SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 22201

Important Instructions to Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more importance (Not applicable for subject English and Communication Skills).
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answer and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.		**Attempt any FIVE of following:**	**10**
	a)	If $f(x)=x^4-2x+7$, find $f(0)+f(2)$	**02**
	Ans	$f(x)=x^4-2x+7$	
		$\therefore f(0)=(0)^4-2(0)+7=7$	½
		$\therefore f(2)=(2)^4-2(2)+7=19$	½
		$\therefore f(0)+f(2)=7+19$	½
		$\therefore f(0)+f(2)=26$	½
		--	
	b)	State whether the function $f(x)=\dfrac{e^x+e^{-x}}{2}$ is odd or even.	**02**
	Ans	$f(x)=\dfrac{e^x+e^{-x}}{2}$	
		$\therefore f(-x)=\dfrac{e^{-x}+e^{-(-x)}}{2}$	½
		$=\dfrac{e^{-x}+e^{x}}{2}$	½
		$=f(x)$	½
		$\therefore$ function is even.	½
		--	
	c)	If $y=\log\left(x^2+2x+5\right)$ then find $\dfrac{dy}{dx}$	**02**

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	c)	$y=\log\left(x^2+2x+5\right)$	
	Ans	$\therefore \frac{dy}{dx}=\frac{1}{x^2+2x+5}\cdot\frac{d}{dx}\left(x^2+2x+5\right)$	1
		$=\frac{1}{x^2+2x+5}\cdot(2x+2)=\frac{2(x+1)}{x^2+2x+5}$	1
		--	
	d)	Evaluate $\int\frac{1-\cos 2x}{1+\cos 2x}\,dx$	**02**
	Ans	$\int\frac{1-\cos 2x}{1+\cos 2x}\,dx$	
		$=\int\frac{2\sin^2 x}{2\cos^2 x}\,dx$	½
		$=\int\tan^2 x\,dx$	½
		$=\int\left(\sec^2 x-1\right)dx$	½
		$=\tan x-x+c$	½
		--	
	e)	Evaluate $\int\frac{1}{2x+5}dx$	**02**
	Ans	$\int\frac{1}{2x+5}dx=\frac{1}{2}\left[\log(2x+5)\right]+c$	2
		OR	
		$\int\frac{1}{2x+5}dx=\frac{1}{2}\int\frac{1}{x+\frac{5}{2}}dx$	
		$=\frac{1}{2}\left[\log\left(x+\frac{5}{2}\right)\right]+c$	2
		--	
	f)	Find the area under the parabola $y^2=4x$, bounded by the lines $x=0, y=0$, $x=4$	**02**
	Ans	Area $A=\int_a^b y\,dx$	
		$=\int_0^4 2\sqrt{x}dx$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	f)	Area $A=\int_0^4 2x^{1/2}dx$	
		$=\left[2\dfrac{x^{3/2}}{\frac{3}{2}}\right]_0^4$	
		$=\left[\dfrac{4}{3}x^{3/2}\right]_0^4$	½
		$=\dfrac{4}{3}\left[4^{3/2}-0\right]$	½
		$=10.667$	½
		---	
	g)	State the trapezoidal rule of numerical integration.	**02**
	Ans	Trapezoidal rule	
		$\int_a^b f(x)dx=\dfrac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+...+y_{n-1})\right]$	2
		where $h=\dfrac{b-a}{n}$	

2.		**Attempt any THREE of the following:**	**12**
	a)	If $x^y=e^{x-y}$ then prove that $\dfrac{dy}{dx}=\dfrac{\log x}{(1+\log x)^2}$	**04**
	Ans	$x^y=e^{x-y}$	
		$\log x^y=\log e^{x-y}$	½
		$\therefore y\log x=(x-y)\log e$	½
		$\therefore y\log x=x-y$	
		$y\log x+y=x$	
		$y(\log x+1)=x$	
		$\therefore y=\dfrac{x}{1+\log x}$	1
		$\therefore \dfrac{dy}{dx}=\dfrac{(1+\log x)\dfrac{d(x)}{dx}-x\dfrac{d(1+\log x)}{dx}}{(1+\log x)^2}$	

