Que. No.	Sub. Que.	Model Answers	Marks	Total Marks

Important Instructions to the Examiners:

1) The Answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more importance. (Not applicable for subject English and Communication Skills.)
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by the candidate and those in the model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's Answers and the model answer.
6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

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1)		**Attempt any TEN of the following:**		
	a)	Solve to find the value of x, if $\begin{vmatrix} 2 & 3 & 1 \\ 6 & x & 2 \\ 4 & x & -2 \end{vmatrix} = 0$		
	Ans.	$\therefore 2(-2x-2x)-3(-12-8)+1(6x-4x)=0$	1	
		$\therefore -8x+60+2x=0$		
		$\therefore -6x+60=0$	½	
		$\therefore -6x=-60 \quad or \quad 6x=60$		
		$\therefore x=10$	½	**2**
		OR		
		$\begin{vmatrix} 2 & 3 & 1 \\ 6 & x & 2 \\ 4 & x & -2 \end{vmatrix} = 2(-2x-2x)-3(-12-8)+1(6x-4x)$	1	
		$= -8x+60+2x$		
		$= -6x+60$	½	
		$\therefore -6x+60=0$		
		$\therefore -6x=-60 \quad or \quad 6x=60$		
		$\therefore x=10$	½	**2**
	b)	If $A=\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix}$, find $3A-2B$.		
	Ans.	$3A=3\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}=\begin{bmatrix} 6 & 9 \\ 12 & 21 \end{bmatrix}$	½	
		$2B=2\begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix}=\begin{bmatrix} 2 & 6 \\ 8 & 12 \end{bmatrix}$	½	
		$\therefore 3A-2B=\begin{bmatrix} 6 & 9 \\ 12 & 21 \end{bmatrix}-\begin{bmatrix} 2 & 6 \\ 8 & 12 \end{bmatrix}$		
		$=\begin{bmatrix} 4 & 3 \\ 4 & 9 \end{bmatrix}$	1	**2**
		OR		
		$\therefore 3A-2B=3\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}-2\begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix}$		
		$=\begin{bmatrix} 6 & 9 \\ 12 & 21 \end{bmatrix}-\begin{bmatrix} 2 & 6 \\ 8 & 12 \end{bmatrix}$	½+½	
		$=\begin{bmatrix} 4 & 3 \\ 4 & 9 \end{bmatrix}$	1	**2**

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1)	c)	If $A=\begin{bmatrix}1 & -5\\ 6 & 4\end{bmatrix}$, $B=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$, find the matrix $AB-2I$, where I is 2×2 identity matrix.		
	Ans.	$\therefore AB=\begin{bmatrix}1 & -5\\ 6 & 4\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$		
		$=\begin{bmatrix}1+0 & 0+5\\ 6+0 & 0-4\end{bmatrix}$		
		$=\begin{bmatrix}1 & 5\\ 6 & -4\end{bmatrix}$	1	
		$\therefore AB-2I=\begin{bmatrix}1 & 5\\ 6 & -4\end{bmatrix}-2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$		
		$=\begin{bmatrix}1 & 5\\ 6 & -4\end{bmatrix}-\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$	½	
		$=\begin{bmatrix}-1 & 5\\ 6 & -6\end{bmatrix}$	½	**2**
		OR		
		$\therefore AB-2I=\begin{bmatrix}1 & -5\\ 6 & 4\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}-2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$		
		$=\begin{bmatrix}1+0 & 0+5\\ 6+0 & 0-4\end{bmatrix}-2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$		
		$=\begin{bmatrix}1 & 5\\ 6 & -4\end{bmatrix}-2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$	1	
		$=\begin{bmatrix}1 & 5\\ 6 & -4\end{bmatrix}-\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$	½	
		$=\begin{bmatrix}-1 & 5\\ 6 & -6\end{bmatrix}$	½	**2**
		---		
	d)	If $A=\begin{bmatrix}1 & 3\\ 2 & 4\end{bmatrix}$, $B=\begin{bmatrix}2 & -1\\ 3 & 2\end{bmatrix}$, verify that $(A+B)^T=A^T+B^T$		
	Ans.	$A+B=\begin{bmatrix}1 & 3\\ 2 & 4\end{bmatrix}+\begin{bmatrix}2 & -1\\ 3 & 2\end{bmatrix}$		
		$=\begin{bmatrix}3 & 2\\ 5 & 6\end{bmatrix}$	½	

