Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
		Important Instructions to the Examiners: 1) The Answers should be examined by key words and not as word-to-word as given in the model answer scheme. 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate. 3) The language errors such as grammatical, spelling errors should not be given more importance. (Not applicable for subject English and Communication Skills.) 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by the candidate and those in the model answer may vary. The examiner may give credit for any equivalent figure drawn. 5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's Answers and the model answer. 6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidate's understanding. 7) For programming language papers, credit may be given to any other program based on equivalent concept.		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)		**Attempt any TEN of the following:**		
	a)	a) Define (i) Transpose Matrix (ii) Orthogonal Matrix		
	Ans.	**Transpose of Matrix:** It is the matrix formed by given matrix by interchanging rows into columns (or equivalently, columns in rows).	1	
		Orthogonal Matrix: A square matrix A is said to orthogonal matrix, if $A \cdot A^T = I$ or $A^T \cdot A = I$.	1	**2**
		Note: The above said definitions are just sample definitions. Students may express the definitions in other words also. As only definitions are asked, no example is required to elaborate the concept. Justify meaning of the definitions and give appropriate marks. Further if instead of definitions only examples are given no marks to be given.		
		---		
	b)	If $A=\begin{bmatrix}3 & -2\\1 & -1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 2\\-1 & 3\end{bmatrix}$, find $\lvert A-B\rvert$		
	Ans.	$A-B=\begin{bmatrix}3 & -2\\1 & -1\end{bmatrix}-\begin{bmatrix}1 & 2\\-1 & 3\end{bmatrix}$		
		$=\begin{bmatrix}3-1 & -2-2\\1+1 & -1-3\end{bmatrix}$	½	
		$=\begin{bmatrix}2 & -4\\2 & -4\end{bmatrix}$	½	
		$\therefore \lvert A-B\rvert = -8+8=0$	1	**2**
		OR		
		$\lvert A-B\rvert=\left\lvert\begin{bmatrix}3 & -2\\1 & -1\end{bmatrix}-\begin{bmatrix}1 & 2\\-1 & 3\end{bmatrix}\right\rvert$		
		$=\left\lvert\begin{bmatrix}3-1 & -2-2\\1+1 & -1-3\end{bmatrix}\right\rvert$	½	
		$=\left\lvert\begin{bmatrix}2 & -4\\2 & -4\end{bmatrix}\right\rvert$	½	
		$=\begin{vmatrix}2 & -4\\2 & -4\end{vmatrix}$	1	
		$=-8+8=0$		**2**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)	c)	If $A=\begin{bmatrix}1&0&1\\2&-1&3\end{bmatrix}$ and $B=\begin{bmatrix}1&-3\\-2&-1\\3&1\end{bmatrix}$, find AB.		
	Ans.	$AB=\begin{bmatrix}1&0&1\\2&-1&3\end{bmatrix}\begin{bmatrix}1&-3\\-2&-1\\3&1\end{bmatrix}$		
		$=\begin{bmatrix}1-0+3&-3-0+1\\2+2+9&-6+1+3\end{bmatrix}$	1	
		$=\begin{bmatrix}4&-2\\13&-2\end{bmatrix}$	1	**2**
		---		
	d)	Resolve into partial fractions: $\frac{x+1}{x^2-x}$		
	Ans.	$\frac{x+1}{x^2-x}=\frac{x+1}{x(x-1)}=\frac{A}{x}+\frac{B}{x-1}$		
		$\therefore \boxed{x+1=(x-1)A+xB}$		
		$Put\ x=0$		
		$\therefore 0+1=(0-1)A+0$		
		$\therefore 1=-A$		
		$\therefore \boxed{-1=A}$	1	
		$Put\ x-1=0 \quad \therefore x=1$		
		$\therefore 1+1=0A+B$		
		$\therefore \boxed{2=B}$	½	
		$\therefore \boxed{\frac{x+1}{x^2-x}=\frac{-1}{x}+\frac{2}{x-1}}$	½	**2**
		Note for partial fraction problems: The problems of partial fractions could also be solved by the method of "equating equal power coefficients". This method is also applicable. Give appropriate marks in accordance with the scheme of marking in the later problems as the solution by this method is not discussed. For the sake of convenience, the solution of the above problem with the help of this method is illustrated hereunder.		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)		$\frac{x+1}{x^2-x}=\frac{x+1}{x(x-1)}=\frac{A}{x}+\frac{B}{x-1}$ $\therefore x+1=(x-1)A+xB$ $\therefore x+1=xA-A+xB$ $\therefore 1\cdot x+1=(A+B)x-A$ $\therefore A+B=1, \quad -A=1$ $\therefore \boxed{A=-1}$ $\therefore B=1-A=1+1$ $\therefore \boxed{B=2}$ $\therefore \boxed{\frac{1}{x^2-x}=\frac{-1}{x}+\frac{2}{x-1}}$	 1 ½ ½	2
		Note: The above problem can also be solved as follows: $\frac{x+1}{x^2-x}=\frac{x+1}{(x-1)x}=\frac{A}{x-1}+\frac{B}{x}$ In this case, we get $\therefore \boxed{A=2} \quad \boxed{B=-1}$ $\therefore \boxed{\frac{x+1}{x^2-x}=\frac{2}{x-1}+\frac{-1}{x}}$ --		
	e)	Define (i) Compound Angle (ii) Allied Angle		
	Ans.	**Compound Angle:** An angle formed by sum or difference of many angles is said to be compound angle. **Allied Angles:** If the sum or difference of two angles is $n\frac{\pi}{2}$, where n is whole number, the angles are allied angles. --	1 1	2
	f)	Prove that $\frac{1}{1-\sin\theta}+\frac{1}{1+\sin\theta}=2\sec^2\theta$		
	Ans.	$\frac{1}{1-\sin\theta}+\frac{1}{1+\sin\theta}$ $=\frac{1+\sin\theta+1-\sin\theta}{(1-\sin\theta)(1+\sin\theta)} \quad or \quad \frac{1+\sin\theta+1-\sin\theta}{1-\sin^2\theta}$ $=\frac{2}{\cos^2\theta}$ $=2\sec^2\theta$	½ 1 ½	2

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)	g)	If $\angle A = 30^\circ$, verify that $\sin 3A = 3\sin A - 4\sin^3 A$		
	Ans.	$LHS = \sin 3A = \sin 3(30^\circ) = 1$ $RHS = 3\sin A - 4\sin^3 A$ $= 3\sin 30^\circ - 4\sin^3 30^\circ$ $= 3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3$ $= 1 \qquad ...(*)$ **Note (*):** Due to the use of advance scientific calculator, writing directly the step (*) is allowed. No marks to be deducted.	½ ½ ½ ½	**2**
	h)	If $2\sin 50^\circ \cos 70^\circ = \sin A - \sin B$, find A and B.		
