WINTER– 16 EXAMINATION
Model Answer

Subject Code: **17105**

Important Instructions to examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
Q. 1		**Attempt any TEN of the following:**	**20**
	a)	Find x, if $\begin{vmatrix} -2 & 0 & 0 \\ -6 & x & 1 \\ -4 & 0 & -1 \end{vmatrix} = -4$	**02**
	Ans	$\begin{vmatrix} -2 & 0 & 0 \\ -6 & x & 1 \\ -4 & 0 & -1 \end{vmatrix} = -4$	
		$\therefore -2(-x-0)-0(6+4)+0(0+4x)=-4$	1
		$\therefore 2x-0+0=-4$	
		$\therefore 2x=-4$	½
		$\therefore x=-2$	½
		--	
	b)	If $A=\begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 5 \end{bmatrix}, B=\begin{bmatrix} 1 & 2 & -6 \\ 0 & -1 & 3 \end{bmatrix}$ find $3A-B$	**02**
	Ans	$3A-B=3\begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 5 \end{bmatrix}-\begin{bmatrix} 1 & 2 & -6 \\ 0 & -1 & 3 \end{bmatrix}$	
		$=\begin{bmatrix} 6 & 9 & 3 \\ 0 & -3 & 15 \end{bmatrix}-\begin{bmatrix} 1 & 2 & -6 \\ 0 & -1 & 3 \end{bmatrix}$	1
		$=\begin{bmatrix} 5 & 7 & 9 \\ 0 & -2 & 12 \end{bmatrix}$	1

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Q. No.	Sub Q. N.	Answer	Marking Scheme
1	c)	If $A=\begin{bmatrix}2 & 4\\ -1 & -2\end{bmatrix}$, show that A^2 is null matrix	**02**
	Ans	$A^2=A.A$	
		$=\begin{bmatrix}2 & 4\\ -1 & -2\end{bmatrix}\begin{bmatrix}2 & 4\\ -1 & -2\end{bmatrix}$	
		$=\begin{bmatrix}4-4 & 8-8\\ -2+2 & -4+4\end{bmatrix}$	1
		$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$	1
		---	
	d)	Show that the matrices $A=\begin{bmatrix}2 & 3\\ -3 & 2\end{bmatrix}$ and $B=\begin{bmatrix}4 & -1\\ 1 & 4\end{bmatrix}$ are commute	**02**
	Ans	$AB=\begin{bmatrix}2 & 3\\ -3 & 2\end{bmatrix}\begin{bmatrix}4 & -1\\ 1 & 4\end{bmatrix}$	
		$=\begin{bmatrix}8+3 & -2+12\\ -12+2 & 3+8\end{bmatrix}$	
		$=\begin{bmatrix}11 & 10\\ -10 & 11\end{bmatrix}$	½
		$BA=\begin{bmatrix}4 & -1\\ 1 & 4\end{bmatrix}\begin{bmatrix}2 & 3\\ -3 & 2\end{bmatrix}$	
		$=\begin{bmatrix}8+3 & 12-2\\ 2-12 & 3+8\end{bmatrix}$	
		$=\begin{bmatrix}11 & 10\\ -10 & 11\end{bmatrix}$	½
		$\therefore AB=BA$	½
		$\therefore$ A and B are commute	½
		---	
	e)	Resolve into the partials $\frac{2}{x^2+x-2}$	**02**
	Ans	$\frac{2}{x^2+x-2}=\frac{2}{(x-1)(x+2)}$	
		$\frac{2}{(x-1)(x+2)}=\frac{A}{x-1}+\frac{B}{x+2}$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
1	e)	$\therefore 2=(x+2)A+(x-1)B$ Put $x=1$ $2=(3)A$ $\therefore A=\frac{2}{3}$ Put $x=-2$ $2=(-3)B$ $\therefore B=\frac{-2}{3}$ $\therefore \frac{2}{(x-1)(x+2)}=\frac{\frac{2}{3}}{x-1}+\frac{\frac{-2}{3}}{x+2}$	 ½ ½ ½
	f) Ans	If $A=\begin{bmatrix}1&2\\5&3\end{bmatrix}, B=\begin{bmatrix}2&6\\-3&4\end{bmatrix}$, find $(AB)^T$ $AB=\begin{bmatrix}1&2\\5&3\end{bmatrix}\begin{bmatrix}2&6\\-3&4\end{bmatrix}$ $=\begin{bmatrix}-4&14\\1&42\end{bmatrix}$ $\therefore (AB)^T=\begin{bmatrix}-4&1\\14&42\end{bmatrix}$	**02** 1 1
	g) Ans	Without using calculator find the value of $\sin 15^0$ $\sin 15^0=\sin(45^0-30^0)$ $=\sin 45^0\cos 30^0-\cos 45^0\sin 30^0$ $=\frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}\frac{1}{2}$ $=\frac{\sqrt{3}-1}{2\sqrt{2}}=$	**02** ½ ½ ½ ½
	h) Ans	Prove that $\cos(\pi+\theta)=-\cos\theta$ $\cos(\pi+\theta)=\cos\pi\cos\theta-\sin\pi\sin\theta$ $=(-1)\cos\theta-(0)\sin\theta$ $=-\cos\theta$	**02** ½ 1 ½

