}{2} \log x \right)$	½		
$\frac{dv}{dx} = \frac{(\sqrt{x})^x}{2} [1 + \log x]$			



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Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	f)	$\frac{du}{dv} = \frac{dx}{dv} \cdot \frac{du}{dx}$ $\frac{du}{dv} = \frac{5^{\sqrt{x}} \log 5}{\frac{(\sqrt{x})^x}{2} [1 + \log x]}$ $\frac{du}{dv} = \frac{5^{\sqrt{x}} \log 5}{\sqrt{x} (\sqrt{x})^x [1 + \log x]}$	½
5.		<p>Attempt any FOUR of the following:</p> <p>a) Evaluate $\lim_{x \rightarrow 0} \frac{\log(e+x) - 1}{x}$</p> <p>Ans $\lim_{x \rightarrow 0} \frac{\log(e+x) - 1}{x} = \lim_{x \rightarrow 0} \frac{\log(e+x) - \log e}{x}$</p> $= \lim_{x \rightarrow 0} \frac{\log\left(\frac{e+x}{e}\right)}{x}$ $= \lim_{x \rightarrow 0} \log\left(1 + \frac{x}{e}\right)^{\frac{1}{x}}$ $= \lim_{x \rightarrow 0} \left[\log\left(1 + \frac{x}{e}\right)^{\frac{e}{x}} \right]^{\frac{1}{e}}$ $= \log e^{\frac{1}{e}} = \frac{1}{e}$	16 04 ½ ½ 1 1 1
	b)	<p>Evaluate $\lim_{x \rightarrow 0} \frac{\sin 3x + 7x}{4x + \sin 2x}$</p> <p>Ans $\lim_{x \rightarrow 0} \frac{\sin 3x + 7x}{4x + \sin 2x}$</p> $= \lim_{x \rightarrow 0} \frac{\sin 3x + 7x}{\frac{x}{4x + \sin 2x}}$	04 1



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No.	Sub Q. N.	Answer	Marking Scheme
5.	b)	$\lim_{x \rightarrow 0} \frac{\left(\frac{\sin 3x}{3} \times 3\right) + 7}{4 + \left(\frac{\sin 2x}{2} \times 2\right)}$ $= \frac{(1 \times 3) + 7}{4 + (1 \times 2)}$ $= \frac{10}{6} = \frac{5}{3} \text{ or } 1.667$	1 1 1
	c)	<p>Using Bisection method find the approximate root of the equation $x^3 - 6x + 3 = 0$ (Perform three iterations)</p>	04
	Ans	$x^3 - 6x + 3 = 0$ $f(x) = x^3 - 6x + 3$ $f(0) = 3 > 0$ $f(1) = -2 < 0$ <p>root is in (0,1)</p> $\therefore x_1 = \frac{0+1}{2} = 0.5$ $\therefore f(0.5) = 0.125 > 0$ <p>\therefore root is in (0.5,1)</p> $\therefore x_2 = \frac{0.5+1}{2} = 0.75$ $\therefore f(0.75) = -1.078 < 0$ <p>\therefore root is in (0.5,0.75)</p> $\therefore x_3 = \frac{0.75+0.5}{2} = 0.625$ <p>OR</p> $x^3 - 6x + 3 = 0$ $f(x) = x^3 - 6x + 3$ $f(0) = 3 > 0$ $f(1) = -2 < 0$ <p>root is in (0,1)</p>	1 1 1 1 1