	Ans.	$2\sin 50^\circ \cos 70^\circ = \sin A - \sin B$ $\therefore \sin(50^\circ + 70^\circ) + \sin(50^\circ - 70^\circ) = \sin A - \sin B$ $\therefore \sin(120^\circ) + \sin(-20^\circ) = \sin A - \sin B$ $\therefore \sin(120^\circ) - \sin(20^\circ) = \sin A - \sin B$ $\therefore A = 120^\circ$ $B = 20^\circ$	½ ½ ½ ½	**2**
		OR $2\sin 50^\circ \cos 70^\circ = \sin A - \sin B$ $\therefore 2\sin 50^\circ \cos 70^\circ = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ $\therefore \frac{A+B}{2} = 70 \quad and \quad \frac{A-B}{2} = 50$ $\therefore A + B = 140$ $\underline{A - B = 100}$ $\therefore A = 120$ $B = 20$	½ ½ ½ ½	**2**
	i)	Express as product: $\cos 4\theta + \cos 8\theta$		
	Ans.	$\cos 4\theta + \cos 8\theta = 2\cos\left(\frac{4\theta + 8\theta}{2}\right)\cos\left(\frac{4\theta - 8\theta}{2}\right)$ $= 2\cos(6\theta)\cos(-2\theta)$ $= 2\cos 6\theta \cos 2\theta$	1 1	**2**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)	j)	Prove that $2\tan^{-1}\left(\frac{1}{3}\right)=\tan^{-1}\left(\frac{3}{4}\right)$		
	Ans.	$2\tan^{-1}\left(\frac{1}{3}\right)=\tan^{-1}\left(\frac{2\left(\frac{1}{3}\right)}{1-\left(\frac{1}{3}\right)^2}\right) \quad ...(*)$	1	
		$=\tan^{-1}\left(\frac{\frac{2}{3}}{\frac{8}{9}}\right)$		
		$=\tan^{-1}\left(\frac{3}{4}\right) \quad ...(**)$	1	**2**
		OR		
		$2\tan^{-1}\left(\frac{1}{3}\right)=\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{3}\right)$		
		$=\tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{3}}{1-\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)}\right) \quad ...(*)$	1	
		$=\tan^{-1}\left(\frac{6}{8}\right)$		
		$=\tan^{-1}\left(\frac{3}{4}\right) \quad ...(**)$	1	**2**
		Note: Due to the use of non-programmable scientific calculator, the value (**) can be calculated directly after the step (*) instead of doing calculations algebraically.		
		--		
	k)	Prove that the lines $5x-12y+1=0$ and $10x-24y-1=0$ are parallel to each other.		
	Ans.	*For* $5x-12y+1=0$,		
		slope $m_1=-\frac{a}{b}=-\frac{5}{-12}=\frac{5}{12}$	½	
		For $10x-24y-1=0$,		
		slope $m_2=-\frac{a}{b}=-\frac{10}{-24}=\frac{5}{12}$	½	
		$\therefore m_1=m_2$	½	
		$\therefore$ the lines are parallel.	½	**2**
		--		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)	l)	Prove that the lines $2x-3y+1=0$ and $3x+2y-5=0$ are perpendicular to each other.		
	Ans.	$For\ \ 2x-3y+1=0,$		
		$slope\ \ m_1=-\frac{a}{b}=-\frac{2}{-3}=\frac{2}{3}$	½	
		$For\ \ 3x+2y-5=0,$		
		$slope\ \ m_2=-\frac{a}{b}=-\frac{3}{2}$	½	
		$\therefore m_1\cdot m_2=-1\ \ or\ \ m_1\cdot m_2+1=0\ \ or\ \ m_1=\frac{-1}{m_2}$	½	
		$\therefore the\ lines\ are\ perpendicular.$	½	**2**
		--		
	m)	Find x, if $\tan^{-1}(1)+\tan^{-1}(x)=0$.		
	Ans.	$\tan^{-1}(1)+\tan^{-1}(x)=0$		
		$\therefore \tan^{-1}(x)=-\tan^{-1}(1)$		
		$=\tan^{-1}(-1)$	1	
		$\therefore x=-1$	1	**2**
		OR		
		$\tan^{-1}(1)+\tan^{-1}(x)=0$		
		$\therefore \tan^{-1}\left(\frac{1+x}{1-1\cdot x}\right)=0$	½	
		$\therefore \frac{1+x}{1-1\cdot x}=\tan 0=0$	½	
		$\therefore 1+x=0$	½	
		$\therefore x=-1$	½	**2**
		OR		
		$\tan^{-1}(1)+\tan^{-1}(x)=0$		
		$\therefore \frac{\pi}{4}+\tan^{-1}(x)=0$	½	
		$\therefore \tan^{-1}(x)=-\frac{\pi}{4}$	½	
		$\therefore x=\tan\left(-\frac{\pi}{4}\right)$	½	
		$\therefore x=-1$	½	**2**
		--		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2)		**Attempt any FOUR of the following:**		
	a)	Find x using Cramer's rule $x+z=4,\ y+z=2,\quad x+y=0$		
	Ans.	$D=\begin{vmatrix}1&0&1\\0&1&1\\1&1&0\end{vmatrix}=1(0-1)-0+1(0-1)$ $=-2$	1½	
		$D_x=\begin{vmatrix}4&0&1\\2&1&1\\0&1&0\end{vmatrix}=4(0-1)-0+1(2-0)$ $=-2$	1½	
		$\therefore x=\dfrac{D_x}{D}=\dfrac{-2}{-2}=1$ ---	1	**4**
	b)	Find y using Cramer's rule: $x-\dfrac{2}{y}+\dfrac{2}{z}=6,\ 3x+\dfrac{4}{y}-\dfrac{1}{z}=1,\quad 4x+\dfrac{1}{y}-\dfrac{3}{z}=4$		
	Ans.	$D=\begin{vmatrix}1&-2&2\\3&4&-1\\4&1&-3\end{vmatrix}=1(-12+1)+2(-9+4)+2(3-16)$ $=-47$	1½	
		$D_{\frac{1}{y}}=\begin{vmatrix}1&6&2\\3&1&-1\\4&4&-3\end{vmatrix}=1(-3+4)-6(-9+4)+2(12-4)$ $=47$	1½	
		$\therefore \dfrac{1}{y}=\dfrac{D_{\frac{1}{y}}}{D}=\dfrac{47}{-47}=-1$	½	
		$\therefore y=-1$	½	**4**
		OR		
		$Put\ \ \dfrac{1}{y}=p$		
		$D=\begin{vmatrix}1&-2&2\\3&4&-1\\4&1&-3\end{vmatrix}=1(-12+1)+2(-9+4)+2(3-16)$ $=-47$	1½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2)		$D_p = \begin{vmatrix} 1 & 6 & 2 \\ 3 & 1 & -1 \\ 4 & 4 & -3 \end{vmatrix} = 1(-3+4) - 6(-9+4) + 2(12-4)$		
		$= 47$	1½	
		$\therefore p = \dfrac{D_p}{D} = \dfrac{47}{-47} = -1$	½	
		$\therefore y = \dfrac{1}{p} = -1$	½	**4**
		--		
	c)	Solve by determinant method: $\dfrac{2}{x-1} + \dfrac{3}{y-3} = 5,\quad \dfrac{3}{x-1} - \dfrac{4}{y-3} = -1$		
	Ans.	$\dfrac{2}{x-1} + \dfrac{3}{y-3} = 5,\quad \dfrac{3}{x-1} - \dfrac{4}{y-3} = -1$		
		$Put\ \dfrac{1}{x-1} = a \qquad \dfrac{1}{y-3} = b$		
		$\therefore 2a + 3b = 5$		
		$3a - 4b = -1$		
		$\therefore D = \begin{vmatrix} 2 & 3 \\ 3 & -4 \end{vmatrix} = -8 - 9 = -17$	1	