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Q. No.	Sub Q. N.	Answer	Marking Scheme
1	i)	If $2\sin 40^0.\cos 10^0 = \sin A+\sin B$ then find A and B	**02**
	Ans	$\sin(40^0+10^0)+\sin(40^0-10^0)=\sin A+\sin B$	1
		$\sin 50^0+\sin 30^0=\sin A+\sin B$	½
		$\therefore A=50^0$ and $B=30^0$	½
		OR	
		$2\sin 40^0.\cos 10^0=\sin A+\sin B$	
		$\because \sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$	
		$\therefore 2\sin 40^0.\cos 10^0=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$	1
		$\therefore \frac{A+B}{2}=40^0$ and $\frac{A-B}{2}=10^0$	½
		$\therefore A+B=80^0$ and $A-B=20^0$	
		$\therefore A=50^0$ and $B=30^0$	½
	j)	Express as product and evaluate: $\sin 81^0-\sin 99^0$	**02**
	Ans	$\sin 81^0-\sin 99^0=2\cos\left(\frac{81^0+99^0}{2}\right)\sin\left(\frac{81^0-99^0}{2}\right)$	1
		$=2\cos(90^0)\sin(-9^0)=0$	½+½
		k) Comment:If Question is attempted and results are as per model answer then give the credit to the student accordingly.	
	k)	Show that the lines $2x+3y-1=0$ and $3x+2y+6=0$ are perpendicular	**02**
	Ans	Slope of $2x+3y-1=0$ is $m_1=\frac{-2}{3}$	½
		Slope of $3x+2y+6=0$ is $m_2=\frac{-3}{2}$	½
		$\therefore m_1\ m_2=\frac{-2}{3}.\frac{-3}{2}=1$	½
		$\therefore$ lines are not perpendicular	½
	l)	Find the equation of line passing through the point $(-3,2)$ and having slope 5/2.	**02**
	Ans	Equation of line is $y-y_1=m(x-x_1)$	
		$y-2=\frac{5}{2}(x+3)$	1
		$5x-2y+19=0$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
2		**Attempt any Four of the following:**	**16**
	a)	If $A=\begin{bmatrix}1&3\\2&1\end{bmatrix}, B=\begin{bmatrix}2&2\\-1&-1\end{bmatrix}, C=\begin{bmatrix}4&-3\\-2&3\end{bmatrix}$ verify that $(A+B)C=AC+BC$	**04**
	Ans	$A+B=\begin{bmatrix}1&3\\2&1\end{bmatrix}+\begin{bmatrix}2&2\\-1&-1\end{bmatrix}=\begin{bmatrix}3&5\\1&0\end{bmatrix}$	½
		$(A+B)C=\begin{bmatrix}3&5\\1&0\end{bmatrix}\begin{bmatrix}4&-3\\-2&3\end{bmatrix}=\begin{bmatrix}2&6\\4&-3\end{bmatrix}$	1
		$AC=\begin{bmatrix}1&3\\2&1\end{bmatrix}\begin{bmatrix}4&-3\\-2&3\end{bmatrix}=\begin{bmatrix}-2&6\\6&-3\end{bmatrix}$	1
		$BC=\begin{bmatrix}2&2\\-1&-1\end{bmatrix}\begin{bmatrix}4&-3\\-2&3\end{bmatrix}=\begin{bmatrix}4&0\\-2&0\end{bmatrix}$	1
		$AC+BC=\begin{bmatrix}2&6\\4&-3\end{bmatrix}$	
		$\therefore (A+B)C=AC+BC$	½
		--	
	b)	Find the inverse of matrix A= $\begin{bmatrix}1&2&4\\-1&2&3\\1&4&1\end{bmatrix}$ using adjoint method	**04**
	Ans	$A=\begin{bmatrix}1&2&4\\-1&2&3\\1&4&1\end{bmatrix}$	
		$\lvert A\rvert=-26\neq 0$	½
		$\therefore A^{-1}$ exists	
		Matrix of minors $=\begin{bmatrix}\begin{vmatrix}2&3\\4&1\end{vmatrix}&\begin{vmatrix}-1&3\\1&1\end{vmatrix}&\begin{vmatrix}-1&2\\1&4\end{vmatrix}\\\begin{vmatrix}2&4\\4&1\end{vmatrix}&\begin{vmatrix}1&4\\1&1\end{vmatrix}&\begin{vmatrix}1&2\\1&4\end{vmatrix}\\\begin{vmatrix}2&4\\2&3\end{vmatrix}&\begin{vmatrix}1&4\\-1&3\end{vmatrix}&\begin{vmatrix}1&2\\-1&2\end{vmatrix}\end{bmatrix}$	1
		$=\begin{bmatrix}-10&-4&-6\\-14&-3&2\\-2&7&4\end{bmatrix}$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
2	b)	Matrix of cofactors $=\begin{bmatrix}-10 & 4 & -6\\ 14 & -3 & -2\\ -2 & -7 & 4\end{bmatrix}$	½
		$\text{Adj.}A=\begin{bmatrix}-10 & 14 & -2\\ 4 & -3 & -7\\ -6 & -2 & 4\end{bmatrix}$	½
		$A^{-1}=\frac{1}{\lvert A\rvert}\text{Adj.}A$	
		$A^{-1}=\frac{1}{-26}\begin{bmatrix}-10 & 14 & -2\\ 4 & -3 & -7\\ -6 & -2 & 4\end{bmatrix}$	1
		--	
	c)	Resolve into partial fractions $\frac{x-5}{x(x+3)(x-2)}$	**04**
	Ans	$\frac{x-5}{x(x+3)(x-2)}=\frac{A}{x}+\frac{B}{x+3}+\frac{C}{x-2}$	½
		$x-5=A(x+3)(x-2)+Bx(x-2)+Cx(x+3)$	
		Put $x=0$	
		$-5=A(3)(-2)$	
		$-5=-6A$	
		$\therefore A=\frac{5}{6}$	1
		Put $x=-3$	
		$-3-5=B(-3)(-3-2)$	
		$-8=B(-3)(-5)$	
		$-8=B(15)$	
		$\therefore B=\frac{-8}{15}$	1
		Put $x=2$	
		$2-5=C(2)(2+3)$	
		$-3=C(2)(5)$	
		$-3=C(10)$	
		$\therefore C=\frac{-3}{10}$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
2	c)	$\therefore \dfrac{x-5}{x(x+3)(x-2)} = \dfrac{\frac{5}{6}}{x} + \dfrac{\frac{-8}{15}}{x+3} + \dfrac{\frac{-3}{10}}{x-2}$	½
		--	
	d)	Prove that $\sqrt{2+\sqrt{2+2\cos 4\theta}} = 2\cos\theta$	**04**
	Ans	$\sqrt{2+\sqrt{2+2\cos 4\theta}}$	
		$= \sqrt{2+\sqrt{2(1+\cos 4\theta)}}$	½
		$= \sqrt{2+\sqrt{2(2\cos^2 2\theta)}}$	1
		$= \sqrt{2+\sqrt{4\cos^2 2\theta}}$	
		$= \sqrt{2+2\cos 2\theta}$	½
		$= \sqrt{2(1+\cos 2\theta)}$	
		$= \sqrt{2(2\cos^2 \theta)}$	1
		$= \sqrt{4\cos^2 \theta}$	
		$= 2\cos\theta$	1
		--	
	e)	Prove that : $\tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) = \cot^{-1}\left(\frac{9}{2}\right)$	**04**
	Ans	$\tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) = \tan^{-1}\left(\dfrac{\frac{1}{7}+\frac{1}{13}}{1-\frac{1}{7}.\frac{1}{13}}\right)$	2
		$= \tan^{-1}\left(\frac{20}{90}\right)$	
		$= \tan^{-1}\left(\frac{2}{9}\right)$	1
		$= \cot^{-1}\left(\frac{9}{2}\right)$	1
		--	
	f)	If $\tan(x+y) = \frac{3}{4}$ and $\tan(x-y) = \frac{1}{3}$, find $\tan 2x$	**04**