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No.	Sub Q. N.	Answer				Marking Scheme
5.	c)	a	b	$x = \frac{a+b}{2}$	$f(x)$	1+1+1
		0	1	0.5	0.125	
		0.5	1	0.75	-1.078	
		0.5	0.75	0.625	-----	
		<p>OR</p> $x^3 - 6x + 3 = 0$ $f(x) = x^3 - 6x + 3$ $f(2) = -1 < 0$ $f(3) = 12 > 0$ <p>root is in (2,3)</p> $\therefore x_1 = \frac{2+3}{2} = 2.5$ $\therefore f(2.5) = 3.625 > 0$ <p>\therefore root is in (2.5,2)</p> $\therefore x_2 = \frac{2.5+2}{2} = 2.25$ $\therefore f(2.25) = 0.891 > 0$ <p>\therefore root is in (2.25,2)</p> $\therefore x_3 = \frac{2.25+2}{2} = 2.125$				1
		<p>OR</p> $x^3 - 6x + 3 = 0$ $f(x) = x^3 - 6x + 3$ $f(2) = -1 < 0$ $f(3) = 12 > 0$ <p>root is in (2,3)</p>				1
		a	b	$x = \frac{a+b}{2}$	$f(x)$	1+1+1
		2	3	2.5	3.625	
		2	2.5	2.25	0.891	
		2	2.25	2.125	-----	



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Q. No.	Sub Q. N.	Answer	Marking Scheme																
5.	c)	<p>OR</p> $x^3 - 6x + 3 = 0$ $f(x) = x^3 - 6x + 3$ $f(-2) = 7 > 0$ $f(-3) = -6 < 0$ <p>root is in $(-3, -2)$</p> $\therefore x_1 = \frac{-2-3}{2} = -2.5$ $\therefore f(-2.5) = 2.375 > 0$ <p>\therefore root is in $(-3, -2.5)$</p> $\therefore x_2 = \frac{-2.5-3}{2} = -2.75$ $\therefore f(-2.75) = -1.297 < 0$ <p>\therefore root is in $(-2.75, -2.5)$</p> $\therefore x_3 = \frac{-2.75-2.5}{2} = -2.625$ <p>OR</p> $x^3 - 6x + 3 = 0$ $f(x) = x^3 - 6x + 3$ $f(-2) = 7 > 0$ $f(-3) = -6 < 0$ <p>root is in $(-3, -2)$</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>a</th> <th>b</th> <th>$x = \frac{a+b}{2}$</th> <th>$f(x)$</th> </tr> </thead> <tbody> <tr> <td>-3</td> <td>-2</td> <td>-2.5</td> <td>2.375</td> </tr> <tr> <td>-3</td> <td>-2.5</td> <td>-2.75</td> <td>-1.297</td> </tr> <tr> <td>-2.75</td> <td>-2.5</td> <td>-2.625</td> <td>-----</td> </tr> </tbody> </table>	a	b	$x = \frac{a+b}{2}$	$f(x)$	-3	-2	-2.5	2.375	-3	-2.5	-2.75	-1.297	-2.75	-2.5	-2.625	-----	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1+1+1</p>
a	b	$x = \frac{a+b}{2}$	$f(x)$																
-3	-2	-2.5	2.375																
-3	-2.5	-2.75	-1.297																
-2.75	-2.5	-2.625	-----																
	d)	Use Regula-Falsi method, to find approximate root of the equation $x^3 - x - 4 = 0$ (Three iterations)	04																
	Ans	$x^3 - x - 4 = 0$																	