		$D_a = \begin{vmatrix} 5 & 3 \\ -1 & -4 \end{vmatrix} = -20 + 3 = -17$	½	
		$D_b = \begin{vmatrix} 2 & 5 \\ 3 & -1 \end{vmatrix} = -2 - 15 = -17$	½	
		$\therefore \dfrac{1}{x-1} = a = \dfrac{D_a}{D} = \dfrac{-17}{-17} = 1$	½	
		$\therefore 1 = x - 1$		
		$\therefore \boxed{2 = x}$	½	
		$\therefore \dfrac{1}{y-3} = b = \dfrac{D_a}{D} = \dfrac{-17}{-17} = 1$	½	
		$\therefore 1 = y - 3$		
		$\therefore \boxed{4 = y}$	½	**4**
		--		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2)	d)	Show that $\sin 50^o - \sin 70^o + \sin 10^o = 0$		
	Ans.	$\sin 50^o - \sin 70^o + \sin 10^o$		
		$= 2\cos 60^o \sin(-10^o) + \sin 10^o$	1	
		$= -2\cos 60^o \sin 10^o + \sin 10^o$		
		$= -2\left(\frac{1}{2}\right)\sin 10^o + \sin 10^o$	1	
		$= -\sin 10^o + \sin 10^o$	1	
		$= 0$	1	**4**
		OR		
		$\sin 50^o - \sin 70^o + \sin 10^o$		
		$= \sin 50^o + \sin 10^o - \sin 70^o$		
		$= \sin 50^o + 2\cos 40^o \sin(-30^o)$		
		$= \sin 50^o - 2\cos 40^o \sin 30^o$	1	
		$= \sin 50^o - 2\cos 40^o\left(\frac{1}{2}\right)$	1	
		$= \sin 50^o - \cos 40^o$		
		$= \sin 50^o - \cos(90^o - 50^o)$		
		$= \sin 50^o - \sin 50^o$	1	
		$= 0$	1	**4**
		OR		
		$\sin 50^o - \sin 70^o + \sin 10^o$		
		$= \sin 50^o + \sin 10^o - \sin 70^o$		
		$= 2\sin 30^o \cos 20^o - \sin 70^o$	1	
		$= 2\left(\frac{1}{2}\right)\cos 20^o - \sin 70^o$	1	
		$= \cos 20^o - \sin 70^o$		
		$= \cos 20^o - \sin(90^o - 20^o)$		
		$= \cos 20^o - \cos 20^o$	1	
		$= 0$	1	**4**
		--		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2)	e)	Prove that $\tan 15^o + \tan 75^o = 4$		
	Ans.	$\tan 15^o = \tan\left(45^o - 30^o\right) = \dfrac{\tan 45^o - \tan 30^o}{1 + \tan 45^o \tan 30^o}$	½	
		$= \dfrac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \cdot \frac{1}{\sqrt{3}}}$	½	
		$= \dfrac{\sqrt{3} - 1}{\sqrt{3} + 1}$	½	
		$\tan 75^o = \tan\left(45^o + 30^o\right) = \dfrac{\tan 45^o + \tan 30^o}{1 - \tan 45^o \tan 30^o}$	½	
		$= \dfrac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \cdot \frac{1}{\sqrt{3}}}$	½	
		$= \dfrac{\sqrt{3} + 1}{\sqrt{3} - 1}$	½	
		$\therefore \tan 15^o + \tan 75^o = \dfrac{\sqrt{3} - 1}{\sqrt{3} + 1} + \dfrac{\sqrt{3} + 1}{\sqrt{3} - 1}$	½	
		$= \dfrac{\left(\sqrt{3} - 1\right)^2 + \left(\sqrt{3} + 1\right)^2}{\left(\sqrt{3}\right)^2 - 1^2}$	½	**4**
		$= 4$		
		OR		
		$\tan 15^o + \tan 75^o = \tan\left(45^o - 30^o\right) + \tan\left(45^o + 30^o\right)$		
		$= \dfrac{\tan 45^o - \tan 30^o}{1 + \tan 45^o \tan 30^o} + \dfrac{\tan 45^o + \tan 30^o}{1 - \tan 45^o \tan 30^o}$	½+½	
		$= \dfrac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \cdot \frac{1}{\sqrt{3}}} + \dfrac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \cdot \frac{1}{\sqrt{3}}}$	½+½	
		$= \dfrac{\sqrt{3} - 1}{\sqrt{3} + 1} + \dfrac{\sqrt{3} + 1}{\sqrt{3} - 1}$	½+½	
		$= \dfrac{\left(\sqrt{3} - 1\right)^2 + \left(\sqrt{3} + 1\right)^2}{\left(\sqrt{3}\right)^2 - 1^2}$		
		$= 4$	1	**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2)	f)	Prove that $\frac{\sin 7x+\sin x}{\cos 5x-\cos 3x}=\sin 2x-\cos 2x\cot x$		
	Ans.	$\frac{\sin 7x+\sin x}{\cos 5x-\cos 3x}=\frac{2\sin\left(\frac{7x+x}{2}\right)\cos\left(\frac{7x-x}{2}\right)}{-2\sin\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)}$		
		$=\frac{2\sin(4x)\cos(3x)}{-2\sin(4x)\sin(x)}$	1	
		$=\frac{\cos(3x)}{-\sin x}$	1	
		$=\frac{\cos(x+2x)}{-\sin x}$		
		$=\frac{\cos x\cos 2x-\sin x\sin 2x}{-\sin x}$	1	
		$=\frac{\cos x\cos 2x}{-\sin x}-\frac{\sin x\sin 2x}{-\sin x}$		
		$=-\cot x\cos 2x+\sin 2x$		
		$OR\ \sin 2x-\cos 2x\cot x$	1	**4**
		---		
3)		**Attempt any FOUR of the following:**		
	a)	Find the matrix X such that $AX=B$, where $A=\begin{bmatrix}-3 & -2 & -1\\ 6 & 4 & 2\\ 9 & 6 & 3\end{bmatrix}$ and $B=\begin{bmatrix}-1\\ 2\\ 3\end{bmatrix}$		
	Ans.	$\lvert A\rvert=-3(12-12)+2(18-18)-1(36-36)=0$		
		For the given matrix A, $\lvert A\rvert=0.$		
		$\therefore A^{-1}$ *does not exist.*		
		$\therefore X=A^{-1}B$ *has indefinite solutions.*		
		OR in short, solution does not exist.		**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)	b)	Find the value of x and y, if $\begin{bmatrix}1 & 2\\3 & 2\end{bmatrix}\begin{bmatrix}x & 5 & -3\\2 & y & 5\end{bmatrix}=\begin{bmatrix}5 & -3 & 7\\7 & 7 & 1\end{bmatrix}$		
	Ans.	$\begin{bmatrix}1 & 2\\3 & 2\end{bmatrix}\begin{bmatrix}x & 5 & -3\\2 & y & 5\end{bmatrix}=\begin{bmatrix}5 & -3 & 7\\7 & 7 & 1\end{bmatrix}$		
		$\therefore\begin{bmatrix}x+4 & 5+2y & -3+10\\3x+4 & 15+2y & -9+10\end{bmatrix}=\begin{bmatrix}5 & -3 & 7\\7 & 7 & 1\end{bmatrix}$	1	
		$\therefore x+4=5 \quad and \quad 5+2y=-3$		
		$\therefore x=1 \quad and \quad 2y=-8$	1	
		$\therefore \boxed{x=1} \quad and \quad \boxed{y=-4}$	1+1	
		OR	or	
		$\therefore 3x+4=7 \quad and \quad 15+2y=7$	1	
		$\therefore 3x=3 \quad and \quad 2y=-8$		
		$\therefore \boxed{x=1} \quad and \quad \boxed{y=-4}$	1+1	**4**
	c)	If $A=\begin{bmatrix}1 & 2 & 2\\2 & 1 & 2\\2 & 2 & 1\end{bmatrix}$, then show that A^2-4A is a scalar matrix.		
