



**Important Instructions to examiners:**

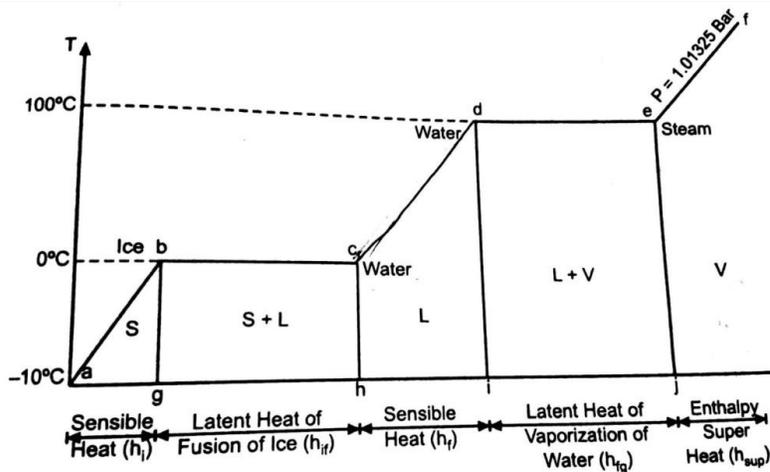
- 1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
- 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
- 3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills).
- 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
- 5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
- 6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
- 7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. No.	Answer	Marking Scheme
1.a)		<b>Attempt any SIX of the following</b>	12
	i	<b>Define</b> 1) <b>Thermodynamic state:</b> It is the condition of the system (when the system is in thermodynamic equilibrium) at any particular moment which can be identified by the statement of its properties such as pressure, temperature, volume, etc. 2) <b>Thermodynamic cycle:</b> When the process or processes performed on the system are in such a way that the final state is identical with initial state, it is known as thermodynamic cycle or cyclic process.	1 1
	ii	<b>State Zeroth law of thermodynamics.</b> <b>Ans:</b> When two systems are each in thermal equilibrium with a third system, then the two systems are also in thermal equilibrium with one another. This law provides the basis for temperature measurement.	2
	iii	<b>Ideal Gas:</b> It is defined as the gas which strictly obeys all the gas laws under all conditions of temperature and pressure. <b>Assumptions:</b> 1) Its evaporation from its liquid state is complete, the gas consists of a large number of molecules, which are in random motion and obey Newton's laws of motion; 2) the volume of the molecules is negligibly small compared to the volume occupied by the gas; and 3) no forces act on the molecules except during elastic collisions of negligible duration. ( <b>Any two assumptions</b> )	1 1
	iv	<b>State Avogadro's law:</b> <b>Ans:</b> Avogadro's law (sometimes referred to as Avogadro's hypothesis or Avogadro's principle) is an experimental gas law relating volume of a gas to the amount of substance of gas present. A modern statement of Avogadro's law is: Avogadro's law states that, "equal volumes of all gases, at the same temperature and pressure, have the same number of molecules". For a given mass of an ideal gas, the volume and amount (moles) of the gas are directly proportional if the temperature and pressure are constant.	1 1
	v	<b>Ans: i) Dryness fraction:-</b> It is defined as a fraction of steam that is in vapour form in liquid vapour is called dryness fraction. Dryness fraction is the ratio of the weight of actual dry steam to the weight of wet steam. $X = W_s / W_s + W = \text{Actual weight of dry steam} / \text{Weight of wet steam}$	1









Consider formation of steam from ice at  $-10^{\circ}\text{C}$  at atmospheric pressure of 1.01325 bar. This is explained from stage a to stage f as shown in fig.

Explanati  
on-2

a-b ( Heating of ice from  $-10^{\circ}\text{C}$  to melting point of ice) :-

In this heating, temp of ice will increase, which can be sensed by thermometer. Therefore it is called as 'sensible heat'. During this heat addition, solid phase exists. This sensible heat is denoted as 'hi '

b-c (Latent heat of fusion of ice):-

Point b is saturation state of ice. Further addition of heat will not increase the temp of ice but ice will start converting into water. Thus b-c is transformation phase. In this region, solid as well as liquid phase exists and heat supplied is called as 'Latent heat' as it is not sensed by thermometer. It is denoted by 'hi f '

c-d (Sensible heating of water from  $0^{\circ}\text{C}$  to saturation temp) :-

In this region water at  $0^{\circ}\text{C}$  is heated to  $100^{\circ}\text{C}$  (Boiling point of water at 1.01325 bar.) This change in temp is sensed by thermometer. So it is known as 'sensible heat' of water. It is denoted by hf . Here only liquid phase exist.

$$\text{Heat supplied} = h_f = mC_p (T_d - T_c) = 1 \times 4.18 (100 - 0) = 4.18 \text{ KJ/ kg}$$

d-e (Latent heat of vaporization of water) :-

At point 'd' water is at saturation state. Further addition of heat will not increase the temp but heat is transferred at constant temp to change the phase from liquid to vapour (steam). This is known as ' Latent Heat of vaporization of water'. It is denoted by ' h\_{fg} '. From steam tables, at 1.01325 bar,  $h_{fg} = 2256.9 \text{ KJ/Kg}$  . In this region, both liquid and vapour phase exist.

e-f (Superheating of steam):-

At point 'e', steam is present in dry saturated state. It does not contain any moisture, as all liquid is converted into steam. Further heating of steam will increase the temp of steam, which is known as superheating. Thus, superheating is defined as heating of





The sole effect external to system could be reduced to raise the weight but heat transfer of the effect is also considered.

