17216

21718

3 Hours / 100 Marks Seat No. | | | | | | | | |

Instructions – (1) All Questions are *Compulsory.*

(2) Answer each next main Question on a new page.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Assume suitable data, if necessary.

(6) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Attempt any TEN of the following: **20**

a) Find x and y if $x(1-i)+y(2+i)+6=0$

b) Define composite function.

c) If $f(x)=x^4-2x+7$ find $f(0)+f(2)$

d) Express in the form of $a+ib$ if $z=\dfrac{1+i}{3-i}$

e) Evaluate $\lim\limits_{x\to 4}\dfrac{2-\sqrt{x}}{4-x}$

f) Evaluate $\lim\limits_{x\to 0}\dfrac{5\sin x+7x}{8x-3\tan x}$

g) Evaluate $\lim\limits_{x\to\infty}\left(1-\frac{7}{2x}\right)^{x}$

h) If $y=\log(1+x^2)$ Find $\frac{dy}{dx}$

P.T.O.

Marks

i) Find dy/dx if $y = \dfrac{\sin x}{1 + \cos x}$

j) Find dy/dx if $x^3 + y^3 = 3axy$

k) Using Gauss seidal method find first iteration for the system of equations:

$$8x + 2y + 3z = 30$$
$$x - 9y + 2z = 1$$
$$2x + 3y + 6z = 31$$

l) Show that the root of the equation $xe^x - 3 = 0$ lies in the interval (1, 2).

2. Attempt any FOUR of the following: 16

a) Find modulus and argument of –3 + 3i.

b) Using De-Movier's Theorem, simplify,

$$\frac{(\cos\theta - l\sin\theta)^5 (\cos 3\theta + l\sin 3\theta)^{-4}}{(\cos 3\theta + l\sin 3\theta)^{-2} (\cos 5\theta - l\sin 5\theta)^3}$$

c) Find all required roots of $(-1)^{1/5}$ using De-Movier's Theorem.

d) If $\cos(A + l\,B) = x + l\,y$ show that $\dfrac{x^2}{\cos^2 A} - \dfrac{y^2}{\sin^2 A} = 1$ and $\dfrac{x^2}{\cos h^2 B} + \dfrac{y^2}{\sin h^2 B} = 1$

e) If $f(x) = y = \dfrac{ax + 1}{5x - a}$ show that $f(y) = x$.

f) If $f(x) = \log\left(\dfrac{x - 1}{x}\right)$, show that $f(y^2) = f(y) + f(-y)$

Marks

3. Attempt any FOUR of the following: 16

a) If $f(x) = \dfrac{x-1}{x+1}$, $x \neq -1$, show that $f\left(\dfrac{x-1}{x+1}\right) = \dfrac{-1}{x}$

b) For what values of x, $f(x) = f(2x+1)$ if $f(x) = x^2 - 3x + 4$

c) Evaluate $\lim\limits_{x \to 3} \left[\dfrac{1}{x-3} - \dfrac{1}{(x^2 - 5x + 6)}\right]$

d) Evaluate $\lim\limits_{x \to \infty} x\left[\sqrt{x^2+1} - \sqrt{x^2-1}\right]$

e) Evaluate $\lim\limits_{x \to 0} \dfrac{\cos 3x - \cos 5x}{x^2}$

f) Evaluate $\lim\limits_{x \to 0} \dfrac{10^x - 2^x - 5^x + 1}{x \tan x}$

4. Attempt any FOUR of the following: 16

a) Using First principle of derivative, find the derivative of log x.

b) If u and v are differentiable functions of x and $y = {}^{u}\!/_{v}$ then

$$\frac{dy}{dx} = \frac{V\dfrac{du}{dx} - U\dfrac{dv}{dx}}{V^2}$$

c) If $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$ then show that $\dfrac{dy}{dx} = 1 - y^2$

d) If $y = \tan^{-1}\left[\dfrac{\sin 2x}{1 - \cos 2x}\right]$ Find $\dfrac{dy}{dx}$

e) Find derivative of $(\sin^{-1}x)^{\cos x}$

f) Differentiate $5^{\sqrt{x}}$ w.r.t. to $\left(\sqrt{x}\right)^x$

Marks

5. Attempt any FOUR of the following: 16

a) Evaluate $\lim\limits_{x \to 0} \dfrac{\log(e + x) - 1}{x}$

b) Evaluate $\lim\limits_{x \to 0} \dfrac{\sin 3x + 7x}{4x + \sin 2x}$

c) Using Bisection method find the approximate root of the equation $x^3 - 6x + 3 = 0$ (Perform three iterations)

d) Use Regular falsi method to find approximate root of the equation $x^3 - x - 4 = 0$ (Three iterations)

e) Use Newton Raphson method to evaluate $\sqrt[3]{20}$ (upto three iterations only)

f) Using Bisection method find the root of the equation $x^3 - 4x - 9$ in the interval (2, 3).

6. Attempt any FOUR of the following: 16

a) If $y = 2\sin 2x - 5\cos 2x$, show that $\dfrac{d^2 y}{dx^2} + 4y = 0$

b) If $y = \log(\log x)$ prove that

$$x\frac{d^2 y}{dx^2} + \frac{dy}{dx} + x\left(\frac{dy}{dx}\right)^2 = 0$$

c) Using Gauss elimination method, solve the equation:

$$2x + 3y + 2z = 2$$
$$10x + 3y + 4z = 16$$
$$3x + 6y + z = -6$$

d) Using Jacobi's method, solve the system of equations:

$10x + 2y + z = 9$; $2x + 20y - 2z = -44$; $-2x + 3y + 10z = 22$

(Perform three iterations).

e) Using Gauss-seidal method, solve the equations:

$5x - y = 9$; $x - 5y + z = -4$; $y - 5z = 15$ (Perform three iterations)

f) Using Jacobi's method, solve the equations

$2x + 3y - 4z = 1$, $5x + 9y + 3z = 17$; $8x - 2y - z = 5$

(Perform three iterations)