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1)		$\therefore (A+B)^T = \begin{bmatrix} 3 & 5 \\ 2 & 6 \end{bmatrix}$ ....(i)	½	
		$\therefore A^T + B^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$		
		$= \begin{bmatrix} 3 & 5 \\ 2 & 6 \end{bmatrix}$ ....(ii)	½	
		$\therefore$ by (i) and (ii),		
		$(A+B)^T = A^T + B^T$	½	**2**
		OR		
		$\therefore (A+B)^T = \left\{ \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} \right\}^T$		
		$= \begin{bmatrix} 3 & 2 \\ 5 & 6 \end{bmatrix}^T$	½	
		$= \begin{bmatrix} 3 & 5 \\ 2 & 6 \end{bmatrix}$ ....(i)	½	
		$\therefore A^T + B^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$		
		$= \begin{bmatrix} 3 & 5 \\ 2 & 6 \end{bmatrix}$ ....(ii)	½	
		$\therefore$ by (i) and (ii),		
		$(A+B)^T = A^T + B^T$	½	**2**
		---		
	e)	Resolve $\frac{1}{1-x^2}$ into partial fractions.		
	Ans.	$\frac{1}{1-x^2} = \frac{1}{(1-x)(1+x)} = \frac{A}{1-x} + \frac{B}{1+x}$		
		$\therefore \boxed{1 = (1+x)A + (1-x)B}$		
		$Put\ 1-x=0 \quad \therefore x=1$		
		$\therefore 1 = (1+1)A + 0$		
		$\therefore \boxed{\frac{1}{2} = A}$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)		$Put\ 1+x=0 \quad \therefore x=-1$		
		$\therefore 1=0+(1+1)B$		
		$\therefore \boxed{\frac{1}{2}=B}$	½	
		$\therefore \boxed{\frac{1}{1-x^2}=\frac{\frac{1}{2}}{1-x}+\frac{\frac{1}{2}}{1+x}}$	½	**2**
		Note for partial fraction problems: The problems of partial fractions could also be solved by the method of "equating equal power coefficients". This method is also applicable. Give appropriate marks in accordance with the scheme of marking in the later problems as the solution by this method is not discussed. For the sake of convenience, the solution of the above problem with the help of this method is illustrated hereunder.		
		$\frac{1}{1-x^2}=\frac{1}{(1-x)(1+x)}=\frac{A}{1-x}+\frac{B}{1+x}$		
		$\therefore 1=(1+x)A+(1-x)B$		
		$\therefore 1=(A+B)+(A-B)x$		
		$\therefore 1+0x=(A+B)+(A-B)x$		
		$\therefore A+B=1 \quad and \quad A-B=0$		
		$\therefore A+B=1$ $A-B=0$ $+$ ————————		
		$\therefore 2A=1$		
		$\therefore \boxed{A=\frac{1}{2}}$	1	
		$\therefore B=A$	½	
		$\therefore \boxed{B=\frac{1}{2}}$		
		$\therefore \therefore \boxed{\frac{1}{1-x^2}=\frac{\frac{1}{2}}{1-x}+\frac{\frac{1}{2}}{1+x}}$	½	**2**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)	f)	Without using calculator find the value of $\sin(-330^\circ)$		
	Ans.	$\sin(-330^\circ) = -\sin 330^\circ$		
		$= -\sin(360^\circ - 30^\circ)$	½	
		$= -\sin(-30^\circ)$	½	
		$= \sin(30^\circ)$	½	
		$= \frac{1}{2} \quad or \quad 0.5$	½	**2**
		OR		
		$\sin(-330^\circ) = -\sin 330^\circ$		
		$= -\sin(270^\circ + 60^\circ)$	½	
		$= -\sin(3\times 90^\circ + 60^\circ)$	½	
		$= +\cos(60^\circ)$	½	
		$= \frac{1}{2} \quad or \quad 0.5$	½	**2**
		OR		
		$\sin(-330^\circ) = \sin(-360^\circ + 30^\circ)$	½	
		$= \sin(30^\circ)$	1	
		$= \frac{1}{2} \quad or \quad 0.5$	½	**2**
	g)	Write the following formulae: (i) $\sin(A+B)$ and (ii) $\cos(A-B)$		
	Ans. i)	$\sin(A+B) = \sin A\cos B + \cos A\sin B$	1	
	ii)	$\cos(A-B) = \cos A\cos B + \sin A\sin B$	1	**2**
	h)	If $\sin A = \frac{1}{2}$, find $\sin 3A$.		
	Ans.	$\sin 3A = 3\sin A - 4\sin^3 A$	1	
		$= 3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3$	½	
		$= 1 \qquad ...(*)$	½	**2**
		Note (*): Due to the use of advance scientific calculator, writing directly the step (*) is allowed. No marks to be deducted.		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)		**OR**		
		Given that $\sin A = \frac{1}{2}$.		
		$\therefore A = \sin^{-1}\left(\frac{1}{2}\right) = 30^o$	1	
		$\therefore \sin 3A = \sin(3\times 30^o) = \sin(90^o)$	½	
		$= 1$	½	**2**
		--		
	i)	Evaluate $2\cos 75^o \cos 15^o$ without using calculator.		
	Ans.	$2\cos 75^o \cos 15^o = \cos(75^o + 15^o) + \cos(75^o - 15^o)$	½	
		$= \cos(90^o) + \cos(60^o)$	½	
		$= 0 + \frac{1}{2}$	½	
		$= \frac{1}{2}$ *or* 0.5	½	**2**
		OR		
		$\cos 75^o = \cos(30^o + 45^o)$		
		$= \cos 30^o \cos 45^o - \sin 30^o \sin 45^o$		
		$= \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} - \frac{1}{2}\cdot\frac{1}{\sqrt{2}}$		
		$= \frac{\sqrt{3}-1}{2\sqrt{2}}$	½	
		$\cos 15^o = \cos(45^o - 30^o)$		
		$= \cos 45^o \cos 30^o + \sin 45^o \sin 30^o$		
		$= \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\cdot\frac{1}{2}$		
		$= \frac{\sqrt{3}+1}{2\sqrt{2}}$	½	
		$2\cos 75^o \cos 15^o = 2\times\frac{\sqrt{3}-1}{2\sqrt{2}}\times\frac{\sqrt{3}+1}{2\sqrt{2}}$	½	
		$= \frac{1}{2}$ *or* 0.5	½	**2**
		--		

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1)	j)	Prove that $\cos^{-1}(-x) = \pi - \cos^{-1} x$.		
	Ans.	$Let\ \cos^{-1}(x) = \theta$		
		$\therefore x = \cos\theta$	½	
		$\therefore -x = -\cos\theta$		
		$\therefore -x = \cos(\pi - \theta)$	½	
		$\therefore \cos^{-1}(-x) = \pi - \theta$	½	
		$= \pi - \cos^{-1} x$	½	**2**
		--		
	k)	Find the slope of a line passing through points $(-1, -2)$ and $(-3, 8)$.		
	Ans.	$slope\quad m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8+2}{-3+1}$	1	
		$= -5$	1	**2**
		--		
	l)	Find the range and the coefficient of range for the following data: 120, 100, 130, 50, 150.		
	Ans.	$Smallest\ \ Value\ \ S = 50, \qquad Largest\ \ Value\ \ L = 150$		
		$\therefore Range = L - S = 150 - 50$		
		$= 100$	1	
		$Coeff.\ of\ \ Range = \frac{L-S}{L+S} = \frac{150-50}{150+50}$	½	
		$= \frac{1}{2}\quad or \quad 0.5$	½	**2**
		--		
2)	a)	The voltage in an electric circuit are related by the following equations: $V_1 + V_2 + V_3 = 9,\ \ V_1 - V_2 + V_3 = 3,\ \ V_1 + V_2 - V_3 = 1$. Find $V_1,\ V_2\ and\ V_3$.		
		Note: As in this problem the method of solution is not mentioned/prescribed and as the problem is to be solved within the prescribed curriculum only, the problem can be solved by two different methods: Cramer's Method and Inverse Matrix Method. But the problem is not supposed to be solved by the method of simultaneous linear equation as prescribed in school algebra.		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2)		$V_1+V_2+V_3=9$ $V_1-V_2+V_3=3$ $V_1+V_2-V_3=1$ $D=\begin{vmatrix}1&1&1\\1&-1&1\\1&1&-1\end{vmatrix}=1(1-1)-1(-1-1)+1(1+1)$ $=4$	1	
		$D_1=\begin{vmatrix}9&1&1\\3&-1&1\\1&1&-1\end{vmatrix}=9(1-1)-1(-3-1)+1(3+1)$ $=8$	½	
		$D_2=\begin{vmatrix}1&9&1\\1&3&1\\1&1&-1\end{vmatrix}=1(-3-1)-9(-1-1)+1(1-3)$ $=12$	½	
		$D_3=\begin{vmatrix}1&1&9\\1&-1&3\\1&1&1\end{vmatrix}=1(-1-3)-1(1-3)+9(1+1)$ $=16$	½	
		$\therefore V_1=\frac{D_1}{D}=\frac{8}{4}=2$	½	
		$V_2=\frac{D_2}{D}=\frac{12}{4}=3$	½	
		$V_3=\frac{D_3}{D}=\frac{16}{4}=4$	½	**4**
		OR		
		$V_1+V_2+V_3=9$ $V_1-V_2+V_3=3$ $V_1+V_2-V_3=1$ $\therefore A=\begin{bmatrix}1&1&1\\1&-1&1\\1&1&-1\end{bmatrix},\quad X=\begin{bmatrix}V_1\\V_2\\V_3\end{bmatrix},\quad B=\begin{bmatrix}9\\3\\1\end{bmatrix}$ $\therefore \lvert A\rvert=\begin{vmatrix}1&1&1\\1&-1&1\\1&1&-1\end{vmatrix}=1(1-1)-1(-1-1)+1(1+1)$ $=4$	1	