Body or system contains heat. Heat is low grade energy.

Entire heat cannot be converted into work.

Application:- Any example related from Conduction, Convection or Radiation.

Heat Received by system is considered as Positive (+Q). Heat rejected by system is considered as Negative (-Q).

**Thermodynamic Work:**

It occurs due to Displacement or motion.

In stable system work transfer will be zero.

The sole effect external to the system could be reduced to rise of a weight. Body or system never contains work.

Heat is high grade energy.

Entire work can be converted into heat.

Application :- Any example where force and displacement occurs

Work done on the System is considered as Negative (-W)

Work done by the System is considered as Positive (+W)

1 mark  
for sign  
conventions

1 mark  
for any  
two  
points

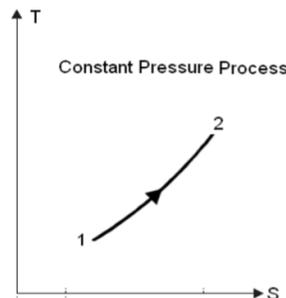
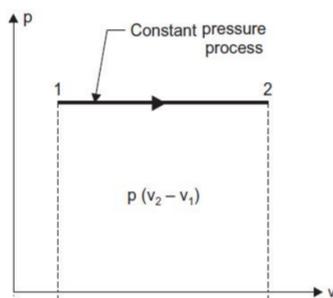
1 mark  
for sign  
conventions

**b. Q. Draw P-V and T-S chart of following gas processes.**

**i) Isobaric process**

**ii) Isothermal process**

**Ans: i) Isobaric process:-**



**ii) Isothermal Process:-**

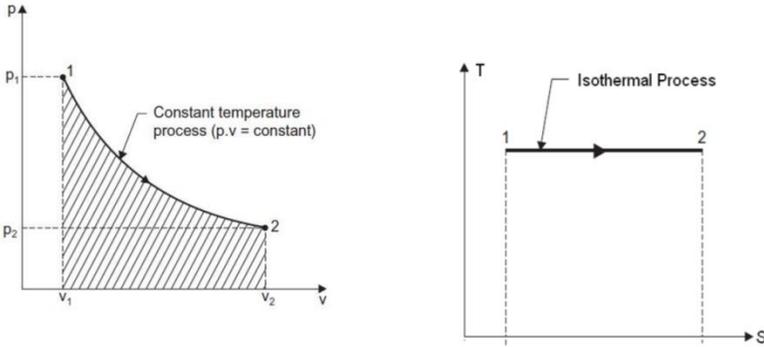
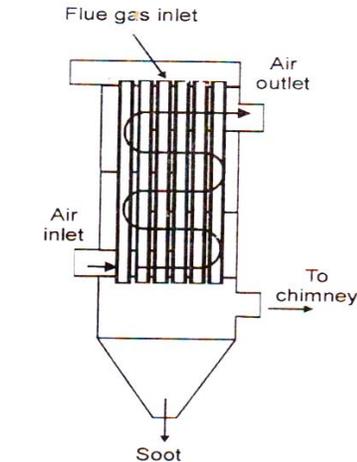
1 mark  
for P-V  
chart

1 mark  
for T-S  
chart

1 mark  
for P-V  
chart

1 mark  
for T-S



		chart
c.	<p><b>Q. Explain with neat sketch working of air preheater in boiler.</b></p> <p><b>Ans:- Construction and Working of air preheater: -</b></p> <p>Air Preheater is an important accessory for steam boiler. It consists of plates or the tubes with hot gases on one sides and air on other side. For air Preheater the cold air required for burning of the fuel is passed from bottom, it comes out after circulation from the top as a hot air which is then passed further for combustion of fuel. Due to arrangement of air Preheater the efficiency of boiler increases and also the smoke is reduced. Since it is comparatively high temperature air coming out from the Preheater, there is better combustion of fuel.</p>  <p style="text-align: center;">Air-Preheater (Tubular type)</p>	2 marks  2 marks for diagram
d.	<p><b>Q. Explain with neat sketch regenerative feed heating system.</b></p> <p><b>Ans:- Construction and Working: -</b></p> <ol style="list-style-type: none"> <li>1. The process of heating feed water is known as “regenerative feed heating”.</li> <li>2. The dry saturated steam, from the boiler enters the turbine at a higher temperature</li> <li>3. Then this high temperature dry steam expands adiabatically to a lower temperature</li> <li>4. The expansion of steam in the same way as that in Rankine and Carnot cycle.</li> <li>5. Now the condensate from the condenser is pumped back and circulated around the</li> </ol>	2 marks



turbine casing.