	Ans.	$A^2=A\cdot A=\begin{bmatrix}1 & 2 & 2\\2 & 1 & 2\\2 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 2\\2 & 1 & 2\\2 & 2 & 1\end{bmatrix}$		
		$=\begin{bmatrix}1+4+4 & 2+2+4 & 2+4+2\\2+2+4 & 4+1+4 & 4+2+2\\2+4+2 & 4+2+2 & 4+4+1\end{bmatrix}$	1	
		$=\begin{bmatrix}9 & 8 & 8\\8 & 9 & 8\\8 & 8 & 9\end{bmatrix}$	1	
		$\therefore A^2-4A=\begin{bmatrix}9 & 8 & 8\\8 & 9 & 8\\8 & 8 & 9\end{bmatrix}-4\begin{bmatrix}1 & 2 & 2\\2 & 1 & 2\\2 & 2 & 1\end{bmatrix}$		
		$=\begin{bmatrix}9-4 & 8-8 & 8-8\\8-8 & 9-4 & 8-8\\8-8 & 8-8 & 9-4\end{bmatrix} \quad or \quad \begin{bmatrix}9 & 8 & 8\\8 & 9 & 8\\8 & 8 & 9\end{bmatrix}-\begin{bmatrix}4 & 8 & 8\\8 & 4 & 8\\8 & 8 & 4\end{bmatrix}$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)		$=\begin{bmatrix}5&0&0\\0&5&0\\0&0&5\end{bmatrix}$	1	
		$\therefore A^2-4A$ *is a scalar matrix.*	1	**4**
		--		
	d)	Verify that $(AB)C=A(BC)$, if $A=\begin{bmatrix}1&2\\-2&3\end{bmatrix},\ B=\begin{bmatrix}2&1\\2&3\end{bmatrix},\ C=\begin{bmatrix}-3&1\\2&0\end{bmatrix}$		
	Ans.	$AB=\begin{bmatrix}1&2\\-2&3\end{bmatrix}\begin{bmatrix}2&1\\2&3\end{bmatrix}$		
		$=\begin{bmatrix}2+4&1+6\\-4+6&-2+9\end{bmatrix}$	½	
		$=\begin{bmatrix}6&7\\2&7\end{bmatrix}$	½	
		$(AB)C=\begin{bmatrix}6&7\\2&7\end{bmatrix}\begin{bmatrix}-3&1\\2&0\end{bmatrix}$		
		$=\begin{bmatrix}-18+14&6+0\\-6+14&2+0\end{bmatrix}$	½	
		$=\begin{bmatrix}-4&6\\8&2\end{bmatrix}$	½	
		$BC=\begin{bmatrix}2&1\\2&3\end{bmatrix}\begin{bmatrix}-3&1\\2&0\end{bmatrix}$		
		$=\begin{bmatrix}-6+2&2+0\\-6+6&2+0\end{bmatrix}$	½	
		$=\begin{bmatrix}-4&2\\0&2\end{bmatrix}$	½	
		$A(BC)=\begin{bmatrix}1&2\\-2&3\end{bmatrix}\begin{bmatrix}-4&2\\0&2\end{bmatrix}$		
		$=\begin{bmatrix}-4+0&2+4\\8+0&-4+6\end{bmatrix}$		
		$=\begin{bmatrix}-4&6\\8&2\end{bmatrix}$	½	
		$\therefore (AB)C=A(BC)$	½	**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)		**OR**		
		$(AB)C=\left\{\begin{bmatrix}1&2\\-2&3\end{bmatrix}\begin{bmatrix}2&1\\2&3\end{bmatrix}\right\}\begin{bmatrix}-3&1\\2&0\end{bmatrix}$		
		$=\begin{bmatrix}2+4&1+6\\-4+6&-2+9\end{bmatrix}\begin{bmatrix}-3&1\\2&0\end{bmatrix}$	½	
		$=\begin{bmatrix}6&7\\2&7\end{bmatrix}\begin{bmatrix}-3&1\\2&0\end{bmatrix}$	½	
		$=\begin{bmatrix}-18+14&6+0\\-6+14&2+0\end{bmatrix}$	½	
		$=\begin{bmatrix}-4&6\\8&2\end{bmatrix}$	½	
		$A(BC)=\begin{bmatrix}1&2\\-2&3\end{bmatrix}\left\{\begin{bmatrix}2&1\\2&3\end{bmatrix}\begin{bmatrix}-3&1\\2&0\end{bmatrix}\right\}$		
		$=\begin{bmatrix}1&2\\-2&3\end{bmatrix}\begin{bmatrix}-6+2&2+0\\-6+6&2+0\end{bmatrix}$	½	
		$=\begin{bmatrix}1&2\\-2&3\end{bmatrix}\begin{bmatrix}-4&2\\0&2\end{bmatrix}$	½	
		$=\begin{bmatrix}-4+0&2+4\\8+0&-4+6\end{bmatrix}$		
		$=\begin{bmatrix}-4&6\\8&2\end{bmatrix}$	½	
		$\therefore (AB)C=A(BC)$	½	**4**
		--		
	e)	Prove that $\dfrac{\operatorname{cosec} A}{\operatorname{cosec} A-1}+\dfrac{\operatorname{cosec} A}{\operatorname{cosec} A+1}=2\cdot \sec^2 A$		
	Ans.	$\dfrac{\operatorname{cosec} A}{\operatorname{cosec} A-1}+\dfrac{\operatorname{cosec} A}{\operatorname{cosec} A+1}=\dfrac{\operatorname{cosec} A(\operatorname{cosec} A+1)+\operatorname{cosec} A(\operatorname{cosec} A-1)}{(\operatorname{cosec} A-1)(\operatorname{cosec} A+1)}$		
		$=\dfrac{\operatorname{cosec}^2 A+\operatorname{cosec} A+\operatorname{cosec}^2 A-\operatorname{cosec} A}{\operatorname{cosec}^2 A-1}$	1	
		$=\dfrac{2\operatorname{cosec}^2 A}{\operatorname{cosec}^2 A-1}$	1	
		$=\dfrac{2\operatorname{cosec}^2 A}{\cot^2 A}$ *or* $=\dfrac{\frac{2}{\sin^2 A}}{\frac{1-\sin^2 A}{\sin^2 A}}$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)		$= 2 \cdot \frac{1}{\sin^2 A} \cdot \frac{\sin^2 A}{\cos^2 A}$ *or* $= \frac{2}{1-\sin^2 A}$	1	
		$= 2 \cdot \frac{1}{\cos^2 A}$ *or* $= \frac{2}{\cos^2 A}$		
		$= 2 \cdot \sec^2 A$ *or* $= 2 \cdot \sec^2 A$	1	**4**
		---		
	f)	In any triangle ABC, prove that $\tan A + \tan B + \tan C = \tan A \tan B \tan C$		
	Ans.	*We have,* $A + B + C = 180^\circ$ *or* π		
		$\therefore A + B = 180^\circ - C$		
		$\therefore \tan(A+B) = \tan(180^\circ - C)$	1	
		$\therefore \frac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$	1	
		$\therefore \tan A + \tan B = -\tan C[1 - \tan A \tan B]$		
		$\therefore \tan A + \tan B = -\tan C + \tan A \tan B \tan C$	1	
		$\therefore \tan A + \tan B + \tan C = \tan A \tan B \tan C$	1	**4**
		Note: Instead of taking $A + B = 180^\circ - C$, the problem can also be solved by taking either $A + C = 180^\circ - B$ or $B + C = 180^\circ - A$.		
		---		
4)		**Attempt any FOUR of the following:**		
	a)	Verify that $AA^{-1} = I$, if $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$		
	Ans.	$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$		
		$\therefore \lvert A \rvert = 1(1-0) - 1(0-1) + 1(0-1) = 1$	½	
		Matrix of Cofactor of A is,		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		$C(A)=\begin{bmatrix} \begin{vmatrix}1&1\\0&1\end{vmatrix} & -\begin{vmatrix}0&1\\1&1\end{vmatrix} & \begin{vmatrix}0&1\\1&0\end{vmatrix} \\ -\begin{vmatrix}1&1\\0&1\end{vmatrix} & \begin{vmatrix}1&1\\1&1\end{vmatrix} & -\begin{vmatrix}1&1\\1&0\end{vmatrix} \\ \begin{vmatrix}1&1\\1&1\end{vmatrix} & -\begin{vmatrix}1&1\\0&1\end{vmatrix} & \begin{vmatrix}1&1\\0&1\end{vmatrix} \end{bmatrix}$	1½	
		$=\begin{bmatrix}1&1&-1\\-1&0&1\\0&-1&1\end{bmatrix}$	½	
		(*) Note: In the matrix *C(A)*, if 1 to 3 elements are wrong (either in sign or value), deduct ½ mark, if 4 to 6 elements are wrong, deduct 1 marks, if 7 to 9 are wrong, deduct all the 2 marks. Further, if all the elements in the last i.e., *adj (A)* are correct, then only give ½ mark.		