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2)		The cofactor matrix of A is, $$C(A)=\begin{bmatrix} \begin{vmatrix}-1 & 1\\ 1 & -1\end{vmatrix} & -\begin{vmatrix}1 & 1\\ 1 & -1\end{vmatrix} & \begin{vmatrix}1 & -1\\ 1 & 1\end{vmatrix} \\ -\begin{vmatrix}1 & 1\\ 1 & -1\end{vmatrix} & \begin{vmatrix}1 & 1\\ 1 & -1\end{vmatrix} & -\begin{vmatrix}1 & 1\\ 1 & 1\end{vmatrix} \\ \begin{vmatrix}1 & 1\\ -1 & 1\end{vmatrix} & -\begin{vmatrix}1 & 1\\ 1 & 1\end{vmatrix} & \begin{vmatrix}1 & 1\\ 1 & -1\end{vmatrix} \end{bmatrix}$$		
		$$=\begin{bmatrix}0 & 2 & 2\\ 2 & -2 & 0\\ 2 & 0 & -2\end{bmatrix} \quad ----------(*)$$	1½	
		$$\therefore adj(A)=\begin{bmatrix}0 & 2 & 2\\ 2 & -2 & 0\\ 2 & 0 & -2\end{bmatrix}$$ $$\therefore A^{-1}=\frac{1}{\lvert A\rvert}adj(A)$$		
		$$=\frac{1}{4}\begin{bmatrix}0 & 2 & 2\\ 2 & -2 & 0\\ 2 & 0 & -2\end{bmatrix}$$	½	
		$\therefore$ the solution is, $$X=A^{-1}B$$ $$=\frac{1}{4}\begin{bmatrix}0 & 2 & 2\\ 2 & -2 & 0\\ 2 & 0 & -2\end{bmatrix}\begin{bmatrix}9\\ 3\\ 1\end{bmatrix}$$ $$=\frac{1}{4}\begin{bmatrix}8\\ 12\\ 16\end{bmatrix}$$ $$=\begin{bmatrix}2\\ 3\\ 4\end{bmatrix}$$		
		$$\therefore V_1=2,\quad V_2=3,\quad V_3=4$$	1	**4**
		Note: 1) (*) In the matrix *C(A)*, if 1 to 3 elements are wrong (either in sign or value), deduct ½ mark, if 4 to 6 elements are wrong, deduct 1 marks, if 7 to 9 are wrong, deduct all the 1½ marks. Further, if all the elements in the last i.e., *adj (A)* are correct, then only give 1 mark.		

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2)		**Note 2)** To find the adj (A), there are various methods are prescribed in the MSBTE Curriculum which are discussed hereunder for the sake of convenience for marks distribution.		
		The matrix of minors is, $M(A)=\begin{bmatrix} \begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} \end{bmatrix}$ $=\begin{bmatrix} 0 & -2 & 2 \\ -2 & -2 & 0 \\ 2 & 0 & -2 \end{bmatrix}$		
		$\therefore$ the matrix of cofactors is, $C(A)=\begin{bmatrix} 0 & 2 & 2 \\ 2 & -2 & 0 \\ 2 & 0 & -2 \end{bmatrix}$	1½	
		$\therefore adj(A)=\begin{bmatrix} 0 & 2 & 2 \\ 2 & -2 & 0 \\ 2 & 0 & -2 \end{bmatrix}$		
		OR		
		$A_{11}=\begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix}$ $A_{12}=-\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}$ $A_{13}=\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}$		
		$A_{21}=-\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}$ $A_{22}=\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}$ $A_{23}=-\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$		
		$A_{31}=\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix}$ $A_{32}=-\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$ $A_{33}=\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}$		
		Note: In the above, if 1 to 3 elements are wrong, deduct ½ mark, if 4 to 6 elements are wrong, deduct 1 marks, and if 7 to 9 are wrong, deduct all the marks. Further, if all the elements in the following matrices C(A) and adj (A) are correct, then only give the marks.		

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2)		$\therefore$ the matrix of cofactors is,		
		$C(A)=\begin{bmatrix}0 & 2 & 2\\2 & -2 & 0\\2 & 0 & -2\end{bmatrix}$	1½	
		$\therefore adj(A)=\begin{bmatrix}0 & 2 & 2\\2 & -2 & 0\\2 & 0 & -2\end{bmatrix}$		
		--		
	b)	If $A=\begin{bmatrix}x & 2 & -5\\3 & 1 & 2y\end{bmatrix}$ and $B=\begin{bmatrix}2y+5 & 6 & -15\\9 & 3 & -6\end{bmatrix}$ and if $3A=B$, find $x,\ y$.		
	Ans.	$Given\ \ 3A=B$		
		$\therefore 3\begin{bmatrix}x & 2 & -5\\3 & 1 & 2y\end{bmatrix}=\begin{bmatrix}2y+5 & 6 & -15\\9 & 3 & -6\end{bmatrix}$		
		$\therefore \begin{bmatrix}3x & 6 & -15\\9 & 3 & 6y\end{bmatrix}=\begin{bmatrix}2y+5 & 6 & -15\\9 & 3 & -6\end{bmatrix}$	1	
		$\therefore 3x=2y+5 \quad and \quad 6y=-6$	1	
		$\therefore x=1 \quad and \quad y=-1$	1+1	**4**
		--		
	c)	If $A=\begin{bmatrix}0 & 1 & -1\\3 & -2 & 3\\2 & -2 & 3\end{bmatrix}$, show that $A^2=I$.		
	Ans.	$A^2=A\cdot A=\begin{bmatrix}0 & 1 & -1\\3 & -2 & 3\\2 & -2 & 3\end{bmatrix}\begin{bmatrix}0 & 1 & -1\\3 & -2 & 3\\2 & -2 & 3\end{bmatrix}$		
		$=\begin{bmatrix}0+3-2 & 0-2+2 & 0+3-3\\0-6+6 & 3+4-6 & -3-6+9\\0-6+6 & 2+4-6 & -2-6+9\end{bmatrix}$	2	
		$=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$	1	
		$=I$	1	**4**

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2)	d)	Using matrix inversion method, solve the equations: $5x+y=13,\quad 3x+2y=5$.		
	Ans.	$5x+\ y=13$ $3x+2y=5$ $\therefore A=\begin{bmatrix}5&1\\3&2\end{bmatrix},\quad X=\begin{bmatrix}x\\y\end{bmatrix},\quad B=\begin{bmatrix}13\\5\end{bmatrix}$		
		$\therefore \lvert A\rvert=\begin{vmatrix}5&1\\3&2\end{vmatrix}=10-3=7$	1	
		$C(A)=\begin{bmatrix}2&-3\\-1&5\end{bmatrix}$	1	
		$\therefore adj(A)=\begin{bmatrix}2&-1\\-3&5\end{bmatrix}$	½	
		$\therefore A^{-1}=\frac{1}{\lvert A\rvert}adj(A)$ $=\frac{1}{7}\begin{bmatrix}2&-1\\-3&5\end{bmatrix}$	½	
		$\therefore$ the solution is, $X=A^{-1}B$ $=\frac{1}{7}\begin{bmatrix}2&-1\\-3&5\end{bmatrix}\begin{bmatrix}13\\5\end{bmatrix}$ $=\frac{1}{7}\begin{bmatrix}21\\-14\end{bmatrix}$ $=\begin{bmatrix}3\\-2\end{bmatrix}$ $\therefore x=3,\quad y=-2$	1	**4**
		Note: To find the adj (A), students may follow any of methods as shown in the question 2 (a). Please give appropriate marks, as per scheme of marking discussed in the question 2 (a). ---		
	d)	Resolve into partial fractions: $\frac{x^2+4x+1}{(x-1)(x+1)(x+3)}$		
	Ans.	$\frac{x^2+4x+1}{(x-1)(x+1)(x+3)}=\frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{x+3}$ $\therefore x^2+4x+1=(x-1)(x+1)(x+3)\left[\frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{x+3}\right]$		