6. The circulated condensate is in direction opposite to the steam flow in turbine.

7. The steam is thus heated before entering the boiler.

8. Such a system of heating is known as regenerative feed heating.

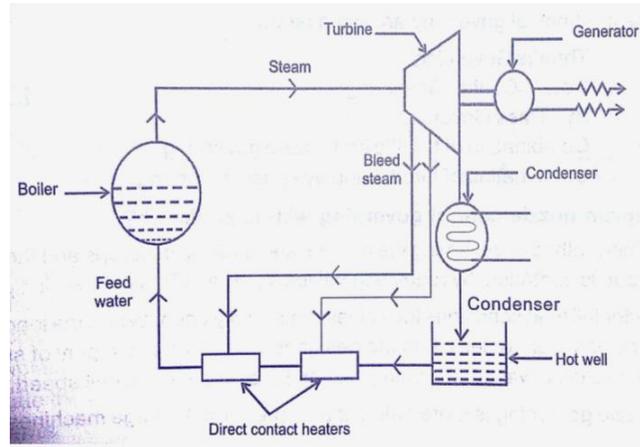
**Advantages of feed heating**

1. The thermal efficiency of boiler increases as heat input decreases.

2. Capacity of Condenser reduced.

3. Reduce fuel consumption.

4. Thermal stress in the boiler reduces as hot feed water is supplied.



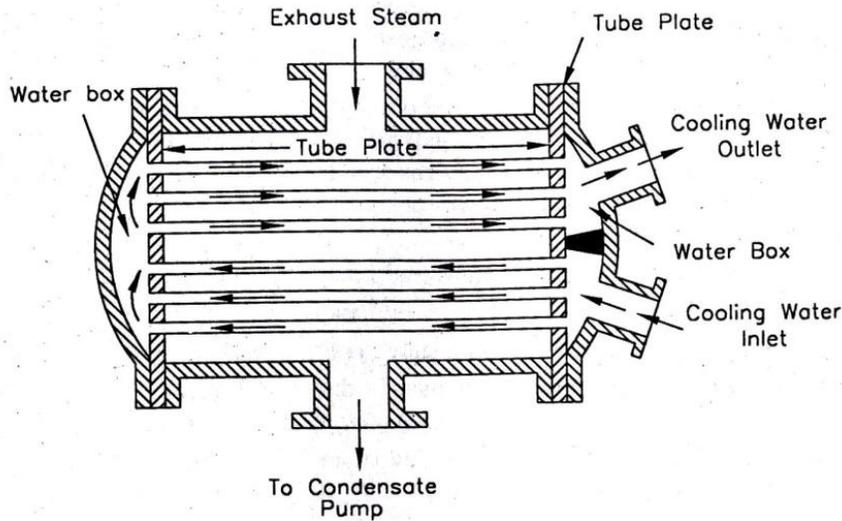
2 marks  
for  
diagram

e. **Q. Explain with neat sketch working of 2 pass surface condenser.**

**Ans:-**

In this type of condenser, the cooling water and exhaust steam do not come in direct contact with each other. This type is generally used where large quantities of inferior water are available and where better quality of feed water to the boiler must be used. It consists of an air tight cylindrical shell closed at each end as shown in the figure. A number of water tubes are fixed in the tube plates which are located between each cover head and shell. The exhaust steam from the prime mover enters at the top of the condenser and surrounds the condenser tubes through which cooling water is circulated under force. The steam gets condensed as it comes in contact with the cold surface of the tubes. The cooling water flows in one direction through the first set of the tubes situated in the lower half of condenser and returns in opposite direction through the second set of the tubes situated in upper half of the condenser. The warm cooling water coming out from the condenser is discharged into cooling tower, river or cooling pond. The condensate is taken out from the condenser by a separate extraction pump and air is removed by an air pump.

2 marks



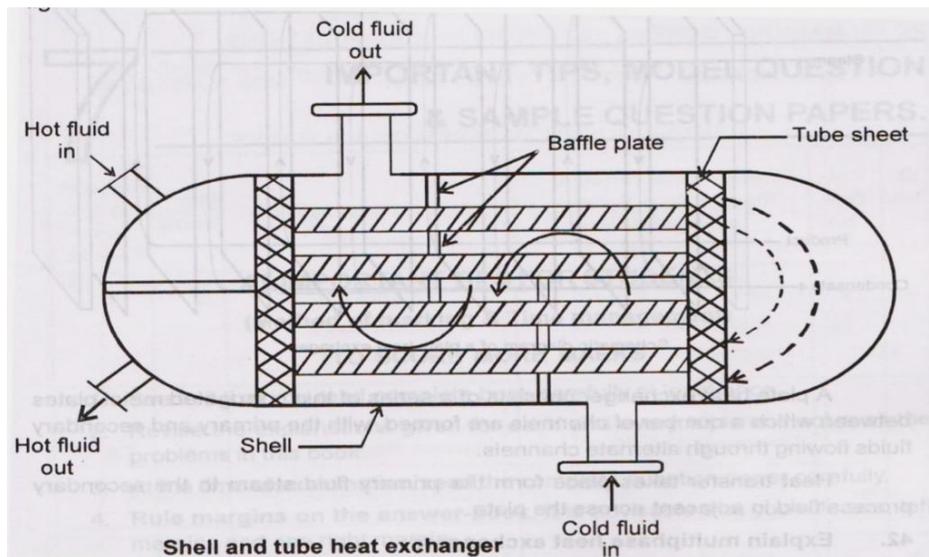
2 marks for diagram

f. Q. Explain the working of shell and tube type heat exchanger with neat sketch.