Q. No.	Sub Q. N.	Answer	Marking Scheme
2	Ans	$\tan(2x) = \tan(x+y+x-y)$	1
		$= \dfrac{\tan(x+y)+\tan(x-y)}{1-\tan(x+y).\tan(x-y)}$	1
		$= \dfrac{\frac{3}{4}+\frac{1}{3}}{1-\frac{3}{4}.\frac{1}{3}}$	1
		$= \dfrac{\frac{9+4}{12}}{\frac{12-3}{12}}$	
		$= \dfrac{13}{9}$	1

3		**Attempt any FOUR of the following:**	**16**
	a)	Solve by Cramer's Rule: $x+y=3, y+z=5, x+z=4$	**04**
	Ans	$D = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 1[1-0]-1[0-1]+0[0-1] = 2$	1
		$D_x = \begin{vmatrix} 3 & 1 & 0 \\ 5 & 1 & 1 \\ 4 & 0 & 1 \end{vmatrix} = 3[1-0]-1[5-4]+0[0-4] = 2$	½
		$D_y = \begin{vmatrix} 1 & 3 & 0 \\ 0 & 5 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 1[5-4]-3[0-1]+0[0-5] = 4$	½
		$D_z = \begin{vmatrix} 1 & 1 & 3 \\ 0 & 1 & 5 \\ 1 & 0 & 4 \end{vmatrix} = 1[4-0]-1[0-5]+3[0-1] = 6$	½
		$x = \dfrac{D_x}{D} = \dfrac{2}{2} = 1$	½
		$y = \dfrac{D_y}{D} = \dfrac{4}{2} = 2$	½
		$z = \dfrac{D_z}{D} = \dfrac{6}{2} = 3$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
3	b)	Express the matrix A as sum of a symmetric and skew symmetric matrices, where $A=\begin{bmatrix}4&2&-3\\1&3&-6\\-5&0&-7\end{bmatrix}$	04
	Ans	$A+A^T=\begin{bmatrix}4&2&-3\\1&3&-6\\-5&0&-7\end{bmatrix}+\begin{bmatrix}4&1&-5\\2&3&0\\-3&-6&-7\end{bmatrix}$	
		$=\begin{bmatrix}8&3&-8\\3&6&-6\\-8&-6&-14\end{bmatrix}$	1
		$A-A^T=\begin{bmatrix}4&2&-3\\1&3&-6\\-5&0&-7\end{bmatrix}-\begin{bmatrix}4&1&-5\\2&3&0\\-3&-6&-7\end{bmatrix}$	
		$=\begin{bmatrix}0&1&2\\-1&0&-6\\-2&6&0\end{bmatrix}$	1
		$\therefore A=\frac{1}{2}\left(A+A^T\right)+\frac{1}{2}\left(A-A^T\right)$	1
		$=\frac{1}{2}\begin{bmatrix}8&3&-8\\3&6&-6\\-8&-6&-14\end{bmatrix}+\frac{1}{2}\begin{bmatrix}0&1&2\\-1&0&-6\\-2&6&0\end{bmatrix}$	
		$=\begin{bmatrix}4&\frac{3}{2}&-4\\\frac{3}{2}&3&-3\\-4&-3&-7\end{bmatrix}+\begin{bmatrix}0&\frac{1}{2}&1\\-\frac{1}{2}&0&-3\\-1&3&0\end{bmatrix}$	1
		---	
	c)	Resolve into partial fraction : $\frac{2x+1}{x^2\left(x+1\right)}$	04
	Ans	$\frac{2x+1}{x^2\left(x+1\right)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+1}$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
3	c)	$2x+1=Ax(x+1)+B(x+1)+Cx^2$	
		Put $x=0$	
		$1=B(0+1)\ B=1$	1
		Put $x=-1$	
		$2(-1)+1=C(-1)^2$	
		$-2+1=C$	
		$C=-1$	1
		Put $x=1$	
		$2(1)+1=A(1)(1+1)+B(1+1)+C(1)^2$	
		$3=A(1)(2)+B(2)+C(1)$	
		$3=2A+2B+C$	
		$3=2A+2(1)+(-1)$	
		$3=2A+1$	
		$3-1=2A$	
		$2=2A\quad \therefore A=1$	1
		$\therefore \frac{2x+1}{x^2(x+1)}=\frac{1}{x}+\frac{1}{x^2}+\frac{-1}{x+1}$	½
		--	
		d) Comment:If Question is attempted and results are as per model answer then give the credit to the student accordingly.	
	d)	Prove that, $\frac{1-\tan^2\theta.\tan\theta}{1+\tan^2\theta.\tan\theta}=\frac{\cos 3\theta}{\cos\theta}$	**04**
	Ans	Consider LHS= $\frac{1-\tan^2\theta.\tan\theta}{1+\tan^2\theta.\tan\theta}$	
		$=\frac{1-\frac{\sin^2\theta}{\cos^2\theta}.\frac{\sin\theta}{\cos\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}.\frac{\sin\theta}{\cos\theta}}$	1
		$=\frac{\frac{\cos^2\theta\cos\theta-\sin^2\theta\sin\theta}{\cos 2\theta\cos\theta}}{\frac{\cos^2\theta\cos\theta+\sin^2\theta\sin\theta}{\cos 2\theta\cos\theta}}$	