		OR	OR	
		Matrix of Cofactor of A is,		
		$M(A)=\begin{bmatrix} \begin{vmatrix}1&1\\0&1\end{vmatrix} & \begin{vmatrix}0&1\\1&1\end{vmatrix} & \begin{vmatrix}0&1\\1&0\end{vmatrix} \\ \begin{vmatrix}1&1\\0&1\end{vmatrix} & \begin{vmatrix}1&1\\1&1\end{vmatrix} & \begin{vmatrix}1&1\\1&0\end{vmatrix} \\ \begin{vmatrix}1&1\\1&1\end{vmatrix} & \begin{vmatrix}1&1\\0&1\end{vmatrix} & \begin{vmatrix}1&1\\0&1\end{vmatrix} \end{bmatrix} \quad ---(*)$		
		$=\begin{bmatrix}1&-1&-1\\1&0&-1\\0&1&1\end{bmatrix}$	1½	
		$\therefore C(A)=\begin{bmatrix}1&1&-1\\-1&0&1\\0&-1&1\end{bmatrix}$	½	
		OR	OR	
		$A_{11}=\begin{vmatrix}1&1\\0&1\end{vmatrix}=1 \quad A_{12}=-\begin{vmatrix}0&1\\1&1\end{vmatrix}=1 \quad A_{13}=\begin{vmatrix}0&1\\1&0\end{vmatrix}=-1$		
		$A_{21}=-\begin{vmatrix}1&1\\0&1\end{vmatrix}=-1 \quad A_{22}=\begin{vmatrix}1&1\\1&1\end{vmatrix}=0 \quad A_{23}=-\begin{vmatrix}1&1\\1&0\end{vmatrix}=1$	1½	
		$A_{31}=\begin{vmatrix}1&1\\1&1\end{vmatrix}=0 \quad A_{32}=-\begin{vmatrix}1&1\\0&1\end{vmatrix}=-1 \quad A_{33}=\begin{vmatrix}1&1\\0&1\end{vmatrix}=1$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		**Note:** In the above, if 1 to 3 elements are wrong, deduct ½ mark, if 4 to 6 elements are wrong, deduct 1 marks, and if 7 to 9 are wrong, deduct all the marks. Further, if all the elements in the following matrices C(A) and adj (A) are correct, then only give the marks.		
		$\therefore adj(A)=\begin{bmatrix}1&-1&0\\1&0&-1\\-1&1&1\end{bmatrix}$		
		$\therefore A^{-1}=\frac{1}{\lvert A\rvert}adj(A)$		
		$=\begin{bmatrix}1&-1&0\\1&0&-1\\-1&1&1\end{bmatrix}$	½	
		$\therefore AA^{-1}=\begin{bmatrix}1&1&1\\0&1&1\\1&0&1\end{bmatrix}\begin{bmatrix}1&-1&0\\1&0&-1\\-1&1&1\end{bmatrix}$		
		$=\begin{bmatrix}1+1-1&-1+0+1&0-1+1\\0+1-1&0+0+1&0-1+1\\1+0-1&-1+0+1&0+0+1\end{bmatrix}$	½	
		$=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$		
		$=I$	½	**4**
		---		
	b)	Express the following equations in the matrix form and solve them using matrix inversion method: $x+3y+2z=6,\quad 3x-2y+5z=5,\quad 2x-3y+6z=7$		
	Ans.	$x+3y+2z=6$ $3x-2y+5z=5$ $2x-3y+6z=7$		
		$\therefore A=\begin{bmatrix}1&3&2\\3&-2&5\\2&-3&6\end{bmatrix},\quad X=\begin{bmatrix}x\\y\\z\end{bmatrix},\quad B=\begin{bmatrix}6\\5\\7\end{bmatrix}$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		$\therefore \|A\| = 1(-12+15)-3(18-10)+2(-9+4) = -31$	½	
		$\therefore adj(A) = \begin{bmatrix} 3 & -24 & 19 \\ -8 & 2 & 1 \\ -5 & 9 & -11 \end{bmatrix}$ ----------- (*)	2	
		(*) Note: Many methods are followed to find $adj(A)$ as discussed in the Q. 4 (a). Please give appropriate marks in accordance with the scheme of marking as discussed therein.		
		$\therefore A^{-1} = \frac{1}{\|A\|} adj(A)$ $= \frac{1}{-31}\begin{bmatrix} 3 & -24 & 19 \\ -8 & 2 & 1 \\ -5 & 9 & -11 \end{bmatrix}$	½	
		$\therefore$ the solution is, $X = A^{-1}B$ $= \frac{1}{-31}\begin{bmatrix} 3 & -24 & 19 \\ -8 & 2 & 1 \\ -5 & 9 & -11 \end{bmatrix}\begin{bmatrix} 6 \\ 5 \\ 7 \end{bmatrix}$ $= \frac{1}{-31}\begin{bmatrix} 31 \\ -31 \\ -62 \end{bmatrix}$ $= \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$	½	
		$\therefore x = -1, \quad y = 1, \quad z = 2$	½	**4**
	c)	Resolve into partial fractions: $\frac{x^3+1}{x^2+2x}$		
	Ans.	$\frac{x^3+1}{x^2+2x} = x-2+\frac{4x+1}{x^2+2x}$	1	
		$\therefore \frac{4x+1}{x^2+2x} = \frac{4x+1}{x(x+2)} = \frac{A}{x}+\frac{B}{x+2}$ $\therefore \boxed{4x+1 = (x+2)A + xB}$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		$Put\ \ x=0$ $\therefore 0+1=(0+2)A+0$ $\therefore 1=2A$ $\therefore \boxed{\frac{1}{2}=A}$	1	
		$Put\ \ x+2=0\quad i.e.,\ \ x=-2$ $\therefore 4(-2)+1=0-2B$ $\therefore -7=-2B$ $\therefore \boxed{\frac{7}{2}=B}$	1	
		$\therefore \frac{4x+1}{x^2+2x}=\frac{\frac{1}{2}}{x}+\frac{\frac{7}{2}}{x+2}$	½	
		$\therefore \boxed{\frac{x^3+1}{x^2+2x}=x-2+\frac{\frac{1}{2}}{x}+\frac{\frac{7}{2}}{x+2}}$	½	**4**
		--		
	d)	Resolve into partial fractions: $\frac{x^2+2x+3}{(x^2+2x+2)(x^2+2x+5)}$		
	Ans.	$Put\ \ x^2+2x=t$ $\frac{x^2+2x+3}{(x^2+2x+2)(x^2+2x+5)}=\frac{t+3}{(t+2)(t+5)}=\frac{A}{t+2}+\frac{B}{t+5}$ $\therefore t+3=(t+5)A+(t+2)B$ $Put\ \ t=-2$ $\therefore -2+3=(-2+5)A+0$ $\therefore \boxed{A=\frac{1}{3}}$	1	
		$Put\ \ t=-5$ $\therefore -5+3=0+(-5+2)B$ $\therefore \boxed{B=\frac{2}{3}}$	1	
		$\therefore \frac{t+3}{(t+2)(t+5)}=\frac{\frac{1}{3}}{t+2}+\frac{\frac{2}{3}}{t+5}$	1	
		$\therefore \boxed{\frac{x^2+2x+3}{(x^2+2x+2)(x^2+2x+5)}=\frac{\frac{1}{3}}{x^2+2x+2}+\frac{\frac{2}{3}}{x^2+2x+5}}$	1	**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		**Note:** Various substitutions are possible in the above example. Interestingly, the corresponding values of A and B still remains the same. e. g., $Put\ x^2+2x+3=t$ $\frac{x^2+2x+3}{(x^2+2x+2)(x^2+2x+5)}=\frac{t}{(t-1)(t+2)}=\frac{A}{t-1}+\frac{B}{t+2}$ $\therefore we\ get,$ $\therefore \boxed{A=\frac{1}{3}}\quad and\quad \boxed{B=\frac{2}{3}}$ Further, if the problem is solved by the students in the following manner, give appropriate marks accordingly. $\frac{x^2+2x+3}{(x^2+2x+2)(x^2+2x+5)}=\frac{Ax+B}{x^2+2x+2}+\frac{Cx+D}{x^2+2x+5}$ $\therefore we\ get,$ $\therefore \boxed{A=0},\ \boxed{B=\frac{1}{3}},\ \boxed{C=0},\ and\ \boxed{D=\frac{2}{3}}$ --		