Ans:

**Shell and tube heat exchanger:** Shell and tube heat exchanger consists of large number of tubes having parallel arrangement inside the shell. The ends of the tube on both sides are connected to a plate called as tube sheet. The whole assembly is called bundle of tubes. The shell should be a house of all the tubes and it should be leakage free. It is closed on both sides. The one fluid flow inside the tube it is called as tube side fluid and the other one which flows outside the tube known as shell side fluid. The transfer of heat takes place between the two fluids through the medium of tube surfaces. Naturally heat flow takes place from hot fluid to the cold fluid. The arrangement of inlet and outlet for hot and cold fluid is shown in figure

2 marks



2 marks

4. Attempt any FOUR of the following



and velocity variation for the same.

**Ans:- Construction and Working:**

1. The impulse turbine consists of one set of nozzle, which is followed by one set of moving blades as shown as in figure.
2. In this type of turbine, power is developed by the impulsive force.
3. The high velocity steam jets are obtained by expansion of steam in the stationary nozzle only
4. Steam then passes at high velocity through moving blade with no drop in pressure but a gradual reduction in velocity.
5. Thus in pure impulse turbine the high velocity jet having nozzle strikes on the blades mounted on the wheel attached to the shaft.
6. These blades change the direction of steam and hence momentum of the jet of steam, which rotates the shaft.
7. The nozzle axis is inclined to an angle to the tangent of the rotor.

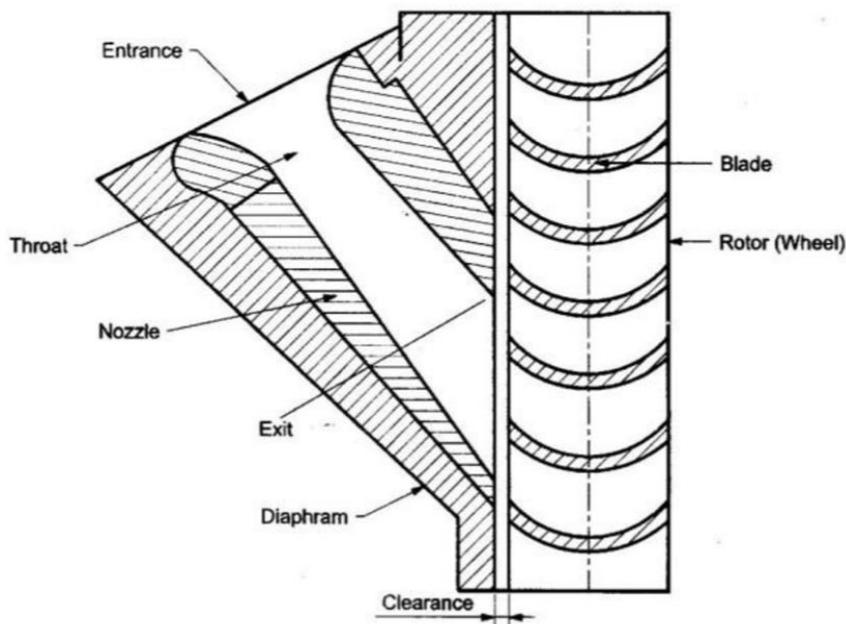


Fig. Simple Impulse Turbine

2 marks  
for  
constructi  
on and  
working

1 mark  
for sketch  
of simple  
impulse  
turbine

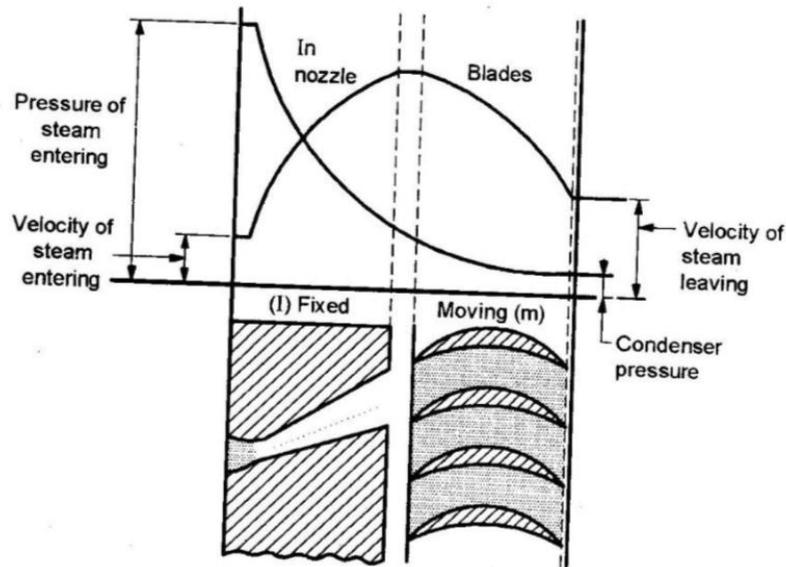


Fig: Pressure and Velocity variation

1 mark  
for sketch  
of  
pressure  
and  
velocity  
variation

- d. Q. A boiler is made of iron plates 12 mm thick. If the temperature of outside surface is 120°C and that of inner is 100°C. Calculate the mass of water evaporated per hour. Assume that area of heating surface is 5 m<sup>2</sup>. K for iron is 84 W/mK and Latent heat of water at 100°C = 2260 KJ.