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2.	a)	$\therefore \frac{dy}{dx}=\frac{(1+\log x)-x\left(\frac{1}{x}\right)}{(1+\log x)^2}$	1
		$\therefore \frac{dy}{dx}=\frac{1+\log x-1}{(1+\log x)^2}$	½
		$\therefore \frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$	½
		--	
	b)	If $x=a(\theta-\sin\theta), y=a(1-\cos\theta)$, then find $\frac{dy}{dx}$ at $\theta=\frac{\pi}{4}$	**04**
	Ans	$x=a(\theta-\sin\theta)$ $\quad$ $y=a(1-\cos\theta)$	
		$\therefore \frac{dx}{d\theta}=a(1-\cos\theta)$ $\quad$ $\frac{dy}{d\theta}=a\sin\theta$	1+1
		$\therefore \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$	
		$\therefore \frac{dy}{dx}=\frac{a\sin\theta}{a(1-\cos\theta)}=\frac{\sin\theta}{(1-\cos\theta)}$ OR $\frac{dy}{dx}=\frac{\sin\theta}{(1-\cos\theta)}=\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}=\cot\frac{\theta}{2}$	1
		at $\theta=\frac{\pi}{4}$	
		$\therefore \frac{dy}{dx}=\frac{\sin\frac{\pi}{4}}{\left(1-\cos\frac{\pi}{4}\right)}=\frac{\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}}$	
		$\therefore \frac{dy}{dx}=\frac{1}{\sqrt{2}-1}$ or 2.414 OR $\frac{dy}{dx}=\cot\frac{\frac{\pi}{4}}{2}=\cot\frac{\pi}{8}=2.414$	1
		--	
	c)	Find maximum and minimum value of $y=x^3-18x^2+96x$	**04**
	Ans	Let $y=x^3-18x^2+96x$	
		$\therefore \frac{dy}{dx}=3x^2-36x+96$	½
		$\therefore \frac{d^2y}{dx^2}=6x-36$	½
		Consider $\frac{dy}{dx}=0$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	c)	$3x^2-36x+96=0$	½
		$x^2-12x+32=0$	
		$\therefore x=8$ or $x=4$	½
		at $x=8$	
		$\frac{d^2y}{dx^2}=6(8)-36=12<0$	½
		$\therefore y$ is minimum at $x=8$	
		$y_{\min}=(8)^3-18(8)^2+96(8)$	
		$=128$	½
		at $x=4$	
		$\frac{d^2y}{dx^2}=6(4)-36=-12<0$	½
		$\therefore y$ is maximum at $x=4$	
		$y_{\max}=(4)^3-18(4)^2+96(4)$	
		$=160$	½
		--	
	d)	Find radius of curvature of the curve $y=x^3$ at $(2,8)$	**04**
	Ans	$y=x^3$	
		$\therefore \frac{dy}{dx}=3x^2$	½
		$\therefore \frac{d^2y}{dx^2}=6x$	½
		at $(2,8)$	
		$\frac{dy}{dx}=3(2)^2=12$	½
		$\frac{d^2y}{dx^2}=6(2)=12$	½
		$\therefore$ Radius of curvature is $\rho=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$	
		$\therefore \rho=\frac{\left[1+(12)^2\right]^{\frac{3}{2}}}{12}$	1
		$\therefore \rho=145.50$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.		**Attempt any THREE of the following:**	**12**
	a)	Find $\frac{dy}{dx}$ if $y = x^x + (\sin x)^x$	**04**
	Ans	Let $u = x^x$	
		$\therefore \log u = \log x^x$	
		$\log u = x \log x$	½
		$\frac{1}{u}\frac{du}{dx} = x\frac{1}{x} + \log x(1)$	1
		$\therefore \frac{du}{dx} = u(1+\log x)$	
		$\therefore \frac{du}{dx} = x^x(1+\log x)$	
		Let $v = (\sin x)^x$	
		$\therefore \log v = \log(\sin x)^x$	
		$\log v = x\log(\sin x)$	½
		$\frac{1}{v}\frac{dv}{dx} = x\frac{1}{\sin x}\cos x + \log(\sin x)(1)$	1
		$\frac{1}{v}\frac{dv}{dx} = x\cot x + \log(\sin x)$	
		$\frac{dv}{dx} = v(x\cot x + \log(\sin x))$	
		$\frac{dv}{dx} = (\sin x)^x(x\cot x + \log(\sin x))$	
		$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$	
		$\therefore \frac{dy}{dx} = x^x(1+\log x) + (\sin x)^x(x\cot x + \log(\sin x))$	1
		--	
	b)	Find $\frac{dy}{dx}$ if $x^2 + 3xy + y^2 = 5$	**04**
	Ans	$x^2 + 3xy + y^2 = 5$	
		$2x + 3\left[x\frac{dy}{dx} + y(1)\right] + 2y\frac{dy}{dx} = 0$	2
		$2x + 3x\frac{dy}{dx} + 3y + 2y\frac{dy}{dx} = 0$	½
		$(3x+2y)\frac{dy}{dx} = -2x - 3y$	½