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Q. No.	Sub Q. N.	Answer	Marking Scheme																								
5.	d)	<p>Let $f(x) = x^3 - x - 4$ $f(1) = -4 < 0$ $f(2) = 2 > 0$ \therefore the root is in (1,2) $x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{1(2) - 2(-4)}{2 - (-4)} = 1.667$ $f(x_1) = -1.035 < 0$ \therefore the root is in (1.667,2) $x_2 = \frac{1.667(2) - 2(-1.035)}{2 - (-1.035)} = 1.781$ $f(x_2) = -0.132 < 0$ \therefore the root is in (1.781,2) $x_3 = \frac{1.781(2) - 2(-0.132)}{2 - (-0.132)} = 1.795$</p> <p>OR</p> <p>Let $f(x) = x^3 - x - 4 = 0$ $f(1) = -4 < 0$ $f(2) = 2 > 0$ \therefore the root is in (1,2)</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>a</th> <th>b</th> <th>f(a)</th> <th>f(b)</th> <th>$x = \frac{af(b) - bf(a)}{f(b) - f(a)}$</th> <th>f(x)</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>2</td> <td></td> <td>2</td> <td>1.667</td> <td>-1.035</td> </tr> <tr> <td>1.667</td> <td>2</td> <td>-1.035</td> <td>2</td> <td>1.781</td> <td>-0.132</td> </tr> <tr> <td>1.781</td> <td>2</td> <td>-0.132</td> <td>2</td> <td>1.795</td> <td>-----</td> </tr> </tbody> </table>	a	b	f(a)	f(b)	$x = \frac{af(b) - bf(a)}{f(b) - f(a)}$	f(x)	1	2		2	1.667	-1.035	1.667	2	-1.035	2	1.781	-0.132	1.781	2	-0.132	2	1.795	-----	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1+1+1</p>
a	b	f(a)	f(b)	$x = \frac{af(b) - bf(a)}{f(b) - f(a)}$	f(x)																						
1	2		2	1.667	-1.035																						
1.667	2	-1.035	2	1.781	-0.132																						
1.781	2	-0.132	2	1.795	-----																						
	e)	Use the Newton-Raphson method to evaluate $\sqrt[3]{20}$ (three iterations)	04																								
	Ans	<p>Let $x = \sqrt[3]{20}$ $\therefore x^3 = 20$ Let $f(x) = x^3 - 20$ $f(2) = -12 < 0$ $f(3) = 7 > 0$</p>																									



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No.	Sub Q. N.	Answer	Marking Scheme
5.	e)	$f'(x) = 3x^2$ Initial root $x_0 = 3$ $\therefore f'(3) = 27$ $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 3 - \frac{f(3)}{f'(3)} = 2.741$ $x_2 = 2.74 - \frac{f(2.741)}{f'(2.741)} = 2.715$ $x_3 = 2.71 - \frac{f(2.715)}{f'(2.715)} = 2.714$ OR Let $(x) = x^3 - 20$ $f(2) = -12 < 0$ $f(3) = 7 > 0$ $f'(x) = 3x^2$ Initial root $x_0 = 3$ $\therefore f'(3) = 27$ $x_i = \frac{xf'(x) - f(x)}{f'(x)}$ $= \frac{3x^3 - x^3 + 20}{3x^2}$ $= \frac{2x^3 + 20}{3x^2}$ $x_1 = 2.741$ $x_2 = 2.715$ $x_3 = 2.714$	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>½</p> <p>1</p> <p>½</p> <p>½</p> <p>½</p>
	f)	Using Bisection method find the root of the equation $x^3 - 4x - 9$ in the interval $(2, 3)$ (Perform three iterations)	04
	Ans	$x^3 - 4x - 9 = 0$ $f(x) = x^3 - 4x - 9$ $f(2) = -9 < 0$ $f(3) = 6 > 0$	