		$=\frac{\cos^2\theta\cos\theta-\sin^2\theta\sin\theta}{\cos^2\theta\cos\theta+\sin^2\theta\sin\theta}$	1
		$\text{RHS}=\frac{\cos 3\theta}{\cos\theta}=\frac{\cos(2\theta+\theta)}{\cos(2\theta-\theta)}$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
3	d)	$=\dfrac{\cos 2\theta\cos\theta-\sin 2\theta\sin\theta}{\cos 2\theta\cos\theta+\sin 2\theta\sin\theta}\neq LHS$	1
		--	
	e)	Prove that, $\dfrac{\cos A}{1-\tan A}+\dfrac{\sin A}{1-\cot A}=\sin A+\cos A$	**04**
	Ans	Consider $\dfrac{\cos A}{1-\tan A}+\dfrac{\sin A}{1-\cot A}$	
		$=\dfrac{\cos A}{1-\dfrac{\sin A}{\cos A}}+\dfrac{\sin A}{1-\dfrac{\cos A}{\sin A}}$	1
		$=\dfrac{\cos A}{\dfrac{\cos A-\sin A}{\cos A}}+\dfrac{\sin A}{\dfrac{\sin A-\cos A}{\sin A}}$	
		$=\dfrac{\cos^2 A}{\cos A-\sin A}+\dfrac{\sin^2 A}{\sin A-\cos A}$	1
		$=\dfrac{\cos^2 A}{\cos A-\sin A}-\dfrac{\sin^2 A}{\cos A-\sin A}$	
		$=\dfrac{\cos^2 A-\sin^2 A}{\cos A-\sin A}$	½
		$=\dfrac{(\cos A-\sin A)(\cos A+\sin A)}{\cos A-\sin A}$	1
		$=\sin A+\cos A$	½
		--	
	f)	Prove that, $\sin(3\theta)=3\sin\theta-4\sin^3\theta$	**04**
	Ans	$\sin 3\theta$	
		$=\sin(\theta+2\theta)$	1
		$=\sin\theta\cos 2\theta+\cos\theta\sin 2\theta$	1
		$=\sin\theta(1-2\sin^2\theta)+\cos\theta(2\sin\theta.\cos\theta)$	
		$=\sin\theta-2\sin^3\theta+2\sin\theta\cos^2\theta$	
		$=\sin\theta-2\sin^3\theta+2\sin\theta(1-\sin^2\theta)$	1
		$=\sin\theta-2\sin^3\theta+2\sin\theta-2\sin^3\theta$	
		$=3\sin\theta-4\sin^3\theta$	1
		--	
4		**Attempt any <u>FOUR</u> of the following:**	**16**
	a)	Using matrix inversion method,solve the equations: $x+3y+3z=12, x+4y+4z=15, x+3y+4z=13$	**04**

<table>
<tr><th>Q. No.</th><th>Sub Q. N.</th><th>Answer</th><th>Marking Scheme</th></tr>
<tr><td>4</td><td>a)
Ans</td><td>

$$|A|=\begin{vmatrix}1&3&3\\1&4&4\\1&3&4\end{vmatrix}=1[16-12]-3[4-4]+3[3-4]$$

$$|A|=1\neq 0 \qquad \therefore A^{-1} \text{ exist}$$

$$A=\begin{bmatrix}1&3&3\\1&4&4\\1&3&4\end{bmatrix}$$

$$\text{Matrix of minors}=\begin{bmatrix}\begin{vmatrix}4&4\\3&4\end{vmatrix}&\begin{vmatrix}1&4\\1&4\end{vmatrix}&\begin{vmatrix}1&4\\1&3\end{vmatrix}\\\begin{vmatrix}3&3\\3&4\end{vmatrix}&\begin{vmatrix}1&3\\1&4\end{vmatrix}&\begin{vmatrix}1&3\\1&3\end{vmatrix}\\\begin{vmatrix}3&3\\4&4\end{vmatrix}&\begin{vmatrix}1&3\\1&4\end{vmatrix}&\begin{vmatrix}1&3\\1&4\end{vmatrix}\end{bmatrix}$$

$$=\begin{bmatrix}4&0&-1\\3&1&0\\0&1&1\end{bmatrix}$$

$$\text{Matrix of cofactors}=\begin{bmatrix}4&0&-1\\-3&1&0\\0&-1&1\end{bmatrix}$$

$$\text{Adj}.A=\begin{bmatrix}4&-3&0\\0&1&-1\\-1&0&1\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}Adj.A=\frac{1}{1}\begin{bmatrix}4&-3&0\\0&1&-1\\-1&0&1\end{bmatrix}$$

$$A^{-1}=\begin{bmatrix}4&-3&0\\0&1&-1\\-1&0&1\end{bmatrix}$$

$$\therefore \begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}4&-3&0\\0&1&-1\\-1&0&1\end{bmatrix}\begin{bmatrix}12\\15\\13\end{bmatrix}$$

$$\therefore \begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}48-45+0\\0+15-13\\-12+0+13\end{bmatrix}$$