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3.	b)	$\frac{dy}{dx}=\frac{-2x-3y}{3x+2y}=\frac{-(2x+3y)}{3x+2y}$	1
		--	
	c)	Evaluate: $\int\frac{\log\left(\tan \frac{x}{2}\right)}{\sin x}dx$	**04**
	Ans	$\int\frac{\log\left(\tan \frac{x}{2}\right)}{\sin x}dx$	
		Put $\log\left(\tan \frac{x}{2}\right)=t$	½
		$\frac{1}{\tan \frac{x}{2}}\sec^2 \frac{x}{2}\left(\frac{1}{2}\right)dx=dt$	½
		$\left(\frac{1}{2}\right)\frac{\cos \frac{x}{2}}{\sin \frac{x}{2}}.\frac{1}{\cos^2 \frac{x}{2}}dx=dt$	
		$\therefore \frac{1}{2\sin\frac{x}{2}\cos\frac{x}{2}}dx=dt$	
		$\therefore \frac{1}{\sin x}dx=dt$	½
		$\therefore \int t\ dt$	1
		$=\frac{t^2}{2}+c$	1
		$=\frac{\left(\log\left(\tan \frac{x}{2}\right)\right)^2}{2}+c$	½
		--	
	d)	Find the equation of tangent to the circle $x^2+y^2+6x-6y-7=0$ at a point it cuts the x-axis	**04**
	Ans	$x^2+y^2+6x-6y-7=0$	
		$\because$ tangent cuts x-axis $\therefore$ $y=0$	
		$\therefore x^2+(0)^2+6x-6(0)-7=0$	
		$\therefore x^2+6x-7=0$	
		$\therefore x=1$ and $x=-7$ $\therefore$ Points are $(1,0)$ and $(-7,0)$	1
		$\because x^2+y^2+6x-6y-7=0$	
		$\therefore 2x+2y\frac{dy}{dx}+6-6\frac{dy}{dx}=0$	½
		$\therefore (2y-6)\frac{dy}{dx}=-2x-6$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	d)	$\therefore \dfrac{dy}{dx} = \dfrac{-2x-6}{2y-6}$	½
		at $(1,0)$	
		Slope $= \dfrac{dy}{dx} = \dfrac{-2(1)-6}{2(0)-6} = \dfrac{-8}{-6} = \dfrac{4}{3}$	½
		$\therefore$ equation is	
		$y - y_1 = m(x - x_1)$	
		$y - 0 = \dfrac{4}{3}(x-1)$	½
		$3y = 4x - 4$	
		$4x - 3y - 4 = 0$	
		at $(-7,0)$	
		Slope $= \dfrac{dy}{dx} = \dfrac{-2(-7)-6}{2(0)-6} = \dfrac{8}{-6} = \dfrac{-4}{3}$	½
		$\therefore$ equation is	
		$y - 0 = \dfrac{-4}{3}(x+7)$	½
		$3y = -4x - 28$	
		$4x + 3y + 28 = 0$	
		--	
		Attempt any THREE of the following:	
4.			**12**
		Evaluate : $\int \dfrac{1}{5+4\cos x} dx$	
	a)		**04**
		$\int \dfrac{1}{5+4\cos x} dx$	
	Ans	Put $\tan\dfrac{x}{2} = t \quad \therefore \cos x = \dfrac{1-t^2}{1+t^2}, \quad dx = \dfrac{2dt}{1+t^2}$	1
		$\therefore \int \dfrac{dx}{5+4\cos x} = \int \dfrac{1}{5+4\left(\dfrac{1-t^2}{1+t^2}\right)} \cdot \dfrac{2dt}{1+t^2}$	½
		$= 2\int \dfrac{1}{t^2+9} dt$	
		$= 2\int \dfrac{1}{t^2+3^2} dt$	1
		$= 2 \times \dfrac{1}{3} \tan^{-1}\left(\dfrac{t}{3}\right) + c$	1
		$= \dfrac{2}{3} \tan^{-1}\left(\dfrac{\tan\frac{x}{2}}{3}\right) + c$	½