Ans:-

Given:- L = 12 mm = 0.012 m ,

T<sub>1</sub> = 120°C,

T<sub>2</sub> = 100°C,

m = ?

A = 5 m<sup>2</sup>,

K = 84 W/mK,

H<sub>fg</sub> = 2260 KJ.

Solution:

Heat transfer from the flat wall is given by,

$$Q = \frac{T_1 - T_2}{\left(\frac{L}{KA}\right)} = \frac{120 - 100}{\left(\frac{0.012}{84 \times 5}\right)} = 700000 \text{ W}$$

$$Q = 700000 \text{ W}$$

We have to calculate mass of water evaporated per hour

$$Q = 700000 \text{ W ( J/s)} = 2520 \times 10^6 \text{ J/hr} = 2520 \times 10^3 \text{ kJ/hr}$$

$$\text{Mass of water evaporated per hour} = 2520 \times 10^3 / 2260$$

$$= 1115 \text{ kg}$$

2 marks

2 marks



<p>e.</p>	<p><b>Q. Find the condenser efficiency, when cooling water enters a condenser at a temperature of 28<sup>0</sup>C and leaves at 39<sup>0</sup>C. The vacuum produced is 705 mm of Hg and barometer reads 760 mm of Hg.</b></p> <p><b>Ans:-</b>          Given:- T<sub>0</sub>= 39<sup>0</sup>C, T<sub>i</sub>= 28<sup>0</sup>C,          Condenser Vacuum= 705 mm of Hg          Barometric Pressure= 760 mm of Hg          Solution:-          Pressure in condenser = Barometric Pressure–Condenser Vacuum          Pressure in condenser = 760 – 705          Pressure in condenser = 55 mm of Hg ..... 1 mark          Pressure in condenser = 55 x 0.00133  <b>Pressure in condenser = 0.07315 bar ..... 1 mark</b>  <b>From steam table</b>, corresponding to pressure of 0.07315 bar, we can find the vacuum temperature (T<sub>s</sub>)    <p style="text-align: right;"><b>T<sub>s</sub>= 39.8<sup>0</sup>C..... 1 mark</b></p> <math display="block">C_{condenser\ efficiency} = \frac{T_0 - T_i}{T_s - T_i}</math> <math display="block">C_{condenser\ efficiency} = \frac{39 - 28}{39.8 - 28}</math> <math display="block">C_{condenser\ efficiency} = 93.22\%</math> <p style="text-align: right;">..... 1 mark</p> <b>Hence condenser efficiency is 93.22%</b> </p>	
<p>f.</p>	<p><b>Q. Define heat transfer. State different modes of heat transfer and explain any one with suitable example.</b></p> <p><b>Ans:-</b>          Heat is energy, which flow from one region to another due to temperature differences.  <b>Heat transfer</b> is defined as the transmission of energy from one region to another as a result of temperature difference.          Modes of heat transfer are          1. Conduction, 2. Convection, 3. Radiation</p> <p>1. <b>Conduction</b> – It is transmission of heat energy between two bodies or two parts of same body through molecules which are more or less stationery e.g. (heating or solid)</p> <p>2. <b>Convection</b> – It is process of heat transfer from higher temperature to lower</p>	<p>1 mark for definition</p> <p>2 marks for classificat ion</p> <p>1 mark for</p>



	<p>temperature due to movement of matter or fluid molecules (density differences) is called convection e.g. heating of water.</p> <p>3. <b>Radiation</b> – It is process of heat transfer between two bodies without any carrying medium through different kind of electro-magnetic wave.</p>	<p>anyone type explanation</p>
Q.5.	<p>Attempt any <b>TWO</b> of the following:</p>	
a)	<p>Write steady flow energy equation stating the meaning of each term in equation and apply it to boiler, steam nozzle and steam turbine.</p>	<p>08 Marks</p>
Sol.	<p>Steady flow process ( open system ) :</p> <p>Hence steady flow equation can be expressed as:</p> <p>Internal Energy at 1 + Potential Energy at 1 + Kinetic Energy at 1 + Flow work at 1 + Heat supplied = Internal Energy at 2 + Potential Energy at 2 + Kinetic Energy at 2 + Flow work at 2 + Work done</p> <p>i.e.</p> <p>Hence the steady flow energy equation is,</p> $h_1 + \frac{c_1^2}{2} + Z_1 g + Q = h_2 + \frac{c_2^2}{2} + Z_2 g + W$ <p>Where,</p> <p><math>h_1</math> &amp; <math>h_2</math> = Enthalpy at inlet and outlet in <math>\frac{J}{Kg}</math></p>	<p>02 Marks for equation</p> <p>02 Marks Boiler equation</p> <p>02 Marks Steam Nozzle</p> <p>02 Marks For</p>



$C_1$  &  $C_2$  = velocity at inlet and out of fluid---- $\frac{m}{s}$

$Z_1$  and  $Z_2$  = height of inlet & outlet above datum

$Q$  = heat supplied per -----Joule

$W$  = work done by 1 kg of fluid----Joule

$PV$  = Flow work-----N-m or Joule

**Boiler:**

Boiler is a steel closed vessel, which converts the water into steam.