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2)		$\therefore \boxed{x^2+4x+1=(x+1)(x+3)A+(x-1)(x+3)B+(x-1)(x+1)C}$		
		$Put\ x=1$		
		$\therefore 1^2+4(1)+1=(1+1)(1+3)A+0+0$		
		$\therefore 6=8A$		
		$\therefore \boxed{\frac{3}{4}=A}$	1	
		$Put\ x=-1$		
		$\therefore (-1)^2+4(-1)+1=0+(-1-1)(-1+3)B+0$		
		$\therefore -2=-4B$		
		$\therefore \boxed{\frac{1}{2}=B}$	1	
		$Put\ x=-3$		
		$\therefore (-3)^2+4(-3)+1=0+0+(-3-1)(-3+1)C$		
		$\therefore -2=8C$		
		$\therefore \boxed{-\frac{1}{4}=C}$	1	
		$\therefore \boxed{\frac{x^2+4x+1}{(x-1)(x+1)(x+3)}=\frac{\frac{3}{4}}{x-1}+\frac{\frac{1}{2}}{x+1}+\frac{-\frac{1}{4}}{x+3}}$	1	**4**
		---		
	f)	Resolve into partial fractions: $\frac{x^2+23x}{(x+3)(x^2+1)}$		
	Ans.	$\frac{x^2+23x}{(x+3)(x^2+1)}=\frac{A}{x+3}+\frac{Bx+C}{x^2+1}$		
		$\therefore x^2+23x=(x+3)(x^2+1)\left[\frac{A}{x+3}+\frac{Bx+C}{x^2+1}\right]$		
		$\therefore \boxed{x^2+23x=(x^2+1)A+(x+3)(Bx+C)}$		
		$Put\ x=-3$		
		$\therefore (-3)^2+23(-3)=((-3)^2+1)A+0$		
		$\therefore -60=10A$		
		$\therefore \boxed{-6=A}$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2)		$Put\ x=0$ $\therefore 0^2+23(0)=(0^2+1)A+(0+3)(0+C)$ $\therefore 0=A+3C$ $\therefore 0=-6+3C$ $\therefore 6=3C$ $\therefore \boxed{2=C}$	1	
		$Put\ x=1$ $\therefore 1^2+23(1)=(1^2+1)A+(1+3)(B+C)$ $\therefore 24=2A+4B+4C$ $\therefore 24=2(-6)+4B+4(2)$ $\therefore 28=4B$ $\therefore \boxed{7=B}$	1	
		$\therefore \boxed{\dfrac{x^2+23x}{(x+3)(x^2+1)}=\dfrac{-6}{x+3}+\dfrac{7x+2}{x^2+1}}$	1	**4**
		Note for Partial Fraction Methods: The above Q. 2 (e) & (f) problems of partial fractions could be solved by the method of "equating equal power coefficients" also. This method, illustrated in the solution of Q. 1 (e), is also applicable. Give appropriate marks in accordance with the scheme of marking. As this method is very tedious and complicated, hardly someone use this method in such cases. So such solution methods for partial fraction problems are not illustrated herein. --		
3)		**Attempt any four**		
	a)	If $A=\begin{bmatrix}2&4&4\\4&2&4\\4&4&2\end{bmatrix}$, show that A^2-8A is a scalar matrix.		
	Ans.	$A^2-8A=A\cdot A-8A=\begin{bmatrix}2&4&4\\4&2&4\\4&4&2\end{bmatrix}\begin{bmatrix}2&4&4\\4&2&4\\4&4&2\end{bmatrix}-8\begin{bmatrix}2&4&4\\4&2&4\\4&4&2\end{bmatrix}$ $=\begin{bmatrix}4+16+16&8+8+16&8+16+8\\8+8+16&16+4+16&16+8+8\\8+16+8&16+8+8&16+16+4\end{bmatrix}-\begin{bmatrix}16&32&32\\32&16&32\\32&32&16\end{bmatrix}$	1+1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)		$=\begin{bmatrix}36&32&32\\32&36&32\\32&32&36\end{bmatrix}-\begin{bmatrix}16&32&32\\32&16&32\\32&32&16\end{bmatrix}$		
		$=\begin{bmatrix}36-16&32-32&32-32\\32-32&36-16&32-32\\32-32&32-32&36-16\end{bmatrix}$		
		$=\begin{bmatrix}20&0&0\\0&20&0\\0&0&20\end{bmatrix}$	1	
		$\therefore A^2-8A$ is a scalar matrix.	1	**4**
		OR		
		$A^2=A\cdot A=\begin{bmatrix}2&4&4\\4&2&4\\4&4&2\end{bmatrix}\begin{bmatrix}2&4&4\\4&2&4\\4&4&2\end{bmatrix}$		
		$=\begin{bmatrix}4+16+16&8+8+16&8+16+8\\8+8+16&16+4+16&16+8+8\\8+16+8&16+8+8&16+16+4\end{bmatrix}$		
		$=\begin{bmatrix}36&32&32\\32&36&32\\32&32&36\end{bmatrix}$	1	
		$8A=8\begin{bmatrix}2&4&4\\4&2&4\\4&4&2\end{bmatrix}=\begin{bmatrix}16&32&32\\32&16&32\\32&32&16\end{bmatrix}$	1	
		$\therefore A^2-8A=\begin{bmatrix}36&32&32\\32&36&32\\32&32&36\end{bmatrix}-\begin{bmatrix}16&32&32\\32&16&32\\32&32&16\end{bmatrix}$		
		$=\begin{bmatrix}36-16&32-32&32-32\\32-32&36-16&32-32\\32-32&32-32&36-16\end{bmatrix}$		
		$=\begin{bmatrix}20&0&0\\0&20&0\\0&0&20\end{bmatrix}$	1	
		$\therefore A^2-8A$ is a scalar matrix.	1	**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)	b)	Resolve into partial fractions: $\dfrac{x^2}{(x^2+1)(x^2+2)}$		
	Ans.	$Put\ \ x^2 = y$ $\dfrac{x^2}{(x^2+1)(x^2+2)} = \dfrac{y}{(y+1)(y+2)} = \dfrac{A}{y+1} + \dfrac{B}{y+2}$ $\therefore y = (y+1)(y+2)\left[\dfrac{A}{y+1} + \dfrac{B}{y+2}\right]$ $\therefore \boxed{y = (y+2)A + (y+1)B}$ $Put\ \ y = -1$ $\therefore -1 = (-1+2)A + 0$ $\therefore \boxed{-1 = A}$	1	
		$Put\ \ y = -2$ $\therefore -2 = 0 + (-2+1)B$ $\therefore -2 = -B$ $\therefore \boxed{2 = B}$	1	
		$\therefore \dfrac{y}{(y+1)(y+2)} = \dfrac{-1}{y+1} + \dfrac{2}{y+2}$	1	
		$\therefore \boxed{\dfrac{x^2}{(x^2+1)(x^2+2)} = \dfrac{-1}{x^2+1} + \dfrac{2}{x^2+2}}$	1	**4**
		---		
	c)	Resolve into partial fractions: $\dfrac{2x+1}{x^2(x+1)}$		
	Ans.	$\dfrac{2x+1}{x^2(x+1)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+1}$ $\therefore 2x+1 = x^2(x+1)\left[\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+1}\right]$ $\therefore \boxed{2x+1 = x(x+1)A + (x+1)B + x^2C}$ $Put\ \ x = 0$ $\therefore 2(0)+1 = 0 + (0+1)B + 0$ $\therefore \boxed{1 = B}$	1	
		$Put\ \ x = -1$ $\therefore 2(-1)+1 = 0 + 0 + (-1)^2 C$ $\therefore \boxed{-1 = C}$	1	