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4.	b)	Evaluate: $\int \frac{x+1}{x\left(x^2-4\right)}\, dx$	**04**
	Ans	$\int \frac{x+1}{x\left(x^2-4\right)}\, dx = \int \frac{x+1}{x(x-2)(x+2)} dx$	
		Let $\frac{x+1}{x(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}$	½
		$x+1 = A(x-2)(x+2) + Bx(x+2) + Cx(x-2)$	
		put $x=0 \quad \therefore A = \frac{-1}{4}$	½
		put $x=2 \quad \therefore B = \frac{3}{8}$	½
		put $x=-2 \quad \therefore C = \frac{-1}{8}$	½
		$\frac{x+1}{x(x-2)(x+2)} = \frac{\frac{-1}{4}}{x} + \frac{\frac{3}{8}}{x-2} + \frac{\frac{-1}{8}}{x+2}$	
		$\int \frac{x+1}{x(x-2)(x+2)} dx = \int \left(\frac{\frac{-1}{4}}{x} + \frac{\frac{3}{8}}{x-2} + \frac{\frac{-1}{8}}{x+2} \right) dx$	½
		$= \frac{-1}{4}\log x + \frac{3}{8}\log(x-2) - \frac{1}{8}\log(x+2) + c$	½+½+½
		--	
	c)	Evaluate: $\int \cos(\log x)\, dx$	**04**
	Ans	$\int \cos(\log x)\, dx$	
		Put $\log x = t \Rightarrow x = e^t$	½
		$\therefore \frac{1}{x} dx = dt$	
		$\therefore dx = xdt$	
		$\therefore dx = e^t dt$	1
		$\therefore \int e^t \cos t dt$	1
		$= \frac{e^t}{1+1}(1\cos t + 1\sin t) + c$	1
		$= \frac{x}{2}\left(\cos(\log x) + \sin(\log x)\right) + c$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	c)	OR	
		$\int \cos(\log x)\ dx$	
		Put $\log x = t \Rightarrow x = e^t$	
		$\therefore \frac{1}{x}dx = dt$	
		$\therefore dx = xdt$	
		$\therefore dx = e^t dt$	1
		$\therefore I = \int e^t \cos t dt$	
		$= \cos t \int e^t dt - \int \left(\int e^t dt \frac{d}{dt} \cos t \right) dt$	
		$= \cos t\ e^t - \int e^t (-\sin t) dt$	1
		$= \cos t\ e^t + \int e^t \sin t dt + c$	
		$= \cos t\ e^t + e^t \sin t - \int e^t \cos t dt + c$	1
		$\therefore I = \cos t\ e^t + e^t \sin t - I + c$	
		$\therefore 2I = \cos t\ e^t + e^t \sin t + c$	
		$\therefore I = \frac{e^t}{2}(\cos t\ + \sin t) + c$	½
		$\therefore I = \frac{x}{2}(\cos(\log x) + \sin(\log x)) + c$	½
		OR	
		$I = \int \cos(\log x)\ dx$	
		$\therefore I = \int \cos(\log x).1\ dx$	½
		$\therefore I = \cos(\log x) \int 1dx - \int \left(\int 1dx \frac{d}{dx} \cos(\log x) \right) dx$	
		$\therefore I = \cos(\log x) x - \int x \left(\frac{-\sin(\log x)}{x} \right) dx$	1
		$\therefore I = x \cos(\log x) + \int \sin(\log x)\ dx$	
		$\therefore I = x \cos(\log x) + \int \sin(\log x).1dx$	½
		$\therefore I = x \cos(\log x) + \left[\sin(\log x) x - \int x \left(\frac{\cos(\log x)}{x} \right) dx \right]$	
		$\therefore I = x \cos(\log x) + x \sin(\log x) - \int \cos(\log x)\ dx$	1
		$\therefore I = x \cos(\log x) + x \sin(\log x) - I + c$	
		$\therefore 2I = x(\cos(\log x) + \sin(\log x)) + c$	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	c)	$\therefore I = \frac{x}{2}\left[\cos(\log x)+\sin(\log x)\right]+c$	1
		----------	
	d)	Evaluate : $\int \frac{1}{x^2+4x+9}\,dx$	**04**
	Ans	$\int \frac{1}{x^2+4x+9}\,dx$	
		Third term $=\left(\frac{1}{2}\times 4\right)^2=4$	1
		$=\int \frac{1}{x^2+4x+4-4+9}\,dx$	1
		$=\int \frac{1}{(x+2)^2+\left(\sqrt{5}\right)^2}\,dx$	1
		$=\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right)+c$	1
		OR	
		$\int \frac{1}{x^2+4x+9}dx$	
		Third term $=\frac{(M.T.)^2}{4(F.T.)}=4$	1
		$=\int \frac{1}{x^2+4x+4-4+9}\,dx$	1
		$=\int \frac{1}{(x+2)^2+\left(\sqrt{5}\right)^2}\,dx$	1
		$=\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right)+c$	1
		----------	
	e)	Evaluate $\int_1^5 \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x+3}}\,dx$	**04**
	Ans	$\int_1^5 \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x+3}}\,dx -------(1)$	
		$I=\int_1^5 \frac{\sqrt{9-(1+5-x)}}{\sqrt{9-(1+5-x)}+\sqrt{(1+5-x)+3}}\,dx$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	e)	$\therefore I=\int_1^5 \frac{\sqrt{x+3}}{\sqrt{x+3}+\sqrt{9-x}}\,dx --------(2)$	½
		add (1) and (2)	
		$I+I=\int_1^5 \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x+3}}\,dx \quad +\int_1^5 \frac{\sqrt{x+3}}{\sqrt{x+3}+\sqrt{9-x}}\,dx$	½
		$\therefore 2I=\int_1^5 \frac{\sqrt{9-x}+\sqrt{x+3}}{\sqrt{9-x}+\sqrt{x+3}}\,dx$	
		$\therefore 2I=\int_1^5 1\,dx$	
		$\therefore 2I=[x]_1^5$	½
		$\therefore 2I=5-1$	
		$\therefore 2I=4$	1
		$I=2$	½
		--	
5.		**Attempt any TWO of the following:**	**12**
	a)	Find the area of the loop of a curve $y^2=x^2(1-x)$.	**06**
	Ans	$y^2=x^2(1-x)$	
		$y=x\sqrt{(1-x)}$	
		at $y=0,\ x^2(1-x)=0$	
		$\therefore x=0\ ,\ 1$	1
		$\therefore A_1=\int_0^1 y\,dx$	
		$=\int_0^1 x\sqrt{1-x}\,dx$	1
		$=\int_0^1 (1-x)\sqrt{x}\,dx$	1
		$=\int_0^1 \left(\sqrt{x}-x^{3/2}\right)dx$	
		$=\left[\frac{2}{3}x^{3/2}-\frac{2}{5}x^{5/2}\right]_0^1$	1
		$=\left[\frac{2}{3}-\frac{2}{5}\right]-0$	
		$=\frac{4}{15}$ or 0.267	1
		$\therefore$ Area of loop $=2\times A_1=2\times\frac{4}{15}=\frac{8}{15}$ or 0.533	1