	e)	Resolve into partial fractions: $\frac{x^2}{(x+1)(x+2)^2}$		
	Ans.	$\frac{x^2}{(x+1)(x+2)^2}=\frac{A}{x+1}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$ $\therefore \boxed{x^2=(x+2)^2A+(x+2)(x+1)B+(x+1)C}$ $Put\ x+1=0\quad i.e.,\ x=-1$ $\therefore (-1)^2=(-1+2)^2A+0+0$ $\therefore \boxed{1=A}$	1	
		$Put\ x+2=0\quad i.e.,\ x=-2$ $\therefore (-2)^2=0+0+(-2+1)C$ $\therefore 4=-C$ $\therefore \boxed{-4=C}$	1	
		$Put\ x=0$ $\therefore 0=(0+2)^2A+(0+2)(0+1)B+(0+1)C$ $\therefore 0=4A+2B+C$ $\therefore 0=4+2B-4$ $\therefore 0=2B$ $\therefore \boxed{0=B}$	1	
		$\therefore \boxed{\frac{x^2}{(x+1)(x+2)^2}=\frac{1}{x+1}+\frac{0}{x+2}+\frac{-4}{(x+2)^2}}$	1	**4**

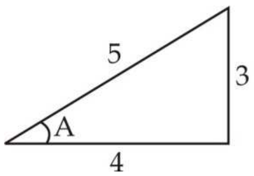
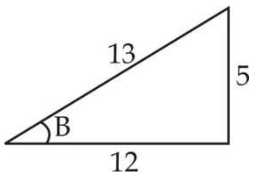
Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)	e)	Resolve into partial fractions: $\dfrac{x^2+1}{x\left(x^2-1\right)}$		
	Ans.	$\dfrac{x^2+1}{x\left(x^2-1\right)}=\dfrac{x^2+1}{x(x+1)(x-1)}=\dfrac{A}{x}+\dfrac{B}{x+1}+\dfrac{C}{x-1}$ $\therefore x^2+1=(x+1)(x-1)A+x(x-1)B+x(x+1)C$ $Put\;\; x=0$ $\therefore 0+1=(0+1)(0-1)A+0+0$ $\therefore 1=-A$ $\therefore \boxed{A=-1}$	1	
		$Put\;\; x=-1$ $\therefore (-1)^2+1=0-1(-1-1)B+0$ $\therefore 2=2B$ $\therefore \boxed{B=1}$	1	
		$Put\;\; x=1$ $\therefore (1)^2+1=0+0+1(1+1)C$ $\therefore 2=2C$ $\therefore \boxed{C=1}$	1	
		$\therefore \boxed{\dfrac{x^2+1}{x\left(x^2-1\right)}=\dfrac{-1}{x}+\dfrac{1}{x+1}+\dfrac{1}{x-1}}$	1	**4**
		Note : If the problem is solved as illustrated below, the solution is consider to be incomplete and marks may be given accordingly. $\dfrac{x^2+1}{x\left(x^2-1\right)}=\dfrac{A}{x}+\dfrac{Bx+C}{x^2-1}$ $Consequently,\; we\;\; get$ $\therefore \boxed{A=-1}$ $\boxed{B=2}\;\; and\;\; \boxed{C=0}$ $\therefore \boxed{\dfrac{x^2+1}{x\left(x^2-1\right)}=\dfrac{-1}{x}+\dfrac{2x}{x^2-1}}$ --		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)		**Attempt any FOUR of the followings:**		
	a)	Prove that $\dfrac{\sin A+\sin 2A+\sin 3A+\sin 4A}{\cos A+\cos 2A+\cos 3A+\cos 4A}=\tan\left(\dfrac{5A}{2}\right)$		
	Ans.	$\dfrac{\sin A+\sin 2A+\sin 3A+\sin 4A}{\cos A+\cos 2A+\cos 3A+\cos 4A}$		
		$=\dfrac{(\sin A+\sin 4A)+(\sin 2A+\sin 3A)}{(\cos A+\cos 4A)+(\cos 2A+\cos 3A)}$		
		$=\dfrac{2\sin\left(\dfrac{5A}{2}\right)\cos\left(\dfrac{-3A}{2}\right)+2\sin\left(\dfrac{5A}{2}\right)\cos\left(\dfrac{-A}{2}\right)}{2\cos\left(\dfrac{5A}{2}\right)\cos\left(\dfrac{-3A}{2}\right)+2\cos\left(\dfrac{5A}{2}\right)\cos\left(\dfrac{-A}{2}\right)}$	1	
		$=\dfrac{2\sin\left(\dfrac{5A}{2}\right)\left[\cos\left(\dfrac{-3A}{2}\right)+\cos\left(\dfrac{-A}{2}\right)\right]}{2\cos\left(\dfrac{5A}{2}\right)\left[\cos\left(\dfrac{-3A}{2}\right)+\cos\left(\dfrac{-A}{2}\right)\right]}$	1	
		$=\dfrac{\sin\left(\dfrac{5A}{2}\right)}{\cos\left(\dfrac{5A}{2}\right)}$	1	
		$=\tan\left(\dfrac{5A}{2}\right)$	1	**4**
		---		
	b)	Prove that $\dfrac{\cot\theta+\operatorname{cosec}\theta-1}{\cot\theta-\operatorname{cosec}\theta+1}=\cot\left(\dfrac{\theta}{2}\right)$		
	Ans.	$\dfrac{\cot\theta+\operatorname{cosec}\theta-1}{\cot\theta-\operatorname{cosec}\theta+1}$		
		$=\dfrac{\dfrac{\cos\theta}{\sin\theta}+\dfrac{1}{\sin\theta}-1}{\dfrac{\cos\theta}{\sin\theta}-\dfrac{1}{\sin\theta}+1}$	½	
		$=\dfrac{\dfrac{\cos\theta+1-\sin\theta}{\sin\theta}}{\dfrac{\cos\theta-1+\sin\theta}{\sin\theta}}$		
		$=\dfrac{\cos\theta+1-\sin\theta}{\cos\theta-1+\sin\theta}$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)		$=\dfrac{2\cos^2\left(\frac{\theta}{2}\right)-2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{-2\sin^2\left(\frac{\theta}{2}\right)+2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}$	1	
		$=\dfrac{\cos\left(\frac{\theta}{2}\right)\left[2\cos\left(\frac{\theta}{2}\right)-2\sin\left(\frac{\theta}{2}\right)\right]}{\sin\left(\frac{\theta}{2}\right)\left[2\cos\left(\frac{\theta}{2}\right)-2\sin\left(\frac{\theta}{2}\right)\right]}$	1	
		$=\cot\left(\frac{\theta}{2}\right)$	1	**4**
		OR		
		$\dfrac{\cot\theta+\cos ec\theta-1}{\cot\theta-\cos ec\theta+1}$		
		$=\dfrac{\cot\theta+\cos ec\theta-\left(\cos ec^2\theta-\cot^2\theta\right)}{\cot\theta-\cos ec\theta+1}$	½	
		$=\dfrac{\left(\cot\theta+\cos ec\theta\right)-\left(\cos ec\theta-\cot\theta\right)\left(\cos ec\theta+\cot\theta\right)}{\cot\theta-\cos ec\theta+1}$		
		$=\dfrac{\left(\cot\theta+\cos ec\theta\right)\left[1-\left(\cos ec\theta-\cot\theta\right)\right]}{\cot\theta-\cos ec\theta+1}$		