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½

½

1</td></tr>
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Q. No.	Sub Q. N.	Answer	Marking Scheme
4	a)	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$	1
		$\therefore x = 3, y = 2, z = 1$	½
		--	
	b)	If $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$ Verify that $(AB)^T = B^T.A^T$	04
	Ans	$AB = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$	
		$AB = \begin{bmatrix} 1+2+2 & 2+2+4 & 3+10+7 \\ 0+2+6 & 0+2+12 & 0+10+21 \\ 0+0+2 & 0+0+4 & 0+0+7 \end{bmatrix}$	
		$AB = \begin{bmatrix} 5 & 8 & 20 \\ 8 & 14 & 31 \\ 2 & 4 & 7 \end{bmatrix}$	1
		$(AB)^T = \begin{bmatrix} 5 & 8 & 2 \\ 8 & 14 & 4 \\ 20 & 31 & 7 \end{bmatrix}$	½
		$B^T.A^T = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 4 \\ 3 & 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$	1
		$B^T.A^T = \begin{bmatrix} 1+2+2 & 0+2+6 & 0+0+2 \\ 2+2+4 & 0+2+12 & 0+0+4 \\ 3+10+7 & 0+10+21 & 0+0+7 \end{bmatrix}$	
		$B^T.A^T = \begin{bmatrix} 5 & 8 & 2 \\ 8 & 14 & 4 \\ 20 & 31 & 7 \end{bmatrix}$	1
		$\therefore (AB)^T = B^T.A^T$	½
		--	

Q. No.	Sub Q. N.	Answer	Marking Scheme
4	c)	Resolve into partial fraction : $\dfrac{2x-3}{(x+1)(x^2+4)}$	**04**
	Ans	$\dfrac{2x-3}{(x+1)(x^2+4)}=\dfrac{A}{x+1}+\dfrac{Bx+C}{x^2+4}$	½
		$2x-3=A(x^2+4)+(Bx+C)(x+1)$	
		Put $x=-1$	
		$2(-1)-3=A((-1)^2+4)$	
		$-5=A(5)$	
		$A=-1$	1
		Put $x=0$	
		$2(0)-3=A((0)^2+4)+(B(0)+C)((0)+1)$	
		$-3=4A+C(1)$	
		$-3=4(-1)+C$	
		$-3+4=C$	
		$C=1$	1
		Put $x=1$	
		$2(1)-3=A((1)^2+4)+(B(1)+C)((1)+1)$	
		$-1=A(5)+(B+C)(2)$	
		$-1=5A+2B+2C$	
		$-1=5(-1)+2B+2(1)$	
		$-1=-5+2B+2$	
		$-1=-3+2B$	
		$2=2B$	
		$B=1$	1
		$\dfrac{2x-3}{(x+1)(x^2+4)}=\dfrac{-1}{x+1}+\dfrac{(1)x+1}{x^2+4}$	½
		--	
	d)	Show that $\dfrac{\sin 5A+2\sin 8A+\sin 11A}{\sin 8A+2\sin 11A+\sin 14A}=\dfrac{\sin 8A}{\sin 11A}$	**04**
	Ans	$\dfrac{\sin 11A+\sin 5A+2\sin 8A}{\sin 14A+\sin 8A+2\sin 11A}$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
4	d)	$=\dfrac{2\sin(8A)\cos(3A)+2\sin 8A}{2\sin(11A)\cos(3A)+2\sin 11A}$	2
		$=\dfrac{2\sin 8A(\cos 3A+1)}{2\sin 11A(\cos 3A+1)}$	1
		$=\dfrac{\sin 8A}{\sin 11A}$	1
		--	
	e)	Show that, $\cos(A+B).\cos(A-B)=\cos^2 A-\sin^2 B$	**04**
	Ans	$\cos(A+B).\cos(A-B)$	
		$=[\cos A\cos B-\sin A\sin B].[\cos A\cos B+\sin A\sin B]$	1
		$=(\cos A\cos B)^2-(\sin A\sin B)^2$	1
		$=\cos^2 A.\cos^2 B-\sin^2 A.\sin^2 B$	
		$=\cos^2 A.(1-\sin^2 B)-(1-\cos^2 A).\sin^2 B$	
		$=\cos^2 A-\cos^2 A.\sin^2 B-\sin^2 B+\cos^2 A.\sin^2 B$	1
		$=\cos^2 A-\sin^2 B$	1
		--	
	f)	Find the equation of the line passing through $(2,5)$ and the point of intersection of $x+y=0$ and $2x-y=9$	**04**
	Ans	$x+y=0$	
		$2x-y=9$	
		$3x=9$	
		$x=3$	½
		$3+y=0$	
		$y=-3$	½
		point of intersection $(3,-3)$	
		$\therefore$ Equation of line is	
		$\dfrac{y-y_1}{y_1-y_2}=\dfrac{x-x_1}{x_1-x_2}$	
		$\dfrac{y-5}{5+3}=\dfrac{x-2}{2-3}$	2