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5.	a)	*OR*	
		$y^2 = x^2(1-x) \quad \therefore y = x\sqrt{(1-x)}$	
		at $y=0,\ x^2(1-x)=0$	
		$\therefore x = 0\ ,\ 1$	1
		$\therefore\ A = \int_0^1 y\,dx$	
		$= \int_0^1 x\sqrt{1-x}\,dx$	1
		put $t = 1-x$, $\therefore dt = -dx$, $\therefore -dt = dx$; when $x \to 0$ to 1, $t \to 1$ to 0	1
		$= -\int_1^0 (1-t)\sqrt{t}\,dt$	
		$= -\int_1^0 \left(\sqrt{t} - t^{3/2}\right)dt$	
		$= -\left[\frac{2}{3}t^{3/2} - \frac{2}{5}t^{5/2}\right]_1^0$	1
		$= -\left[0 - \left(\frac{2}{3} - \frac{2}{5}\right)\right]$	
		$= \frac{4}{15}$ or 0.267	1
		$\therefore$ Area of loop $= 2 \times A_1 = 2 \times \left(\frac{4}{15}\right) = \frac{8}{15}$ or 0.533	1
		--	
	b)	**Attempt the following:**	**06**
	(i)	Form the differential equation of $y = a\sin x + b\cos x$	**03**
	Ans	$y = a\sin x + b\cos x$	
		$\therefore \frac{dy}{dx} = a\cos x - b\sin x$	1
		$\therefore \frac{d^2y}{dx^2} = -a\sin x - b\cos x$	1
		$\therefore \frac{d^2y}{dx^2} = -(a\sin x + b\cos x)$	½
		$\frac{d^2y}{dx^2} = -y$	½
		$\frac{d^2y}{dx^2} + y = 0$	