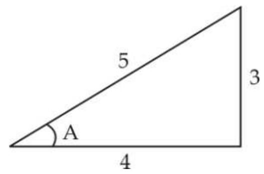
Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)		$Put\ x=1$ $\therefore 2(1)+1=1(1+1)A+(1+1)B+1^2C$ $\therefore 3=2A+2B+C$ $\therefore 3=2A+2(1)-1$ $\therefore 3=2A+1$ $\therefore 2=2A$ $\therefore \boxed{1=A}$ $\therefore \boxed{\frac{2x+1}{x^2(x+1)}=\frac{1}{x}+\frac{1}{x^2}+\frac{-1}{x+1}}$	 1 1	 **4**
	d)	Prove that $\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$		
	Ans.	$\tan(A-B)=\frac{\sin(A-B)}{\cos(A-B)}$ $=\frac{\sin A\cos B-\cos A\sin B}{\cos A\cos B+\sin A\sin B}$ $=\frac{\frac{\sin A\cos B-\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\cos B+\sin A\sin B}{\cos A\cos B}}$ $=\frac{\frac{\sin A\cos B}{\cos A\cos B}-\frac{\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B}+\frac{\sin A\sin B}{\cos A\cos B}}$ $=\frac{\tan A-\tan B}{1+\tan A\tan B}$	 1 1 1 1	 **4**
	e)	Prove that $\frac{\sin A+2\sin 2A+\sin 3A}{\cos A+2\cos 2A+\cos 3A}=\tan 2A$		
	Ans.	$\frac{\sin A+2\sin 2A+\sin 3A}{\cos A+2\cos 2A+\cos 3A}=\frac{\sin A+\sin 3A+2\sin 2A}{\cos A+\cos 3A+2\cos 2A}$ $=\frac{2\sin 2A\cos(-A)+2\sin 2A}{2\cos 2A\cos(-A)+2\cos 2A}$ $=\frac{\sin 2A[2\cos(-A)+2]}{\cos 2A[2\cos(-A)+2]}$ $=\frac{\sin 2A}{\cos 2A}$ $=\tan 2A$	 1 1 1 1	 **4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)		**OR**		
		$\frac{\sin A+2\sin 2A+\sin 3A}{\cos A+2\cos 2A+\cos 3A}=\frac{(\sin A+\sin 2A)+(\sin 2A+\sin 3A)}{(\cos A+\cos 2A)+(\cos 2A+\cos 3A)}$		
		$=\frac{2\sin\frac{3A}{2}\cos\left(-\frac{A}{2}\right)+2\sin\frac{5A}{2}\cos\left(-\frac{A}{2}\right)}{2\cos\frac{3A}{2}\cos\left(-\frac{A}{2}\right)+2\cos\frac{5A}{2}\cos\left(-\frac{A}{2}\right)}$	1	
		$=\frac{2\cos\left(-\frac{A}{2}\right)\left[\sin\frac{3A}{2}+\sin\frac{5A}{2}\right]}{2\cos\left(-\frac{A}{2}\right)\left[\cos\frac{3A}{2}+\cos\frac{5A}{2}\right]}$		
		$=\frac{\sin\frac{3A}{2}+\sin\frac{5A}{2}}{\cos\frac{3A}{2}+\cos\frac{5A}{2}}$	1	
		$=\frac{2\sin 2A\cos\left(-\frac{A}{2}\right)}{2\cos 2A\cos\left(-\frac{A}{2}\right)}$	1	
		$=\tan 2A$	1	**4**
		--		
	f)	Prove that $\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{13}\right)=\cot^{-1}\left(\frac{9}{2}\right)$		
	Ans.	$\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{13}\right)=\tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{13}}{1-\frac{1}{7}\cdot\frac{1}{13}}\right)$ ...(*)	2	
		$=\tan^{-1}\left(\frac{20}{90}\right)$		
		$=\tan^{-1}\left(\frac{2}{9}\right)$ ...(**)	1	
		$=\cot^{-1}\left(\frac{9}{2}\right)$	1	**4**
		Note: Due to advance use of calculator, students may directly write the step (**) after step (*) which is permissible.		