		$=\dfrac{\left(\cot\theta+\cos ec\theta\right)\left[1-\cos ec\theta+\cot\theta\right]}{\cot\theta-\cos ec\theta+1}$	½	
		$=\cot\theta+\cos ec\theta$	½	
		$=\dfrac{\cos\theta}{\sin\theta}+\dfrac{1}{\sin\theta}$	½	
		$=\dfrac{\cos\theta+1}{\sin\theta}$		
		$=\dfrac{2\cos^2\left(\frac{\theta}{2}\right)}{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}$	1	
		$=\cot\left(\frac{\theta}{2}\right)$	1	**4**
		OR		
		$\dfrac{\cot\theta+\cos ec\theta-1}{\cot\theta-\cos ec\theta+1}$		
		$=\dfrac{\cot\theta+\cos ec\theta-1}{\cot\theta-\cos ec\theta+1}\times\dfrac{\cot\theta-\cos ec\theta-1}{\cot\theta-\cos ec\theta-1}$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)		$=\frac{(\cot\theta-1)^2-\cos ec^2\theta}{(\cot\theta-\cos ec\theta)^2-1^2}$	1	
		$=\frac{\cot^2\theta-2\cot\theta+1-\cos ec^2\theta}{\cot^2\theta-2\cot\theta\cos ec\theta+\cos ec^2\theta-1}$		
		$=\frac{-2\cot\theta+1-1}{\cot^2\theta-2\cot\theta\cos ec\theta+\cot^2\theta}$		
		$=\frac{-2\cot\theta}{2\cot^2\theta-2\cot\theta\cos ec\theta}$	1	
		$=\frac{-2\cot\theta}{2\cot\theta(\cot\theta-\cos ec\theta)}$		
		$=\frac{-1}{\cot\theta-\cos ec\theta}$		
		$=\frac{-1}{\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}}$		
		$=\frac{-\sin\theta}{\cos\theta-1}$		
		$=\frac{-2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{-2\sin^2\left(\frac{\theta}{2}\right)}$	1	
		$=\cot\left(\frac{\theta}{2}\right)$	1	**4**
		--		
	c)	Prove that $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$		
	Ans.	$\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)=\tan^{-1}\left[\frac{1/2+1/5}{1-(1/2)(1/5)}\right]$ ...(*)	1	
		$=\tan^{-1}\left(\frac{7}{9}\right)$ ...(**)	1	
		$\therefore\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{8}\right)$		
		$=\tan^{-1}\left(\frac{7}{9}\right)+\tan^{-1}\left(\frac{1}{8}\right)$		
		$=\tan^{-1}\left[\frac{7/9+1/8}{1-(7/9)(1/8)}\right]$ ...(*)	1	

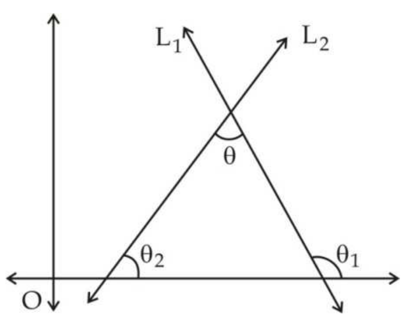
Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)		$= \tan^{-1}\left(\frac{65}{65}\right)$		
		$= \tan^{-1}(1)$ ...(**)	½	
		$= \frac{\pi}{4}$	½	**4**
		Note: Due to the use of non-programmable scientific calculator, the value (**) can be calculated directly after the step (*) instead of doing calculations algebraically. This may happen in the following alternative solutions also.		
		OR		
		$\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{8}\right)=\tan^{-1}\left[\frac{1/5+1/8}{1-(1/5)(1/8)}\right]$	1	
		$= \tan^{-1}\left(\frac{13}{39}\right)$		
		$= \tan^{-1}\left(\frac{1}{3}\right)$	1	
		$\therefore \tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{8}\right)$		
		$= \tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)$		
		$= \tan^{-1}\left[\frac{1/2+1/3}{1-(1/2)(1/3)}\right]$	1	
		$= \tan^{-1}\left(\frac{5}{5}\right)$		
		$= \tan^{-1}(1)$	½	
		$= \frac{\pi}{4}$	½	**4**
		OR		
		$\therefore \tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{8}\right)$		
		$= \tan^{-1}\left[\frac{1/2+1/5}{1-(1/2)(1/5)}\right]+\tan^{-1}\left(\frac{1}{8}\right)$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)		$=\tan^{-1}\left(\frac{7}{9}\right)+\tan^{-1}\left(\frac{1}{8}\right)$	1	
		$=\tan^{-1}\left[\frac{7/9+1/8}{1-(7/9)(1/8)}\right]$	1	
		$=\tan^{-1}\left(\frac{65}{65}\right)$		
		$=\tan^{-1}(1)$	½	
		$=\frac{\pi}{4}$	½	**4**
		---		
	d)	Prove that $\cos^{-1}\left(\frac{4}{5}\right)-\cos^{-1}\left(\frac{12}{13}\right)=\cos^{-1}\left(\frac{63}{65}\right)$		
	Ans.	$Let\quad A=\cos^{-1}\left(\frac{4}{5}\right)$ $\qquad B=\cos^{-1}\left(\frac{12}{13}\right)$		
		$\therefore \cos A=\frac{4}{5}$ $\qquad \cos B=\frac{12}{13}$		
		 		
		$\cos(A-B)=\cos A\cos B+\sin A\sin B$	1	
		$=\frac{4}{5}\times\frac{12}{13}+\frac{3}{5}\times\frac{5}{13}\quad ---(*)$	1	
		$=\frac{48}{65}+\frac{15}{65}$		
		$=\frac{48+15}{65}$		
		$=\frac{63}{65}\quad ---(**)$	1	
		$\therefore A-B=\cos^{-1}\left(\frac{63}{65}\right)$	½	
		$\therefore \cos^{-1}\left(\frac{4}{5}\right)-\cos^{-1}\left(\frac{12}{13}\right)=\cos^{-1}\left(\frac{63}{65}\right)$	½	**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)		**Note:** Due to the use of advance scientific calculators which is permissible in the exam, students may write the step (**) directly after step (*). Writing such step are to be considered. **Note:** To evaluate value of sin A and sin B, many times the relation between sine ratio and cosine ratio is used, instead of using Triangle Method as illustrated in the above solution. As the main object is to find the values, please consider these methods also. This is illustrated hereunder: $\sin A=\sqrt{1-\cos^2 A}=\sqrt{1-\frac{16}{25}}=\frac{3}{5}$ $\sin B=\sqrt{1-\sin^2 B}=\sqrt{1-\frac{144}{169}}=\frac{5}{13}$ ---		
	e)	Find the principal value of i) $\cos\left[\frac{\pi}{2}-\sin^{-1}\left(\frac{1}{2}\right)\right]$ ii) $\cos^{-1}\left(-\frac{1}{2}\right)-\sin^{-1}\left(\frac{1}{2}\right)$		
	Ans.			