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5.	b)(ii)	Solve : $\frac{dy}{dx}+\frac{y}{x}=x^2$	**03**
	Ans	$\frac{dy}{dx}+\frac{y}{x}=x^2$ Comparing with $\frac{dy}{dx}+Py=Q$	
		$\therefore P=\frac{1}{x}$ and $Q=x^2$	½
		$IF=e^{\int\frac{1}{x}dx}=e^{\log x}=x$	1
		$\therefore$ Solution is $y\cdot IF=\int Q\cdot IFdx+c$	
		$y\cdot x=\int x\cdot^2 xdx+c$	½
		$xy=\int x^3dx+c$	
		$xy=\frac{x^4}{4}+c$	1
		--	
	c)	A resistance of 100Ω and inductance of 0.1 henries are connected in series with a battary of 20 volts. find the current in the circuit at any instant , if the relation between L,R and E is $L\frac{di}{dt}+Ri=E$	**06**
	Ans	$L\frac{di}{dt}+Ri=E$	
		$\therefore \frac{di}{dt}+\frac{R}{L}i=\frac{E}{L}$ Comparing with $\frac{dy}{dx}+Py=Q$	
		$\therefore P=\frac{R}{L}$ and $Q=\frac{E}{L}$	½
		$IF=e^{\int\frac{R}{L}dt}=e^{\frac{R}{L}t}$	1
		$\therefore$ Solution is $i\cdot IF=\int Q\cdot IFdt+c$	
		$i\cdot e^{\frac{R}{L}t}=\int\frac{E}{L}e^{\frac{R}{L}t}dt+c$	½
		$i\cdot e^{\frac{R}{L}t}=\frac{E}{L}\frac{e^{\frac{R}{L}t}}{\frac{R}{L}}+c$	1
		$i\cdot e^{\frac{R}{L}t}=\frac{E}{R}e^{\frac{R}{L}t}+c$	
		Initially at t = 0 , i = 0 $\therefore$ c = $\frac{-E}{R}$	1
		$\therefore i\cdot e^{\frac{R}{L}t}=\frac{E}{R}e^{\frac{R}{L}t}+\left(\frac{-E}{R}\right)$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	c)	$i=\frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)$	1
		When R=100 , L = 0.1 , E= 20	
		$i=\frac{20}{100}\left(1-e^{-\frac{100}{0.1}t}\right)$	1
		$i=0.2\left(1-e^{-1000t}\right)$	
6.		**Attempt any TWO of the following:**	**12**
	a)(i)	Using trapezoidal rule, evaluate $\int_0^6 f(x)\,dx$	**03**
	Ans	$\int_a^b f(x)\,dx=\frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+...+y_{n-1})\right]$	
		$a=0, b=6$ and $h=1$	
		$\therefore \int_0^6 f(x)\,dx=\frac{1}{2}\left[(1+0.1428)+2(0.5+0.3333+0.25+0.2+0.6666)\right]$	2
		$=2.5213$	1
	a)(ii)	Using Simpson's $1/3^{rd}$ rule, evaluate $\int_1^2 \frac{1}{x}dx$ given by	**03**
	Ans	$\int_a^b f(x)\,dx=\frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+...+y_{n-1})+2(y_2+y_4+...+y_{n-2})\right]$	
		Let $y=f(x)=\frac{1}{x}$ $a=1, b=2$ and $h=0.25$	
		$\therefore \int_1^2 f(x)\,dx=\frac{0.25}{3}\left[(1+0.5)+4(0.8+0.5714)+2(0.6666)\right]$	2
		$=0.6932$	1