Applying energy equation,

$$Z_1 g + U_1 + \frac{c_1^2}{2} + P_1 V_1 + Q = Z_2 g + U_2 + \frac{c_2^2}{2} + P_2 V_2 + W$$

$$Z_1 = Z_2 = 0$$

$$\text{K.E.} = 0$$

$$W = 0$$

Hence equation is,

$$P_1 V_1 + U_1 + Q = P_2 V_2 + U_2$$

$$h_1 + Q = h_2$$

$$Q = h_2 - h_1$$

**Steam Nozzle:**

It is a passage of varying cross-sectional area by means of which pressure energy of flowing fluid is change into K.E. It is used for producing high velocity jet.

steam  
turbine

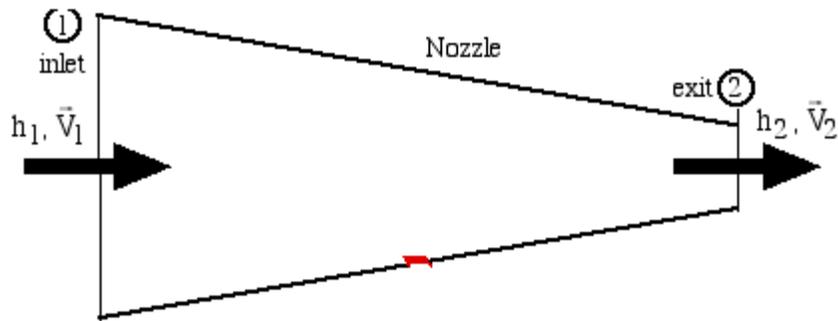


Fig. Nozzle

Apply the Steady flow energy equation on steam nozzle,

$$h_1 + \frac{c_1^2}{2} + Z_1 g + Q = h_2 + \frac{c_2^2}{2} + Z_2 g + W$$

For Steam nozzle,

$$\text{P.E.} = 0,$$

$$W = 0,$$

$$Q = 0,$$

Apply on equation 1;

We get,

$$h_1 + \frac{c_1^2}{2} = h_2 + \frac{c_2^2}{2}$$

$$\frac{c_2^2}{2} = h_1 - h_2 + \frac{c_1^2}{2}$$

$$c_2^2 = 2(h_1 - h_2) + c_1^2$$

$$c_2 = \sqrt{2(h_1 - h_2) + c_1^2}$$

Using above equation exit velocity of fluid can be calculated here velocity is in m/s and enthalpy (h) in Joules.

**Steam Turbine:**



In a steam turbine steam or gas is passed through the turbine and a part of its energy is converted into work in the turbine. This output of the turbine runs a generator to produce electricity, the steam or gas leaves the turbine at lower pressure and temperature.

Applying energy equation to the system,

$$Z_1 g + U_1 + \frac{c_1^2}{2} + P_1 V_1 + Q = Z_2 g + U_2 + \frac{c_2^2}{2} + P_2 V_2 + W$$

But in this case,

$$Z_1 = Z_2 = 0$$

$$\Delta Z = 0$$

$$h_1 + \frac{c_1^2}{2} - Q = h_2 + \frac{c_2^2}{2} + W$$

The sign of Q is negative because heat comes out of the boundary while W is positive because work is done by the system.

b) **What is compounding of steam turbine? Explain with sketch pressure compounding.**

**8 Marks**

**Sol. Compounding:**

- ✓ Compounding is the method adopted to reduce the speed of turbine rotor at the same time to utilize internal energy of steam effectively.
- ✓ In pressure compounding arrangement of blades and nozzles are made as below;

N-M-N-M-N-M

Where;

N= Nozzle

M= Moving blade

- ✓ Nozzle is reduced the pressure and increase the velocity.
- ✓ Moving blade absorb the kinetic energy of steam.
- ✓ Figure shows the rings of fixed nozzles incorporated between the rings of moving blades. The steam at boiler pressure enters the first set of nozzles and expands

**4 Marks**



partially. The kinetic energy of the steam thus obtain is absorbed by the moving blades (stage 1). The steam then expands partially in second set of nozzles where its pressure again falls and the velocity increases; the kinetic energy so obtained is absorbed by the second ring of moving blades(stage 2). This is repeated in stage 3 and steam finally leaves the turbine at low velocity and pressure. The number of stages depends on the number of rows of nozzles through which the steam must pass. The changes in pressure and velocity are shown in figure.

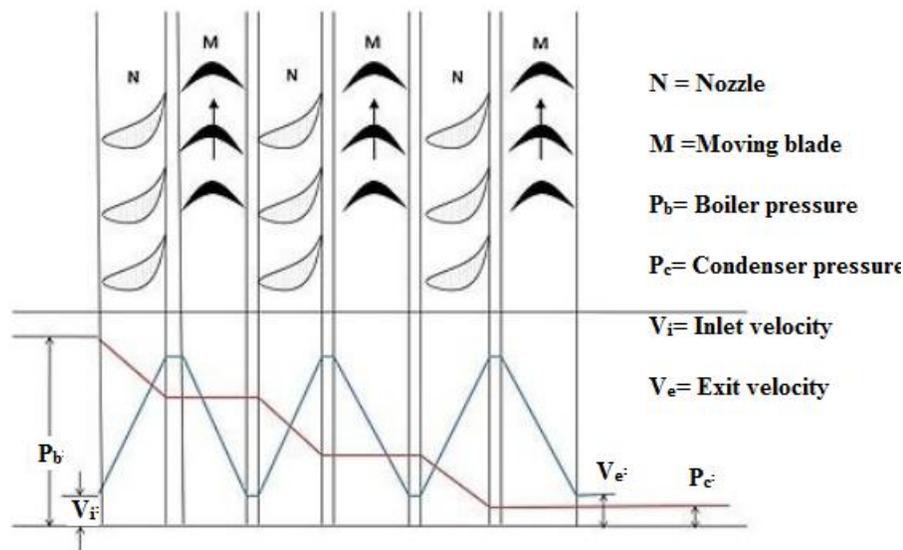


Figure: Pressure compounding

4 Marks

c)