Q. No.	Sub Q. N.	Answer	Marking Scheme
4	f)	$\frac{y-5}{8}=\frac{x-2}{-1}$ $-y+5=8x-16$ $8x+y-21=0$	1
		OR $x+y=0$ $2x-y=9$ $3x=9$ $x=3$	½
		$3+y=0$ $y=-3$	½
		$\therefore$ Point of intersection is $(3,-3)$ $\therefore$ Slope m $=\frac{y_2-y_1}{x_2-x_1}=\frac{-3-5}{3-2}=-8$	1
		equation is, $y-y_1=m(x-x_1)$ $y-5=-8(x-2)$ OR $y+3=-8(x-3)$	1
		$8x+y-21=0$	1
5		**Attempt any FOUR of the following:**	**16**
	a)	Resolve into partial fractions: $\frac{x^4}{x^3+1}$	**04**
	Ans	$x^3+1\overline{)x^4}$ quotient x x^4+x $-x$	
		$\therefore \frac{x^4}{x^3+1}=x+\frac{-x}{x^3+1}=x-\frac{x}{x^3+1}$	1
		Consider $\frac{x}{x^3+1}=\frac{x}{(x+1)(x^2-x+1)}$	

WINTER – 16 EXAMINATION

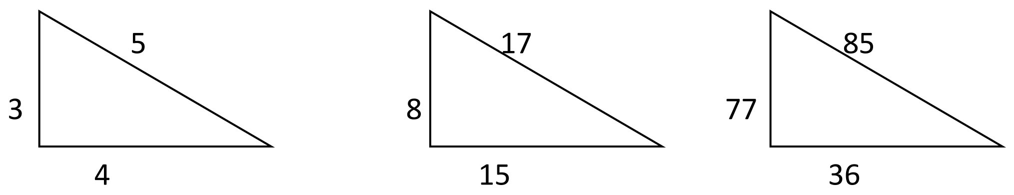
Model Answer Subject Code: 17105

Q. No.	Sub Q. N.	Answer	Marking Scheme
5	a)	$\frac{x}{(x+1)(x^2-x+1)}=\frac{A}{x+1}+\frac{Bx+C}{x^2-x+1}$	½
		$x=A(x^2-x+1)+(Bx+C)(x+1)$	
		Put $x=-1$	
		$-1=A\left((-1)^2-(-1)+1\right)$	
		$-1=A(1+1+1)$	
		$-1=A(3)$	
		$\therefore A=-\frac{1}{3}$	½
		Put $x=0$	
		$0=A(0-0+1)+(B(0)+C)(0+1)$	
		$0=A+C$	
		$0=-\frac{1}{3}+C$	
		$\therefore C=\frac{1}{3}$	½
		Put $x=1$	
		$1=A\left((1)^2-(1)+1\right)+(B(1)+C)((1)+1)$	
		$1=A(1)+(B+C)(2)$	
		$1=A+2B+2C$	
		$1=-\frac{1}{3}+2B+2\left(\frac{1}{3}\right)$	
		$2B=1+\frac{1}{3}-\frac{2}{3}$	
		$2B=\frac{2}{3}\qquad\therefore B=\frac{1}{3}$	1
		$\frac{1}{(x+1)(x^2-x+1)}=\frac{-\frac{1}{3}}{x+1}+\frac{\left(\frac{1}{3}\right)x+\frac{1}{3}}{x^2-x+1}$	
		$\therefore\ \frac{x^4}{x^3+1}=x-\frac{x}{x^3+1}=x-\left(\frac{-\frac{1}{3}}{x+1}+\frac{\left(\frac{1}{3}\right)x+\frac{1}{3}}{x^2-x+1}\right)$	½