Table for 6 a)(i):

x	0	1	2	3	4	5	6
$f(x)$	1	0.5	0.3333	0.25	0.2	0.6666	0.1428

Table for 6 a)(ii):

x	1	1.25	1.5	1.75	2
$y=f(x)$	1	0.8	0.6666	0.5714	0.5

SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 22201

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	b)	Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ Using Simpson's $1/3^{rd}$ rule divide the interval $[0,1]$ into six equal parts.Find approximate value of π.	**06**
	Ans	Let $y=\frac{1}{1+x^2}$ $\quad a=0, b=1$ and $n=6$	
		$\therefore h=\frac{b-a}{n}=\frac{1-0}{6}=\frac{1}{6}$	1

x	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1
$y=\frac{1}{1+x^2}$	1	$\frac{36}{37}$	$\frac{9}{10}$	$\frac{4}{5}$	$\frac{9}{13}$	$\frac{36}{61}$	$\frac{1}{2}$

Marks: 2

Using Simpson's $1/3^{rd}$ rule

$$\int_a^b f(x)dx=\frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+...+y_{n-1})+2(y_2+y_4+...+y_{n-2})\right]$$

$$\therefore \int_0^1 f(x)dx=\frac{\cancel{1}/6}{3}\left[\left(1+\frac{1}{2}\right)+4\left(\frac{36}{37}+\frac{4}{5}+\frac{36}{61}\right)+2\left(\frac{9}{10}+\frac{9}{13}\right)\right]$$ 1

$$=0.7854$$ 1

$$\because \int_0^1 \frac{1}{1+x^2}dx=0.7854$$

$$\therefore \left[\tan^{-1}x\right]_0^1=0.7854$$ ½

$$\left[\tan^{-1}(1)\right]-\left[\tan^{-1}(0)\right]=0.7854$$

$$\frac{\pi}{4}=0.7854$$

$$\pi=3.142$$ ½

OR

Let $y=\frac{1}{1+x^2}$ $\quad a=0, b=1$ and $n=6$

$$\therefore h=\frac{b-a}{n}=\frac{1-0}{6}=\frac{1}{6}=0.1667$$ 1

x	0	0.1667	0.3334	0.5001	0.6668	0.8335	1
$y=\frac{1}{1+x^2}$	1	0.9730	0.9	0.8	0.6922	0.5901	0.5

Marks: 2

Using Simpson's $1/3^{rd}$ rule

$$\int_a^b f(x)dx=\frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+...+y_{n-1})+2(y_2+y_4+...+y_{n-2})\right]$$

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	b)	$\therefore \int_0^1 f(x)\,dx = \frac{0.1667}{3}\left[(1+0.5)+4(0.9730+0.8+0.5901)+2(0.9+0.6922)\right]$	1
		$= 0.7855$	1
		$\int_0^1 \frac{1}{1+x^2}\,dx = 0.7855$	
		$\left[\tan^{-1} x\right]_0^1 = 0.7855$	½
		$\left[\tan^{-1}(1)\right]-\left[\tan^{-1}(0)\right] = 0.7855$	
		$\frac{\pi}{4} = 0.7855$	
		$\pi = 3.142$	½
		--	
	c)	Evaluate $\int_0^6 \frac{1}{1+x^2}\,dx$ Using Simpson's $3/8^{th}$ rule.	**06**
	Ans	Consider $n=6$	
		$y = \frac{1}{1+x^2}$ $a=0, \quad b=6$	
		$\therefore h = \frac{b-a}{n} = \frac{6-0}{6} = 1$	1
		(see table below)	2
		Using Simpson's $3/8^{th}$ rule.	
		$\int_a^b f(x)\,dx = \frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2+y_4...+y_{n-1})+2(y_3+y_6+...+y_{n-3})\right]$	
		$\therefore \int_0^6 \frac{1}{1+x^2}\,dx = \frac{3(1)}{8}\left[(1+0.0270)+3(0.5+0.2+0.0588+0.0385)+2(0.1)\right]$	2
		$\therefore \int_0^6 \frac{1}{1+x^2}\,dx = 1.3571$	1
		<u>Note</u>: If the student has considered any value of n and attempted to solve give appropriate marks.	

x	0	1	2	3	4	5	6
$y = \frac{1}{1+x^2}$	1	0.5	0.2	0.1	0.0588	0.0385	0.0270

Q. No.	Sub Q. N.	Answer	Marking Scheme
		Important Note *In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.*	