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No.	Sub Q. N.	Answer	Marking Scheme																
5.	f)	root is in (2,3)	1																
		$\therefore x_1 = \frac{2+3}{2} = 2.5$	1																
		$\therefore f(2.5) = -3.375 < 0$																	
		\therefore root is in (2.5,3)																	
		$\therefore x_2 = \frac{2.5+3}{2} = 2.75$	1																
		$\therefore f(2.75) = 0.797 > 0$																	
		\therefore root is in (2.75,2.5)																	
		$\therefore x_3 = \frac{2.75+2.5}{2} = 2.625$	1																
		OR																	
		$x^3 - 4x - 9 = 0$																	
		$f(x) = x^3 - 4x - 9$																	
		$f(2) = -9 < 0$																	
		$f(3) = 6 > 0$																	
		root is in (2,3)	1																
		<table border="1" style="width: 100%; text-align: center;"> <thead> <tr> <th>a</th> <th>b</th> <th>$x = \frac{a+b}{2}$</th> <th>$f(x)$</th> </tr> </thead> <tbody> <tr> <td>2</td> <td>3</td> <td>2.5</td> <td>-3.375</td> </tr> <tr> <td>2.5</td> <td>3</td> <td>2.75</td> <td>0.797</td> </tr> <tr> <td>2.5</td> <td>2.75</td> <td>2.625</td> <td>-----</td> </tr> </tbody> </table>	a	b	$x = \frac{a+b}{2}$	$f(x)$	2	3	2.5	-3.375	2.5	3	2.75	0.797	2.5	2.75	2.625	-----	1+1+1
		a	b	$x = \frac{a+b}{2}$	$f(x)$														
2	3	2.5	-3.375																
2.5	3	2.75	0.797																
2.5	2.75	2.625	-----																
6.	a)	Attempt any <u>FOUR</u> of the following:	16																
		If $y = 2 \sin 2x - 5 \cos 2x$ show that $\frac{d^2y}{dx^2} + 4y = 0$	04																
		Ans $y = 2 \sin 2x - 5 \cos 2x$																	
		$\therefore \frac{dy}{dx} = 4 \cos 2x + 10 \sin 2x$	1																
		$\therefore \frac{d^2y}{dx^2} = -8 \sin 2x + 20 \cos 2x$	1																
		$\therefore \frac{d^2y}{dx^2} = -4(2 \sin 2x - 5 \cos 2x)$	$\frac{1}{2}$																



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No.	Sub Q. N.	Answer	Marking Scheme
6.	a)	$\therefore \frac{d^2y}{dx^2} = -4y$ $\therefore \frac{d^2y}{dx^2} + 4y = 0$ <p>OR</p> $y = 2 \sin 2x - 5 \cos 2x$ $\therefore \frac{dy}{dx} = 4 \cos 2x + 10 \sin 2x$ $\therefore \frac{d^2y}{dx^2} = -8 \sin 2x + 20 \cos 2x$ $L.H.S. = \frac{d^2y}{dx^2} + 4y$ $= -8 \sin 2x + 20 \cos 2x + 4(2 \sin 2x - 5 \cos 2x)$ $= -8 \sin 2x + 20 \cos 2x + 8 \sin 2x - 20 \cos 2x$ $= 0$	<p>½</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>
	b)	<p>If $y = \log(\log x)$ show that $x \frac{d^2y}{dx^2} + \frac{dy}{dx} + x \left(\frac{dy}{dx} \right)^2 = 0$</p>	04
	Ans	$y = \log(\log x)$ $\frac{dy}{dx} = \frac{1}{\log x} \cdot \frac{1}{x} = \frac{1}{x \log x}$ $\frac{d^2y}{dx^2} = \frac{(x \log x) \cdot 0 - 1 \left(x \frac{1}{x} + \log x \cdot 1 \right)}{x^2 (\log x)^2}$ $\frac{d^2y}{dx^2} = \frac{-(1 + \log x)}{x^2 (\log x)^2}$ $L.H.S. = x \frac{d^2y}{dx^2} + \frac{dy}{dx} + x \left(\frac{dy}{dx} \right)^2$ $= x \left[\frac{-(1 + \log x)}{x^2 (\log x)^2} \right] + \frac{1}{x \log x} + x \left(\frac{1}{x \log x} \right)^2$ $= \frac{-1}{x (\log x)^2} + \frac{-1}{x (\log x)} + \frac{1}{x (\log x)} + \frac{1}{x (\log x)^2}$ $= 0$ <p>OR</p> $y = \log(\log x)$	<p>1</p> <p>1</p> <p>1</p> <p>1</p>