WINTER – 16 EXAMINATION

Model Answer Subject Code: 17105

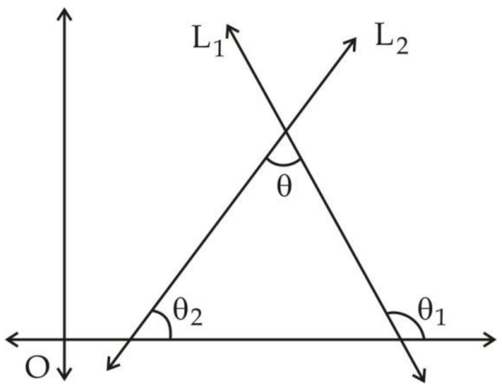
Q. No.	Sub Q. N.	Answer	Marking Scheme
5	b)	$A=\begin{bmatrix}2 & -2\\3 & 1\end{bmatrix}, B=\begin{bmatrix}-1 & 5\\4 & -3\end{bmatrix}, C=\begin{bmatrix}7 & -5\\0 & 5\end{bmatrix}$	**04**
		verify that : $(AB)C = A(BC)$	
	Ans	$AB=\begin{bmatrix}2 & -2\\3 & 1\end{bmatrix}\begin{bmatrix}-1 & 5\\4 & -3\end{bmatrix}$	
		$=\begin{bmatrix}-2-8 & 10+6\\-3+4 & 15-3\end{bmatrix}$	
		$=\begin{bmatrix}-10 & 16\\1 & 12\end{bmatrix}$	1
		$(AB)C=\begin{bmatrix}-10 & 16\\1 & 12\end{bmatrix}\begin{bmatrix}7 & -5\\0 & 5\end{bmatrix}$	
		$=\begin{bmatrix}-70+0 & 50+80\\7+0 & -5+60\end{bmatrix}$	
		$=\begin{bmatrix}-70 & 130\\7 & 55\end{bmatrix}$	1
		$BC=\begin{bmatrix}-1 & 5\\4 & -3\end{bmatrix}\begin{bmatrix}7 & -5\\0 & 5\end{bmatrix}$	
		$=\begin{bmatrix}-7+0 & 5+25\\28+0 & -20-15\end{bmatrix}$	
		$=\begin{bmatrix}-7 & 30\\28 & -35\end{bmatrix}$	1
		$A(BC)=\begin{bmatrix}2 & -2\\3 & 1\end{bmatrix}\begin{bmatrix}-7 & 30\\28 & -35\end{bmatrix}$	
		$=\begin{bmatrix}-14-56 & 60+70\\-21+28 & 90-35\end{bmatrix}$	
		$=\begin{bmatrix}-70 & 130\\7 & 55\end{bmatrix}$	1
		$\therefore (AB)C = A(BC)$	
		--	
	c)	Prove that : $\sin^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{8}{17}\right)=\sin^{-1}\left(\frac{77}{85}\right)$	**04**
	Ans	Let $\sin^{-1}\left(\frac{3}{5}\right)=A$ $\quad\therefore \sin A=\frac{3}{5}$	
		$\cos^2 A = 1-\sin^2 A$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
5	c)	$\cos^2 A = 1-\left(\frac{3}{5}\right)^2$	
		$= 1-\frac{9}{25}$	
		$=\frac{16}{25}$	
		$\cos A = \frac{4}{5}$	1
		Let $\sin^{-1}\left(\frac{8}{17}\right) = B \therefore \sin B = \frac{8}{17}$	
		$\cos^2 B = 1-\sin^2 B$	
		$= 1-\left(\frac{8}{17}\right)^2$	
		$= 1-\frac{64}{289}$	
		$=\frac{225}{289}$	
		$\cos B = \frac{15}{17}$	1
		$\sin(A+B) = \sin A.\cos B + \cos A.\sin B$	
		$\sin(A+B) = \left(\frac{3}{5}\right)\left(\frac{15}{17}\right)+\left(\frac{4}{5}\right)\left(\frac{8}{17}\right) = \frac{77}{85}$	1
		$A+B = \sin^{-1}\left(\frac{77}{85}\right)$	
		$\therefore \sin^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{8}{17}\right) = \sin^{-1}\left(\frac{77}{85}\right)$	1
		OR	
			
		$\sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{3}{4}\right)$ $\sin^{-1}\left(\frac{8}{17}\right) = \tan^{-1}\left(\frac{8}{15}\right)$ $\sin^{-1}\left(\frac{77}{85}\right) = \tan^{-1}\left(\frac{77}{36}\right)$	½+½+½
		$\text{LHS} = \sin^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{8}{17}\right)$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
5	c)	$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{8}{15}\right)$	
		$= \tan^{-1}\left(\frac{\frac{3}{4}+\frac{8}{15}}{1-\frac{3}{4}.\frac{8}{15}}\right)$	1
		$= \tan^{-1}\left(\frac{\frac{45+32}{60}}{\frac{60-24}{60}}\right)$	
		$= \tan^{-1}\left(\frac{77}{36}\right)$	1
		$= \sin^{-1}\left(\frac{77}{85}\right)$	½
		$= \text{RHS}$	
		--	
	d)	Without using calculator prove that $\sin\left(-420^0\right).\cos\left(390^0\right) + \cos\left(-660^0\right).\sin\left(330^0\right) = -1$	**04**
	Ans	$\sin(-420) = -\sin(420) = -\sin(4\times 90+60) = -\sin 60 = -\frac{\sqrt{3}}{2}$	1
		$\cos(390) = \cos(4\times 90+30) = \cos 30 = \frac{\sqrt{3}}{2}$	½
		$\cos(-660) = \cos(660) = \cos(7\times 90+30) = \sin 30 = \frac{1}{2}$	1
		$\sin(330) = \sin(3\times 90+60) = -\cos 60 = -\frac{1}{2}$	½
		$\text{LHS} = \sin(-420).\cos(390) + \cos(-660).\sin(330)$	
		$= \left(-\frac{\sqrt{3}}{2}\right).\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)$	
		$= \left(\frac{-3}{4}\right) + \left(\frac{-1}{4}\right)$	
		$= \frac{-4}{4}$	
		$= -1$	1
		$= \text{RHS}$	
		OR	

Q. No.	Sub Q. N.	Answer	Marking Scheme
5	d)	$\text{LHS} = \sin(-420).\cos(390)+\cos(-660).\sin(330)$	
		$= -\sin(360+60).\cos(360+30)+\cos(720-60).\sin(360-30)$	1
		$= -\sin 60.\cos 30+\cos 60(-\sin 30)$	1
		$= -\sin 60.\cos 30-\cos 60.\sin 30$	½
		$= -\sin(60+30)$	1
		$= -\sin 90 = -1 = \text{RHS}$	½
		--	
	e)	Prove that $\sin C+\sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$	**04**
	Ans	We know that	
		$\sin(A+B)+\sin(A-B) = 2\sin A\cos B$	1
		Let $A+B = C$	
		$A-B = D$	1
		$\therefore 2A = C+D$	
		$\therefore A = \frac{C+D}{2}$	1
		$\therefore B = \frac{C-D}{2}$	
		$\therefore \sin C+\sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$	1
	f)	f) Comment: Question is incomplete.If any student has considered appropriate data and attempted to solve then consider accordingly.If Question is considered as follows: Find the equation of line passing through the point of intersection of the lines $4x+3y=8$ and $x+y=1$ parallel to the line $5x-7y=3$	
	Ans	consider $4x+3y=8$ and $x+y=1$	
		$4x+3y=8$	
		$3x+3y=3$	
		$-\quad -\quad -$	
		$x=5 \qquad \therefore y=-4$	½+½
		point of intersection is $(5,-4)$	
		slope of line $5x-7y=3$ is $\text{m} = \frac{5}{7}$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
5	f)	$\therefore$ Equation of line is	
		$y - y_1 = m(x - x_1)$	
		$y + 4 = \frac{5}{7}(x - 5)$	1
		$5x - 7y - 53 = 0$	1