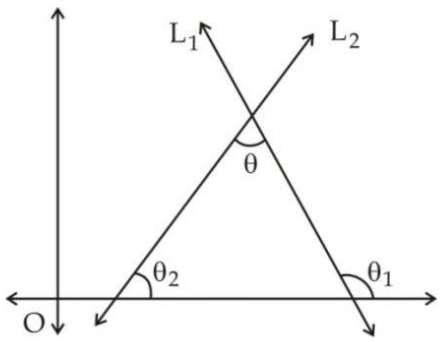
Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		**Attempt any four:**		
	a)	Prove that $\sqrt{2+\sqrt{2+2\cos 4\theta}} = 2\cos\theta$		
	Ans.	$\sqrt{2+\sqrt{2+2\cos 4\theta}} = \sqrt{2+\sqrt{2\left(1+\cos 4\theta\right)}}$		
		$= \sqrt{2+\sqrt{2\left(2\cos^2 2\theta\right)}}$	1	
		$= \sqrt{2+\sqrt{4\cos^2 2\theta}}$		
		$= \sqrt{2+2\cos 2\theta}$	1	
		$= \sqrt{2\left(1+\cos 2\theta\right)}$		
		$= \sqrt{2\left(2\cos^2\theta\right)}$	1	
		$= \sqrt{4\cos^2\theta}$		
		$= 2\cos\theta$	1	**4**
	b)	Prove that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$		
	Ans.	$\cos 3\theta = \cos\left(\theta+2\theta\right)$		
		$= \cos\theta\cos 2\theta - \sin\theta\sin 2\theta$	1	
		$= \cos\theta\cdot\left(2\cos^2\theta-1\right) - \sin\theta\cdot 2\sin\theta\cos\theta$	1	
		$= \cos\theta\cdot\left(2\cos^2\theta-1\right) - 2\sin^2\theta\cdot\cos\theta$		
		$= 2\cos^3\theta - \cos\theta - 2\left(1-\cos^2\theta\right)\cos\theta$	1	
		$= 4\cos^3\theta - 3\cos\theta$	1	**4**
	c)	Prove that $\dfrac{\sin 7x+\sin x}{\cos 5x-\cos 3x} = \sin 2x - \cos 2x\cot x$		
	Ans.	$\dfrac{\sin 7x+\sin x}{\cos 5x-\cos 3x} = \dfrac{2\sin\left(\frac{7x+x}{2}\right)\cos\left(\frac{7x-x}{2}\right)}{-2\sin\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)}$		
		$= \dfrac{2\sin\left(4x\right)\cos\left(3x\right)}{-2\sin\left(4x\right)\sin\left(x\right)}$	1	
		$= \dfrac{\cos\left(3x\right)}{-\sin x}$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		$=\frac{\cos(x+2x)}{-\sin x}$		
		$=\frac{\cos x\cos 2x-\sin x\sin 2x}{-\sin x}$	1	
		$=\frac{\cos x\cos 2x}{-\sin x}-\frac{\sin x\sin 2x}{-\sin x}$		
		$=-\cot x\cos 2x+\sin 2x$	1	**4**
		$OR\ \ \sin 2x-\cot x\cos 2x$		
		--		
	d)	Prove that $\sin 20^\circ\sin 40^\circ\sin 60^\circ\sin 80^\circ=\frac{3}{16}$		
	Ans.	$\sin 20^\circ\sin 40^\circ\sin 60^\circ\sin 80^\circ=\sin 20^\circ\sin 40^\circ\frac{\sqrt{3}}{2}\sin 80^\circ$	½	
		$=\frac{\sqrt{3}}{2}\cdot\frac{-1}{2}(-2\sin 20^\circ\sin 40^\circ)\sin 80^\circ$		
		$=-\frac{\sqrt{3}}{4}(\cos 60^\circ-\cos 20^\circ)\sin 80^\circ$	½	
		$=-\frac{\sqrt{3}}{4}\left(\frac{1}{2}-\cos 20^\circ\right)\sin 80^\circ$	½	
		$=-\frac{\sqrt{3}}{4}\left(\frac{1}{2}\sin 80^\circ-\sin 80^\circ\cos 20^\circ\right)$		
		$=-\frac{\sqrt{3}}{4}\left(\frac{1}{2}\sin 80^\circ-\frac{1}{2}\cdot 2\sin 80^\circ\cos 20^\circ\right)$		
		$=-\frac{\sqrt{3}}{4}\cdot\frac{1}{2}\left[\sin 80^\circ-(\sin 100^\circ+\sin 60^\circ)\right]$	½	
		$=-\frac{\sqrt{3}}{8}\left[\sin 80^\circ-\sin 100^\circ-\frac{\sqrt{3}}{2}\right]$	½	
		$=-\frac{\sqrt{3}}{8}\left[2\cos 90^\circ\sin 20^\circ-\frac{\sqrt{3}}{2}\right]$	½	
		$=-\frac{\sqrt{3}}{8}\left[0-\frac{\sqrt{3}}{2}\right]$	½	
		$=\frac{3}{16}$	½	**4**
		Note: 1) If the above problem is proved, using the values of $\sin 20^\circ,\ \sin 40^\circ,\ \sin 80^\circ$ with the help of calculator, no marks to be given because under the constraint of the MSBTE Curriculum, it is expected that such problems are to be solved without using calculator.		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		**Note 2)** The above problem may also be solved by making various combinations of sine ratios. Consequently the solutions vary in accordance with the combinations. Please give the appropriate marks in accordance with the scheme of marking. For the sake of convenience one of the solutions is illustrated hereunder.		
		$\sin 20^o \sin 40^o \sin 60^o \sin 80^o = \sin 20^o \sin 40^o \frac{\sqrt{3}}{2} \sin 80^o$	½	
		$= \frac{\sqrt{3}}{2} \cdot \frac{-1}{2}\left(-2\sin 40^o \sin 80^o\right)\sin 20^o$		
		$= -\frac{\sqrt{3}}{4}\left(\cos 120^o - \cos 40^o\right)\sin 20^o$	½	
		$= -\frac{\sqrt{3}}{4}\left(\cos\left(90^o + 30^o\right) - \cos 40^o\right)\sin 20^o$		
		$= -\frac{\sqrt{3}}{4}\left(-\sin 30^o - \cos 40^o\right)\sin 20^o$	½	
		$= -\frac{\sqrt{3}}{4}\left(-\frac{1}{2} - \cos 40^o\right)\sin 20^o$	½	
		$= -\frac{\sqrt{3}}{4}\left(-\frac{1}{2}\sin 20^o - \sin 20^o \cos 40^o\right)$		
		$= -\frac{\sqrt{3}}{4}\left(-\frac{1}{2}\sin 20^o - \frac{1}{2} \cdot 2\sin 20^o \cos 40^o\right)$		
		$= \frac{\sqrt{3}}{4} \cdot \frac{1}{2}\left[\sin 20^o + \sin 60^o + \sin\left(-20^o\right)\right]$	½	
		$= \frac{\sqrt{3}}{8}\left[\sin 20^o + \frac{\sqrt{3}}{2} - \sin 20^o\right]$	½	
		$= \frac{\sqrt{3}}{8}\left[\frac{\sqrt{3}}{2}\right]$	½	
		$= \frac{3}{16}$	½	**4**
	e)	Prove that $\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{27}{11}\right)$		
	Ans.	*Let* $A = \cos^{-1}\left(\frac{4}{5}\right)$ $B = \tan^{-1}\left(\frac{3}{5}\right)$		
		$\therefore \cos A = \frac{4}{5}$ *and* $\tan B = \frac{3}{5}$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)				
		$\therefore \tan A = \frac{3}{4}$ ----------- (*)		
		$\therefore A = \tan^{-1}\left(\frac{3}{4}\right)$	1	
		$\therefore \cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{3}{5}\right)$		
		$= \tan^{-1}\left(\frac{\frac{3}{4}+\frac{3}{5}}{1-\frac{3}{4}\cdot\frac{3}{5}}\right)$	1	
		$= \tan^{-1}\left(\frac{\frac{15+12}{20}}{\frac{20-9}{20}}\right)$	1	
		$= \tan^{-1}\left(\frac{27}{11}\right)$	1	**4**
		Note: To evaluate value of tan A, various methods are used by students, such as 'using the relation between sin A and tan A' or 'first to find sin A using cos A and find tan A' etc., instead of using Triangle Method as illustrated in the above solution. As main object is to find the value of tan A, please consider these methods also.		
		--		
	f)	Prove that $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi$		
	Ans.	$\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi + \tan^{-1}\left(\frac{1+2}{1-1\cdot 2}\right) + \tan^{-1}(3)$	1	
		$= \pi + \tan^{-1}(-3) + \tan^{-1}(3)$	1	
		$= \pi - \tan^{-1}(3) + \tan^{-1}(3)$	1	