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No.	Sub Q. N.	Answer	Marking Scheme	
6.	b)	$\therefore \frac{dy}{dx} = \frac{1}{\log x} \cdot \frac{1}{x}$	1	
		$\therefore x \log x \frac{dy}{dx} = 1$ -----(1)		
		$\therefore x \log x \frac{d^2y}{dx^2} + x \frac{1}{x} \frac{dy}{dx} + \log x \frac{dy}{dx} = 0$	1	
		$\therefore x \log x \frac{d^2y}{dx^2} + \frac{dy}{dx} + \log x \frac{dy}{dx} = 0$		
		$\therefore x \frac{d^2y}{dx^2} + \frac{1}{\log x} \frac{dy}{dx} + \frac{dy}{dx} = 0$	1	
			$\therefore x \frac{d^2y}{dx^2} + \left(x \frac{dy}{dx} \right) \frac{dy}{dx} + \frac{dy}{dx} = 0$	
			$\therefore x \frac{d^2y}{dx^2} + \frac{dy}{dx} + x \left(\frac{dy}{dx} \right)^2 = 0$	1

		c)	Using Gauss-elimination method, Solve the equations $2x + 3y + 2z = 2$, $10x + 3y + 4z = 16$, $3x + 6y + z = -6$	04
		Ans	$2x + 3y + 2z = 2$ $10x + 3y + 4z = 16$ $3x + 6y + z = -6$ $10x + 15y + 10z = 10$ $30x + 9y + 12z = 48$ $10x + 3y + 4z = 16$ and $30x + 60y + 10z = -60$ ----- ----- $12y + 6z = -6$ $-51y + 2z = 108$ $\therefore 2y + z = -1$ $4y + 2z = -2$ $51y + 2z = 108$ ----- $55y = 110$ $\therefore x = 1$ $y = -2$ $z = 3$	1 1 1 1



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No.	Sub Q. N.	Answer	Marking Scheme
6.	d)	Using Jacobi's method solve the system of equations: (Perform three iterations) $10x + 2y + z = 9$, $2x + 20y - 2z = -44$, $-2x + 3y + 10z = 22$	04
	Ans	$x = \frac{1}{10}(9 - 2y - z)$ $y = \frac{1}{20}(-44 - 2x + 2z)$ $z = \frac{1}{10}(22 + 2x - 3y)$ Starting with $x_0 = y_0 = z_0 = 0$ $x_1 = 0.9$ $y_1 = -2.2$ $z_1 = 2.2$ $x_2 = 1.12$ $y_2 = -2.07$ $z_2 = 3.04$ $x_3 = 1.01$ $y_3 = -2.008$ $z_3 = 3.045$	1 1 1 1
6.	e)	Using Gauss-seidal method solve the system of equations: $5x - y = 9$, $x - 5y + z = -4$, $y - 5z = 15$ (Perform three iterations)	04
	Ans	$x = \frac{1}{5}(9 + y)$ $y = \frac{1}{5}(4 + x + z)$ $z = \frac{1}{-5}(15 - y)$ Starting with $x_0 = y_0 = z_0 = 0$ $x_1 = 1.8$ $y_1 = 1.16$ $z_1 = -2.768$	1



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6.	e)	$x_2 = 2.032$	1
		$y_2 = 0.653$	
		$z_2 = -2.869$	
	f)	$x_3 = 1.931$	1
		$y_3 = 0.612$	
		$z_3 = -2.878$	
f)	Using Jacobi's method solve the system of equations: $2x + 3y - 4z = 1$, $5x + 9y + 3z = 17$, $8x - 2y - z = 5$ (Perform three iterations)	04	
Ans	$x = \frac{1}{8}(5 + 2y + z)$ $y = \frac{1}{9}(17 - 5x - 3z)$ $z = -\frac{1}{4}(1 - 2x - 3y)$	1	
	Starting with $x_0 = y_0 = z_0 = 0$ $x_1 = 0.625$ $y_1 = 1.889$ $z_1 = -0.25$ $x_2 = 1.066$ $y_2 = 1.625$ $z_2 = 1.479$ $x_3 = 1.216$ $y_3 = 0.804$ $z_3 = 1.502$	1 1 1	



Important Note

In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.