6		**Attempt any <u>FOUR</u> of the following:**	**16**
	a)	If M_1 and M_2 are the slopes of the lines, then prove that the angle between the two lines is $Q = \tan^{-1}\left\|\frac{m_1 - m_2}{1 + m_1 \cdot m_2}\right\|$	**04**
	Ans	Let θ_1 = Inclination of L_1	
		θ_2 =Inclination of L_2	
		$\therefore$ Slope of L_1 is $m_1 = \tan\theta_1$ Slope of L_2 is $m_2 = \tan\theta_2$	
			1
		$\therefore$ from figure,	
		Let $\theta = Q = \theta_1 - \theta_2$	
		$\therefore \tan Q = \tan(\theta_1 - \theta_2)$	1
		$= \frac{\tan\theta_1 - \tan\theta_2}{1 + \tan\theta_1 \tan\theta_2}$	1
		$\therefore \tan Q = \frac{m_1 - m_2}{1 + m_1 \cdot m_2}$	
		Since Q is acute	
		$\therefore \tan Q = \left\|\frac{m_1 - m_2}{1 + m_1 \cdot m_2}\right\|$	
		$\therefore Q = \tan^{-1}\left\|\frac{m_1 - m_2}{1 + m_1 \cdot m_2}\right\|$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
6	b)	Find the equation of the straight which passes through the point of intersection of lines $2x+3y=13, 5x-y=7$ and perpendicular to the line $2x-5y+7=0$	04
	Ans	consider $2x+3y=13, 5x-y=7$	
		$2x+3y=13$	
		$15x-3y=21$	
		$17x=34$	
		$x=2$	½
		$\therefore y=3$	½
		$\therefore$ point of intersection is $(2,3)$	
		slope of line $2x-5y+7=0$ is $\frac{2}{5}$	½
		$\therefore$ Slope of required line is $m=\frac{-5}{2}$	½
		$\therefore$ Equation of line is	
		$y-y_1=m(x-x_1)$	
		$y-3=\frac{-5}{2}(x-2)$	1
		$5x+2y-16=0$	1
	c)	Find the perpendicular distance between the parallel lines $3x+2y-6=0$ and $6x+4y-24=0$	04
	Ans	Given $3x+2y-6=0$, $3x+2y-12=0$	
		Perpendicular distance between the parallel lines is	
		$p=\left\lvert\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right\rvert$	1
		$=\left\lvert\frac{-6+12}{\sqrt{(3)^2+(2)^2}}\right\rvert$	2
		$=\frac{6}{\sqrt{13}}$	1
	d)	Find the length of the perpendicular from $(-3,-4)$ onto the line $4x-3y+20=0$	04
	Ans	Perpendicular distance $p=\left\lvert\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right\rvert$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
6	d)	$p=\left\|\dfrac{(4)(-3)+(-3)(-4)+20}{\sqrt{(4)^2+(-3)^2}}\right\|$ $=\left\|\dfrac{-12+12+20}{\sqrt{16+9}}\right\|$ $=\left\|\dfrac{20}{5}\right\|$ $=4$	2 1
	e) Ans	Resolve into partial fractions : $\dfrac{\sin\theta+1}{(\sin\theta+2)(\sin\theta+3)}$ $\dfrac{\sin\theta+1}{(\sin\theta+2)(\sin\theta+3)}$ put $\sin\theta=t$ $\dfrac{t+1}{(t+2)(t+3)}=\dfrac{A}{t+2}+\dfrac{B}{t+3}$ $t+1=A(t+3)+B(t+2)$ Put $t=-2$ $-2+1=A(-2+3)$ $A=-1$ Put $t=-3$ $-3+1=B(-3+2)$ $B=2$ $\therefore\dfrac{t+1}{(t+2)(t+3)}=\dfrac{-1}{t+2}+\dfrac{2}{t+3}$ $\therefore\dfrac{\sin\theta+1}{(\sin\theta+2)(\sin\theta+3)}=\dfrac{-1}{\sin\theta+2}+\dfrac{2}{\sin\theta+3}$	**04** 1 1 1 1
	f) Ans	If $\sin\theta=\dfrac{-4}{5},\pi<\theta<\dfrac{3\pi}{2}$ Find $(i)\sin2\theta,(ii)\cos2\theta,(iii)\tan2\theta$ $\sin\theta=\dfrac{-4}{5}$ $\cos\theta=\sqrt{1-\sin^2\theta}$	**04**

Q. No.	Sub Q. N.	Answer	Marking Scheme
6	f)	$\cos\theta = \sqrt{1-\left(\frac{4}{5}\right)^2}$ $= \sqrt{\frac{9}{25}}$ $= \frac{3}{5}$ but θ lies in 3^{rd} quadrant $\therefore \cos\theta = \frac{-3}{5}$	1
		$\sin 2\theta = 2\sin\theta.\cos\theta = 2\left(\frac{-4}{5}\right)\left(\frac{-3}{5}\right)$ $\therefore \sin 2\theta = \frac{24}{25}$	1
		$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $= \left(\frac{-3}{5}\right)^2 - \left(\frac{-4}{5}\right)^2$ $= \frac{9}{25} - \frac{16}{25}$ $= \frac{-7}{25}$	1
		$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{\frac{24}{25}}{\frac{-7}{25}} = \frac{-24}{7}$	1
		--- ***Important Note*** *In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.* --- ---	