	i)	$\cos\left[\frac{\pi}{2}-\sin^{-1}\left(\frac{1}{2}\right)\right]=\cos\left[\frac{\pi}{2}-\frac{\pi}{6}\right]$	1	
		$=\cos\left[\frac{\pi}{3}\right]$		
		$=\frac{1}{2}$ *or* 0.5	1	
		OR	OR	
		$\cos\left[\frac{\pi}{2}-\sin^{-1}\left(\frac{1}{2}\right)\right]=\cos\left[90^o-30^o\right]$	1	
		$=\cos\left[60^o\right]$		
		$=\frac{1}{2}$ *or* 0.5	1	
		OR	OR	
		$\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$	1	
		$\therefore\cos\left[\frac{\pi}{2}-\sin^{-1}\left(\frac{1}{2}\right)\right]=\cos\left[\frac{\pi}{2}-\frac{\pi}{6}\right]$		
		$=\cos\left[\frac{\pi}{3}\right]=\frac{1}{2}$ *or* 0.5	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)	e) ii)	$\cos^{-1}\left(-\frac{1}{2}\right)-\sin^{-1}\left(\frac{1}{2}\right)$		
		$=\pi-\cos^{-1}\left(\frac{1}{2}\right)-\sin^{-1}\left(\frac{1}{2}\right)$ *or* $180^{o}-\cos^{-1}\left(\frac{1}{2}\right)-\sin^{-1}\left(\frac{1}{2}\right)$	½	
		$=\pi-\frac{\pi}{3}-\frac{\pi}{6}$ *or* $180^{o}-60^{o}-30^{o}$	½+½	
		$=\frac{6\pi-2\pi-\pi}{6}$		
		$=\frac{\pi}{2}$ *or* 90^{o}	½	
		OR	OR	
		$\cos^{-1}\left(-\frac{1}{2}\right)=\pi-\cos^{-1}\left(\frac{1}{2}\right)$ *or* $180^{o}-\cos^{-1}\left(\frac{1}{2}\right)$	½	
		$=\pi-\frac{\pi}{3}$ *or* $180^{o}-60^{o}$		
		$=\frac{2\pi}{3}$ *or* 120^{o}	½	
		$\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$ *or* 30^{o}	½	
		$\cos^{-1}\left(-\frac{1}{2}\right)-\sin^{-1}\left(\frac{1}{2}\right)=\frac{2\pi}{3}-\frac{\pi}{6}$ *or* $120^{o}-30^{o}$		
		$=\frac{\pi}{2}$ *or* 90^{o}	½	4
		--		
	f)	$x>0,\ y>0$, prove that $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$		
	Ans.	*Put* $\tan^{-1}x=A$ *and* $\tan^{-1}y=B$		
		$\therefore\ x=\tan A$ *and* $y=\tan B$		
		$\therefore\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$	1	
		$=\frac{x+y}{1-xy}$	1	
		$\therefore A+B=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$	1	
		$\therefore\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$	1	4

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		**Attempt any FOUR of the following:**		
	a)	Find the angle between the lines $2x+3y=13$ and $2x-5y+7=0$.		
	Ans.	$For\ line\ \ 2x+3y-13=0$ $slope\ \ m_1=-\frac{A}{B}=-\frac{2}{3}$	1	
		$For\ line\ \ 2x-5y+7=0$ $slope\ \ m_2=-\frac{A}{B}=-\frac{2}{-5}=\frac{2}{5}$	1	
		$\therefore \tan\theta=\left\|\frac{m_1-m_2}{1+m_1\cdot m_2}\right\|$ $=\left\|\frac{-\frac{2}{3}-\frac{2}{5}}{1+\left(-\frac{2}{3}\right)\cdot\left(\frac{2}{5}\right)}\right\|$	1	
		$=\left\|-\frac{16}{11}\right\|$ $=\frac{16}{11}\quad or\quad 1.4545$	½	
		$\therefore \theta=\tan^{-1}\left(\frac{16}{11}\right)\quad or\quad \tan^{-1}(1.4545)$ $or\quad 55.491^{o}\quad or\quad 0.969^{C}$	½	**4**
		--		
	b)	Find the equation of line passing through the point of intersection of the lines $4x+3y=8,\ x+y=1$ and parallel to the line $5x-7y=3$.		
	Ans.	$4x+3y=8,\ x+y=1$ $\therefore 4x+3y=8$ $3x+3y=3$ $-\quad-\quad-$ $x=5$	1	
		$\therefore y=1-x=1-5=-4$ $\therefore$ the point is $(5,\ -4)$.	1	
		$Now\ \ for\ 5x-7y=3$ $slope\ m_0=-\frac{A}{B}=-\frac{5}{-7}=\frac{5}{7}$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		$\therefore slope\ of\ the\ line\ m = m_0 = \frac{5}{7}$	½	
		$\therefore the\ equation\ is$ $y - y_1 = m(x - x_1)$		
		$\therefore y + 4 = \frac{5}{7}(x - 5)$	½	
		$\therefore 7y + 28 = 5x - 25$		
		$\therefore 5x - 7y - 53 = 0 \quad or \quad -5x + 7y + 53 = 0$	½	**4**
		---		
	c)	Find the length of the perpendicular from $(3, 2)$ on the line $4x - 6y = 5$.		
	Ans.	$Given\ \ 4x - 6y - 5 = 0$ $\therefore A = 4,\ \ B = -6,\ \ C = -5$ $\therefore the\ length\ of\ the\ perpendicular\ is,$ $p = \left\| \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}} \right\|$		
		$= \left\| \frac{4(3) - 6(2) - 5}{\sqrt{4^2 + (-6)^2}} \right\|$	2	
		$= \frac{5}{\sqrt{52}} \quad or \quad 0.693$	2	**4**
		Note: If –ve sign is left with the answer, 1 mark is to be deducted.		
		---		
	d)	If m_1 and m_2 are the slopes of two lines, prove that the acute angle between the lines is $\theta = \tan^{-1}\left\| \frac{m_1 - m_2}{1 + m_1 \cdot m_2} \right\|$		
	Ans.		1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		Let θ_1 = Angle of inclination of L_1 θ_2 = Angle of inclination of L_2 $\therefore$ Slope of L_1 is $m_1 = \tan\theta_1$ Slope of L_2 is $m_2 = \tan\theta_2$		
		$\therefore$ *from figure,* $\theta = \theta_1 - \theta_2$	½	
		$\therefore \tan\theta = \tan(\theta_1 - \theta_2)$	½	
		$= \dfrac{\tan(\theta_1) - \tan(\theta_2)}{1 + \tan(\theta_1)\tan(\theta_2)}$	½	
		$= \dfrac{m_1 - m_2}{1 + m_1 \cdot m_2}$	½	
		$\therefore \theta = \tan^{-1}\dfrac{m_1 - m_2}{1 + m_1 \cdot m_2}$	½	
		For angle to be acute, $\theta = \tan^{-1}\left\lvert\dfrac{m_1 - m_2}{1 + m_1 \cdot m_2}\right\rvert$	½	**4**
		--		
	e)	Find the equation of a line passing through (2, 5) and the point of intersection of $x + y = 0,\ 2x - y = 9$.		
	Ans.	$x + y = 0$ $\underline{2x - y = 9}$ $\therefore\ 3x = 9$		
		$\therefore x = 3$ $y = -3$	1 1	
		$\therefore$ Point of intersection $= (3, -3)$ $\therefore$ *equation is,* $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$		
		$\therefore \dfrac{y - 5}{-3 - 5} = \dfrac{x - 2}{3 - 2}$	1	
		$\therefore 8x + y - 21 = 0$	1	
		OR		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		**OR**	OR	
		$\therefore$ Point of intersection $=(3,\ -3)$		
		$\therefore$ *Slope* $m=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{-3-5}{3-2}=-8$	½	
		$\therefore$ *equation is,*		
		$y-y_1=m(x-x_1)$		
		$\therefore y-5=-8(x-2)$	½	
		$\therefore 8x+y-21=0$	1	**4**
		---		
	f)	Find the equation of the line which is perpendicular to the line $5x-2y=7$ and passes through the mid-point of the line joining the points (2, 7) and (-4, 1).		
	Ans.	Slope of $5x-2y=7$ is		
		$m_0=-\dfrac{5}{-2}=\dfrac{5}{2}$	½	
		$\therefore$ slope of required line is		
		$m=-\dfrac{1}{m_0}=-\dfrac{2}{5}$	1	
		Mid-point $=\left(\dfrac{2-4}{2},\ \dfrac{7+1}{2}\right)=(-1,\ 4)$	1	
		$\therefore$ the equation is		
		$y-y_1=m(x-x_1)$		
		$\therefore y-4=-\dfrac{2}{5}(x+1)$	1	
		$\therefore 5y-20=-2x-2$		
		$\therefore 2x+5y-18=0$	½	**4**

		Important Note		
		In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. **In such case, FIRST SEE whether the method falls within the scope of the curriculum**, and THEN ONLY give appropriate marks in accordance with the scheme of marking.		