A  $\text{CO}_2$  gas expand adiabatically from a pressure and volume of 7 bar and  $0.03 \text{ m}^3$  respectively to a pressure of 1.4 bar.

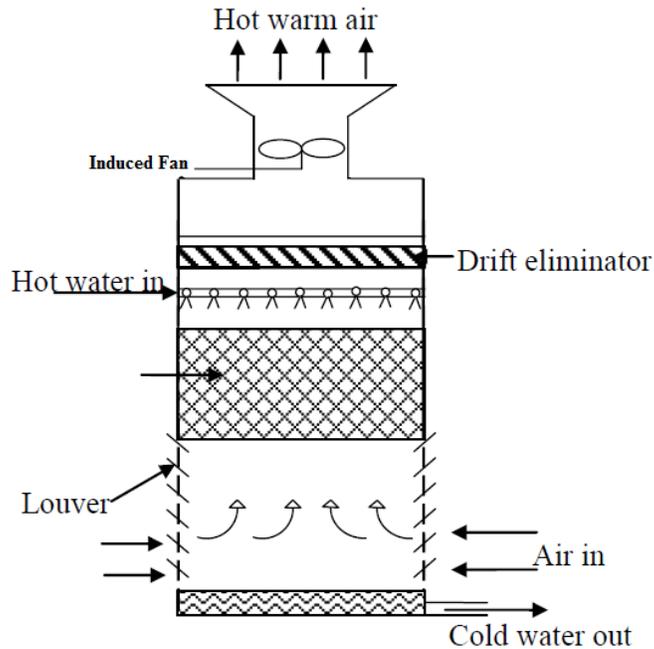
8 Marks

Determine:

- Final volume
- Work done
- Changes in internal energy
- Heat transfer

Take  $C_p = 1.046 \text{ KJ/Kg}^0\text{K}$  and  $C_v = 0.752 \text{ KJ/Kg}^0\text{K}$





02 Marks  
for Fig.

02 Marks  
for  
Working

**Figure: Induced draught tower**

One or more fans are installed at the top of the tower. Depending on the air inlet and flow pattern, induced draft towers are of two types, cross-flow and counter flow towers.

Major advantages of countercurrent induced draft cooling tower (a) Relatively dry air contacts the coldest water at the bottom of the cooling tower (b) Humid air is in contact with the warm water and hence maximum average driving force prevails for both heat and mass transfer.

Disadvantage of induced draft towers compared to forced draft towers It consumes more horse power. Cross-flow induced draft cooling tower requires less motor horse power than countercurrent induced draft cooling towers.

b)

**Draw the sketch of Loeffler boiler. Show the path of flue gasses. Describe the working of boiler also.**

8 Marks



Sol.

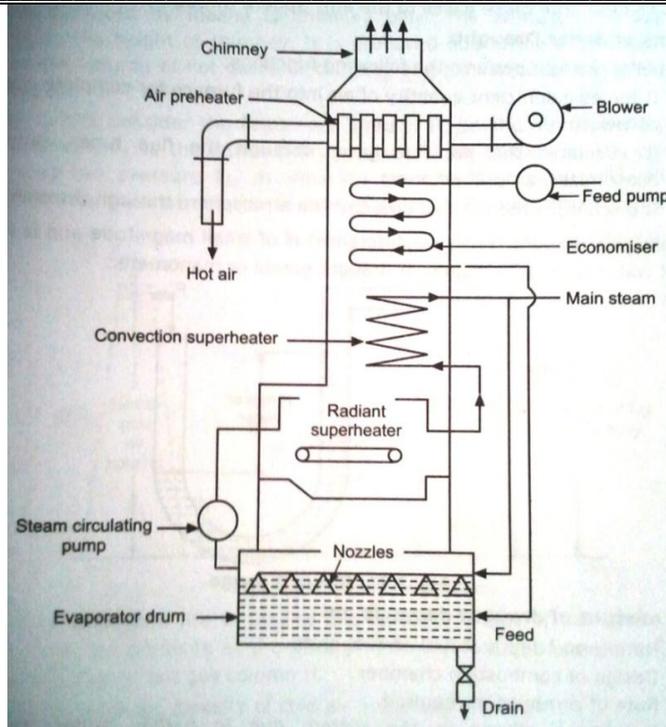


Figure: Loeffler Boiler

**Working:**

This is water tube boiler using a forced circulation. In this boiler water is heated mainly by means of superheated steam. The steam will act as heat carrying and heat absorbing medium. Thus, boiler uses the circulation of steam instead of water and difficulty of deposition of salt and sediment in boiler tubes is completely eliminated.

