17301

Instructions – (1) All Questions are *Compulsory.*

(2) Answer each next main Question on a new page.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Assume suitable data, if necessary.

(6) Use of Non-programmable Electronic Pocket Calculator is permissible.

(7) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. **Attempt any TEN of the following:** **20**

a) Find the gradient of the curve $y = \sqrt{x^3}$ at $x = 4$.

b) Divide 100 into two parts such that their product is maximum.

c) Evaluate : $\int \dfrac{x^2}{4 + x^2} \, dx$

d) Evaluate : $\int \log x \, dx$

e) Evaluate : $\int \tan^3 x \, dx$

f) Evaluate : $\int \dfrac{dx}{(x + 1)(x + 2)}$

Marks

g) Evaluate : $\int \dfrac{\sin x}{\sin 2x}\, dx$

h) If $\int\limits_{0}^{a} 3x^2\, dx = 8$ find the value of 'a',

i) Find the area bounded by $y = x^2 - 9,\ x = 0,\ x = 3$ and the X − axis.

j) Find order and degree of the differential equation

$$\left(\dfrac{d^2 y}{dx^2}\right)^{2/3} = \sqrt{\left(y + \dfrac{dy}{dx}\right)}$$

k) Form a D.E. if $y = a \cos (x + b)$

l) Verify that $y = 4 \sin 3x$ is a solution of the differential equation

$$\dfrac{d^2 y}{dx^2} + 9y = 0$$

m) Find the probability of the occurrence of the digit 3 when an unbiased dice is thrown.

n) A coin is tossed 3 times. What is the probability that appears an odd number of times?

2. **Attempt any <u>FOUR</u> of the following:** **16**

a) Determine a and b such that slope of curve $2y^3 = ax^2 + b$ at $(1,\ -1)$ is same as the slope of $x + y = 0$.

b) Find maximum and minimum value of the function $y = 2x^3 - 21x^2 + 36x - 20$

c) Find radius of curvature of
$y = \log(\sin x)$ at $x = \pi/2$

d) Evaluate : $\int \sin^{-1} x\, dx$

e) Evaluate : $\displaystyle\int \frac{\cos x \, dx}{(1 + \sin x)(2 + \sin x)(3 + \sin x)}$

f) Evaluate : $\displaystyle\int \frac{dx}{1 + 2(x + 2)^2}$

3. **Solve any FOUR of the following:**   16

a) Evaluate : $\displaystyle\int_0^1 \frac{dx}{1 - x + x^2}$

b) Evaluate : $\displaystyle\int_0^{\pi/2} \frac{dx}{9 + 16 \cos^2 x}$

c) Find the area included between the curves $y^2 = 4ax$ and $x^2 = 4ay$.

d) Solve $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$

e) Solve : $\sin^{-1}\left(\dfrac{dy}{dx}\right) = x + y$

f) Solve : $(y^2 - x^2) \, dx - 2xy \, dy = 0$

4. **Attempt any FOUR of the following:**   16

a) Evaluate : $\displaystyle\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} \, dx$

b) Evaluate : $\displaystyle\int_0^{\pi} \frac{dx}{5 + 4 \cos x}$

c) Find the area of the circle $x^2 + y^2 = 9$ using integration.

d) Solve : $x\dfrac{dy}{dx} + y = x^3$

e) Solve : $\left[y\left(1 + \dfrac{1}{x}\right) + \cos y\right] dx + \left[x + \log x - x \sin y\right] dy = 0$

f) Verify that $y = e^{m \sin^+ x}$ is the solution of the differential equation

$$(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - m^2 y = 0$$

5. **Attempt any FOUR of the following:** 16

a) Two unbiased dice are thrown. Find the probability that the sum of the numbers obtained on two dice is neither a multiple of 2 nor a multiple of 3.

b) Probability that a bomb dropped from a plane hits a target is 0.4. Two bombs can destroy a bridge, if in all 6 bombs are dropped, find the probability that the bridge will be destroyed.

c) If the probability of a bad reaction from a certain injection is 0.001. Determine the chance that out of 2000 individuals more than two will get a bad reaction. (Given : e = 2.718)

d) Evaluate : $\int \dfrac{\sin (x + a)}{\sin x} \, dx$

e) Evaluate : $\int\limits_0^{\pi/2} \log (\sin x) \, dx$

f) Evaluate : $\int x \sin x \cos x \, dx$

6. **Attempt any FOUR of the following:** 16

a) It is given that mean and variance of a binomial distribution are 2 and 4/3 respectively what is the probability of obtaining

    (i)   Exactly two successes

    (ii) Less than two successes.

b) A card is drawn from a pack of 100 cards numbered 1 to 100 find the probability of drawing a number which is a square.

c) Divide 20 into two parts so that the product of the square of the one and the cube of the other may be the greatest possible.

Marks

d) Find the equation of tangent to the curve $x = \dfrac{1}{t}$; $y = t - \dfrac{1}{t}$ when $t = 2$.

e) Given $p(A) = \dfrac{1}{4}$ $p(B) = \dfrac{1}{3}$ and $p(A \cup B) = \dfrac{1}{2}$

Evaluate :

(i) $p(A/B)$

(ii) $p(B/A)$

(iii) $p(A \cap B')$

(iv) $p(A/B')$

f) In a certain examination 500 students appeared. Mean score is 68 with S.D. 8 Find the number of students scoring

(i) Less than 50

(ii) Scoring more than 60

Given Area between $z = 0$ and $z = 2.25$ is 0.4878

Area between $z = 0$ and $z = 1$ is 0.3413

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