		$= \pi$	1	**4**
		OR		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		$\tan^{-1}(1)+\tan^{-1}(2)+\tan^{-1}(3)=\tan^{-1}(1)+\pi+\tan^{-1}\left(\frac{2+3}{1-2\cdot 3}\right)$	1	
		$=\tan^{-1}(1)+\pi+\tan^{-1}(-1)$	1	
		$=\tan^{-1}(1)+\pi-\tan^{-1}(1)$	1	
		$=\pi$	1	**4**
		--		
5)		**Attempt any four:**		
	a)	Prove that $\sin(A+B)\sin(A-B)=\sin^2 A-\sin^2 B$		
	Ans.	$\sin(A+B)\sin(A-B)=-\frac{1}{2}\left[-2\sin(A+B)\sin(A-B)\right]$		
		$=-\frac{1}{2}\left[\cos[(A+B)+(A-B)]-\cos[(A+B)-(A-B)]\right]$	1	
		$=-\frac{1}{2}\left[\cos 2A-\cos 2B\right]$	1	
		$=-\frac{1}{2}\left[1-2\sin^2 A-1+2\sin^2 B\right]$	1	
		$=\sin^2 A-\sin^2 B$	1	**4**
		OR		
		$\sin(A+B)\sin(A-B)$		
		$=[\sin A\cos B+\cos A\sin B][\sin A\cos B-\cos A\sin B]$	1	
		$=(\sin A\cos B)^2-(\cos A\sin B)^2$		
		$=\sin^2 A\cos^2 B-\cos^2 A\sin^2 B$	1	
		$=\sin^2 A\left[1-\sin^2 B\right]-\left[1-\sin^2 A\right]\sin^2 B$	1	
		$=\sin^2 A-\sin^2 A\sin^2 B-\sin^2 B+\sin^2 A\sin^2 B$		
		$=\sin^2 A-\sin^2 B$	1	**4**
		--		
	b)	Prove that $\sin C+\sin D=2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$		
	Ans.	We know that,		
		$2\sin A\cos B=\sin(A+B)+\sin(A-B)$	1	
		$Put \quad A+B=C$		
		$A-B=D$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)		$\therefore A=\frac{C+D}{2}$ *and* $B=\frac{C-D}{2}$	1	
		$\therefore \sin C+\sin D=2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$	1	**4**
		--		
	c)	Prove that $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$, if xy < 1.		
	Ans.	*Put* $\tan^{-1}x=A$ *and* $\tan^{-1}y=B$		
		$\therefore \quad x=\tan A$ *and* $y=\tan B$		
		$\therefore \tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$	1	
		$=\frac{x+y}{1-xy}$	1	
		$\therefore A+B=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$	1	
		$\therefore \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$	1	**4**
		--		
	d)	Prove that if θ is the acute angle between the lines with slopes m_1 and m_2, then $\tan\theta=\left\|\frac{m_1-m_2}{1+m_1\cdot m_2}\right\|$.		
	Ans.		½	
		Let θ_1 = Angle of inclination of L_1 θ_2 = Angle of inclination of L_2		
		$\therefore$ Slope of L_1 is $m_1=\tan\theta_1$ Slope of L_2 is $m_2=\tan\theta_2$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)		$\therefore$ *from figure,* $\theta = \theta_1 - \theta_2$ $\therefore \tan\theta = \tan(\theta_1 - \theta_2)$ $= \dfrac{\tan(\theta_1) - \tan(\theta_2)}{1 + \tan(\theta_1)\tan(\theta_2)}$ $= \dfrac{m_1 - m_2}{1 + m_1 \cdot m_2}$ *For angle to be acute,* $\tan\theta = \left\lvert \dfrac{m_1 - m_2}{1 + m_1 \cdot m_2} \right\rvert$	 1 1 1	 **4**
		--		
	e)	Find the equation of the line which passes through the point of intersection of the lines $2x + 3y = 13,\ 5x - y = 7$ and perpendicular to the line $2x - 5y + 7 = 0$.		
	Ans.	$2x + 3y = 13,\ 5x - y = 7$ $\therefore\ 2x + 3y = 13$ $\underline{15x - 3y = 21}$ $\therefore\ 17x = 34$ $\therefore x = 2$ $y = 3$ $\therefore$ Point of intersection $= (2, 3)$ *Slope of line* $2x - 5y + 7 = 0$ *is* $m_0 = -\dfrac{A}{B} = -\dfrac{2}{-5} = \dfrac{2}{5}$ $\therefore$ *Slope of required line is* $m = -\dfrac{5}{2}$ $\therefore$ *the equation is,* $y - y_1 = m(x - x_1)$ $\therefore y - 3 = -\dfrac{5}{2}(x - 2)$ $\therefore 2y - 6 = -5x + 10$ $\therefore 5x + 2y - 16 = 0 \quad or \quad 5x + 2y = 16$	 1 ½ ½ 1 1	 **4**
		--		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)	f)	Find the length of the perpendicular from $(3,\ 2)$ on the line $4x-6y-5=0$.		
	Ans.	$Given\ \ 4x-6y-5=0$ $\therefore A=4,\quad B=-6,\quad C=-5$ $\therefore$ *the length of the perpendicular is,* $p=\left\|\dfrac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right\|$		
		$=\left\|\dfrac{4(3)-6(2)-5}{\sqrt{4^2+(-6)^2}}\right\|$	2	
		$=\dfrac{5}{\sqrt{52}}\quad or\quad 0.693$ **Note:** If –ve sign is left with the answer, 1 mark is to be deducted.	1+1	**4**
		---		
6)		**Attempt any four:**		
	a)	Find the perpendicular distance between the parallel lines $5x-12y+1=0$ and $10x-24y=1$.		
	Ans.	$Given\ 5x-12y+1=0\quad and\quad 10x-24y=1$ $\therefore 10x-24y+2=0\quad and\quad 10x-24y-1=0$	1	
		$\therefore A=10,\quad B=-24,\quad C_1=2\quad and\quad C_2=-1$ $\therefore p=\left\|\dfrac{C_1-C_2}{\sqrt{A^2+B^2}}\right\|$		
		$=\left\|\dfrac{2+1}{\sqrt{10^2+(-24)^2}}\right\|$	1	
		$=\dfrac{3}{26}\qquad or\qquad 0.115$	1+1	**4**
		Note: If the –ve value is written by the student (i.e., $-\dfrac{3}{26}\quad or\quad -0.115$, deduct 1 mark. **OR** $Given\ 5x-12y+1=0\quad and\quad 10x-24y=1$		
		$\therefore 5x-12y+1=0\quad and\quad 5x-12y-\dfrac{1}{2}=0$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		$\therefore A=5,\ \ B=-12,\ \ C_1=1\ \ and\ \ C_2=-\dfrac{1}{2}$		
		$\therefore p=\left\|\dfrac{C_1-C_2}{\sqrt{A^2+B^2}}\right\| = \left\|\dfrac{1+\frac{1}{2}}{\sqrt{5^2+(-12)^2}}\right\|$	1	
		$=\dfrac{3}{26}$ *or* 0.115	1+1	**4**
		---		
	b)	Find the equation of straight line passing through the points $(-4,\ 6)$ and $(8,\ -3)$.		
	Ans.	$\dfrac{y-y_1}{y_2-y_1}=\dfrac{x-x_1}{x_2-x_1}$		
		$\therefore \dfrac{y-6}{-3-6}=\dfrac{x+4}{8+4}$	1	
		$\therefore \dfrac{y-6}{-9}=\dfrac{x+4}{12}$	1	
		$\therefore 12(y-6)=-9(x+4)$		
		$\therefore 12y-72=-9x-36$	1	
		$\therefore 9x+12y-36=0$ *or* $-9x-12y+36=0$		
		or $3x+4y-12=0$ *or* $-3x-4y+12=0$	1	**4**
		OR		
		$\therefore slope\ of\ \ line\ is\ m=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{-3-6}{8+4}=-\dfrac{3}{4}$	1	
		$\therefore the\,equation\,is,$		
		$y-y_1=m(x-x_1)$		
		$\therefore y-6=-\dfrac{3}{4}(x+4)$ *or* $y+3=-\dfrac{3}{4}(x-8)$	1	
		$\therefore 4y-24=-3x-12$ *or* $4y+12=-3x+24$	1	
		$\therefore 3x+4y-12=0$ *or* $-3x-4y+12=0$	1	**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)	c)	Find the mean deviation from mean for following distribution:		