Loeffler boiler consist of evaporator drum, which may be placed at any convenient point outside the furnace setting. The feed water pumps feed the water to economizer, which is placed in the path of flue gases. The economizer extracts sensible heat from flue gases and hot water at temp. close to saturation temp. is passed to evaporator drum. From superheater big portion of steam (about 3/4) is trapped off for external use and remainder portion (about 1/4) is passed to evaporator drum.

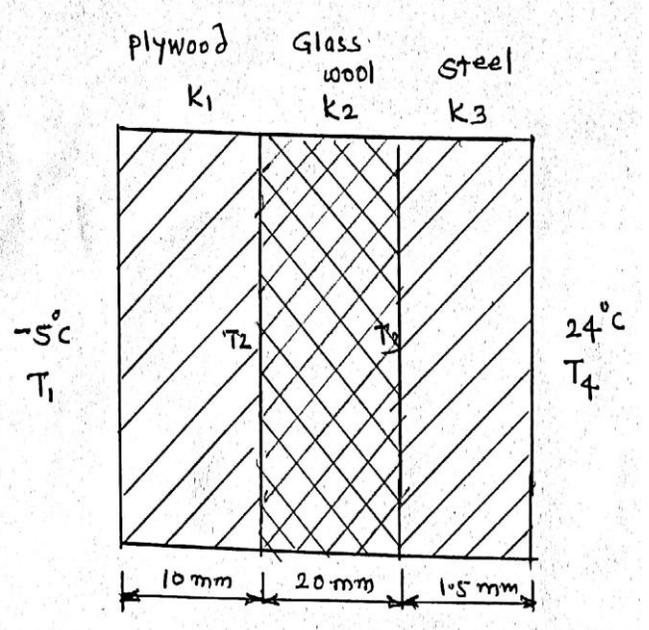
The evaporator drum which is used provided with set of nozzle through which steam enters in evaporator drum. Nozzles are made of special design to avoid priming and noise. The steam from evaporator drum is passed to superheater through circulating pump. The air preheater maybe placed in path of flue gases to supply the hot air in combustion chamber.

Loeffler boiler has steam-generating capacity of 100 tons/hour at 140 bar pressure.

04 Marks  
for  
Figure

04 Marks  
for  
Working



c)	(i)	State Stefan's Boltzman law.	4 Marks
Sol.		<p>Stefan-Boltzmann law state that the total radiant heat energy emitted from a surface is proportional to the fourth power of its absolute temperature.</p> $E = \sigma T^4$ <p>Where; E = Radiant heat energy emitted from surface in <math>W/m^2</math></p> <p>T= Absolute temperature in K</p> <p>And</p> <p><math>\sigma</math> = Stefan's Boltzman Constant</p> $= 5.67 \times 10^{-8} W/m^2K^4$	02 Marks for Statements 02 Marks for Formula and Value
	(ii)	<p>A wall of refrigerated van of 1.5 mm of steel at outer surface, 10 mm plywood at the inner surface and 2 cm of glass wool in between. Calculate the rate of heat flow, if the temperature of inside and outside surfaces are <math>-5^{\circ}C</math> and <math>24^{\circ}C</math>. Take; K (for steel ) = <math>23.2 W/mK</math></p> <p>K (for Glass wool) = <math>0.14 W/mk</math>, K (for Plywood) = <math>0.052 W/mK</math></p>	4 Marks
Sol.		 <p>The diagram shows a cross-section of a composite wall with three layers. From left to right: Plywood (10 mm thick, thermal conductivity <math>K_1</math>), Glass wool (20 mm thick, thermal conductivity <math>K_2</math>), and Steel (1.5 mm thick, thermal conductivity <math>K_3</math>). The inner surface of the plywood is at <math>-5^{\circ}C</math> (<math>T_1</math>). The interface between plywood and glass wool is at <math>T_2</math>. The interface between glass wool and steel is at <math>T_3</math>. The outer surface of the steel is at <math>24^{\circ}C</math> (<math>T_4</math>). The thicknesses are indicated at the bottom: 10 mm for plywood, 20 mm for glass wool, and 1.5 mm for steel.</p>	2 Marks For Formula 2 Marks for Ans.



$$K_1 \text{ for plywood} = 0.052 \text{ W/mK}$$

$$K_2 \text{ for Glass wool} = 0.14 \text{ W/mK}$$

$$K_3 \text{ for steel} = 23.2 \text{ W/mK}$$

$$T_1 = -5 + 273 = 268 \text{ K}$$

$$T_4 = 24 + 273 = 297 \text{ K}$$

$$\begin{aligned} \frac{Q}{A} &= \frac{T_1 - T_4}{\frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3}} \\ &= \frac{268 - 297}{\frac{0.01}{0.052} + \frac{0.02}{0.14} + \frac{0.0095}{23.2}} \\ &= \frac{-29}{0.192 + 0.142 + 0.00064} \\ &= -86.66 \text{ W/m}^2 \end{aligned}$$

The negative sign indicate that heat flow from outside to inside.

If  $T_1$  as  $24^\circ\text{C}$  and  $T_4$  as  $-5^\circ\text{C}$  is considered, answer is  $86.6 \text{ W/m}^2$