Marks	0-10	10-20	20-30	30-40	40-50
No. of Students	5	8	15	16	6

Ans.

Class	xi	f_i	$f_i x_i$	$D_i = \lvert x_i - \bar{x} \rvert$	$f_i D_i$
0-10	5	5	25	22	110
10-20	15	8	120	12	96
20-30	25	15	375	2	30
30-40	35	16	560	8	128
40-50	45	6	270	18	108
		50	**1350**		**472**

1+1

$$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{1350}{50} = 27$$

1

$$M.D. = \frac{\sum f_i D_i}{N}$$

$$= \frac{472}{50}$$

$$= 9.44$$

1 **4**

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d) Find the S. D. of following data:

Marks	0-10	10-20	20-30	30-40	40-50
No. of Students	5	8	15	16	6

Ans.

Class	xi	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
0-10	5	3	15	25	75
10-20	15	5	75	225	1125
20-30	25	8	200	625	5000
30-40	35	3	105	1225	3675
40-50	45	1	45	2025	2025
		20	**440**		**11900**

1+1

$$S.D. = \sqrt{\frac{\sum f_i x_i^2}{N} - \left(\frac{\sum f_i x_i}{N}\right)^2}$$

$$= \sqrt{\frac{11900}{20} - \left(\frac{440}{20}\right)^2}$$

1

$$= 10.536$$

1 **4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		**OR**		
		(table below)	1+1	
		$A=25 \quad h=10, \quad d_i=\frac{x_i-A}{h}$		
		$S.D.=h\times\sqrt{\frac{\sum f_i d_i^2}{N}-\left(\frac{\sum f_i d_i}{N}\right)^2}$		
		$=10\times\sqrt{\frac{24}{20}-\left(\frac{-6}{20}\right)^2}$	1	
		$=10.536$	1	**4**
		Note: Students may take any another value for A in the above/ below example. So the above table and corresponding values vary accordingly. But the final answer will be the same.		
	e)	The two sets of observations are given below:		
		Set I: $\bar{x}=82.5$, $\sigma=7.3$; **Set II**: $\bar{x}=48.75$, $\sigma=8.35$		
		Which of the two sets is more consistent?		
	Ans.	$C.V.(I)=\frac{\sigma}{\bar{x}}\times 100=\frac{7.3}{82.5}\times 100=8.848$	1	
		$C.V.(II)=\frac{\sigma}{\bar{x}}\times 100=\frac{8.35}{48.75}\times 100=17.12$	1	
		$\therefore C.V.(I)<C.V.(II)$	1	
		$\therefore$ *Set I* is more consistent.	1	**4**

Class	xi	f_i	d_i	$f_i d_i$	d_i^2	$f_i d_i^2$
0-10	5	3	-2	-6	4	12
10-20	15	5	-1	-5	1	5
20-30	25	8	**0**	0	**0**	0
30-40	35	3	1	3	1	3
40-50	45	1	2	2	4	4
		20		**-6**		**24**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)	f)	Find the variance and coefficient of variance of the following:		

Class Interval	0-10	10-20	20-30	30-40	40-50
Frequencies	14	23	27	21	15

Ans.

Class	xi	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
0-10	5	14	70	25	350
10-20	15	23	345	225	5175
20-30	25	27	675	625	16875
30-40	35	21	735	1225	25725
40-50	45	15	675	2025	30375
			2500		**78500**

1

$$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{2500}{100} = 25$$ 1

$$S.D. = \sqrt{\frac{\sum f_i x_i^2}{N} - \left(\frac{\sum f_i x_i}{N}\right)^2}$$

$$= \sqrt{\frac{78500}{100} - \left(\frac{2500}{100}\right)^2}$$

$$= 12.649$$ ½

$$\therefore Variance = (S.D.)^2$$

$$= 12.649^2$$

$$= 159.997$$ ½

$$Coeff.\ of\ Variance = \frac{S.D.}{\bar{x}} \times 100$$

$$= \frac{12.649}{25} \times 100$$

$$= 50.596$$ 1 **4**

OR

$$\therefore Variance = \frac{\sum f_i x_i^2}{N} - \left(\frac{\sum f_i x_i}{N}\right)^2$$

$$= \frac{78500}{100} - \left(\frac{2500}{100}\right)^2$$

$$= 160$$ 1

$$Coeff.\ of\ Variance = \frac{S.D.}{\bar{x}} \times 100$$

$$= \frac{12.649}{25} \times 100$$

$$= 50.596$$ 1 **4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		**OR**		

Class	xi	f_i	d_i	$f_i d_i$	d_i^2	$f_i d_i^2$
0-10	5	14	-2	-28	4	56
10-20	15	23	-1	-23	1	23
20-30	**25**	**27**	**0**	**0**	**0**	**0**
30-40	35	21	1	21	1	**21**
40-50	45	15	2	30	4	60
		100		**0**		**160**

1

$$A = 25, \quad h = 10, \quad d_i = \frac{x_i - A}{h}$$

$$\therefore \bar{x} = A + \frac{\sum f_i d_i}{N} \times h$$

$$= 25 + \frac{0}{100} \times 10$$

$$= 25$$

1

$$S.D. = h \times \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2}$$

$$= 10 \times \sqrt{\frac{160}{100} - \left(\frac{0}{100}\right)^2}$$

$$= 12.649$$

½

$$\therefore Variance = (S.D.)^2$$

$$= 12.649^2$$

$$= 159.997$$

½

$$Coeff.\ of\ Variance = \frac{S.D.}{\bar{x}} \times 100$$

$$= \frac{12.649}{25} \times 100$$

$$= 50.596$$

1 4

OR

$$\therefore Variance = h^2 \left[\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2\right]$$

$$= h^2 \left[\frac{160}{100} - \left(\frac{0}{100}\right)^2\right]$$

$$= 159.997$$

1

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		$Coeff.\ of\ Variance = \frac{S.D.}{\overline{x}} \times 100 = \frac{12.649}{25} \times 100$ $= 50.596$	1	**4**
		--		
		-- --		
		Important Note In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. **In such case, FIRST SEE whether the method falls within the scope of the curriculum**, and THEN ONLY give appropriate marks in accordance with the scheme of marking.		
		-- --		