17105

15116

3 Hours / 100 Marks Seat No. | | | | | | | | |

Instructions : (1) All Questions are *compulsory*.

(2) Answer each next main Question on a new page.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Assume suitable data, if necessary.

(6) Use of Non-Programmable Electronic Pocket Calculator is permissible.

(7) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Attempt any TEN of the following : **20**

(a) Solve $\begin{vmatrix} 1 & x & x^2 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{vmatrix} = \begin{vmatrix} 5 & 4 \\ 5 & 4 \end{vmatrix}$

(b) If $A = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix}$, find $2A + 3B - 4I$, where I is the unit matrix of order two.

(c) If $A = \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$, prove that $A^2 - 3A = 2I$, where I is the unit matrix of order two.

(d) If $A = \begin{bmatrix} 1 & 2 \\ 5 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 6 \\ -3 & 4 \end{bmatrix}$ then verify that $(AB)' = B'A'$

(e) Resolve into partial fraction $\frac{1}{x^2 + 3x + 2}$

(f) Prove that $\cos 2\theta = \cos^2\theta - \sin^2\theta$.

(g) Without using calculator, find the value of $\sin 15°$

(h) Prove that $\frac{\tan 420° + \tan 300°}{1 - \tan 420° \tan 660°} = 0$

P.T.O.

(i) If sin 80° + sin 50° = 2 sinA cosA, then find A and B.

(j) Prove that : sin (n + 1)A . sin (n + 2)A + cos (n + 1) A. cos (n + 2)A = cos A

(k) Find the distance between parallel lines $3x + 2y - 6 = 0$ and $3x + 2y - 12 = 0$

(l) Evaluate $\begin{vmatrix} 3 & 4 & 2 \\ 12 & 16 & 8 \\ -5 & -6 & 0 \end{vmatrix}$

2. Attempt any FOUR of the following : 16

(a) Solve the following equations by using Cramer's rule $x + y - z = 0$, $2x + y + 3z = 9$, $x - y + z = 2$.

(b) Find x, y, z, if $\left\{\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix} + 2\begin{bmatrix} 3 & 0 & 2 \\ 1 & 4 & 5 \\ 2 & 1 & 0 \end{bmatrix}\right\}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

(c) If $A = \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix}$, show that $A^2 - 8A$ is a scalar matrix.

(d) If $A = \begin{bmatrix} -2 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 1 & 1 \end{bmatrix}$ show that the matrix AB is non-singular.

(e) Resolve into partial fraction $\frac{3x - 1}{(x - 4)(2x + 1)(x - 1)}$

(f) Solve by Cramer's rule $x + y + z = 6$; $2x - y + 3z = 9$; $x + 2y + 3z = 14$.

3. Attempt any FOUR of the following : 16

(a) Using matrix inversion method, solve the following equations :

$x + 3y + 2z = 6$, $3x - 2y + 5z = 5$, $2x - 3y + 6z = 7$.

(b) Resolve into partial fraction $\frac{2x - 3}{(x + 1)(x^2 + 4)}$

(c) Resolve into partial fraction $\frac{5 \cos x - 3}{(\cos x + 1)(\cos x - 3)}$

(d) Resolve into partial fraction $\frac{x^4}{x^3 + 1}$

(e) Find inverse of the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

(f) If I is an unit matrix of order 3 and $A = \begin{bmatrix} 1 & 2 & 6 \\ 7 & 4 & 10 \\ 1 & 3 & 5 \end{bmatrix}$,

then find $A^2 - 3A + I$.

4. Attempt any FOUR of the following : **16**

(a) Prove that $\cos(A + B) = \cos A \cos B - \sin A \sin B$

(b) Prove that $\cos 3A = 4\cos^3 A - 3\cos A$.

(c) Without using calculator show that $\cos 20° \cos 40° \cos 60° \cos 80° = \frac{1}{16}$

(d) Without using calculator show that $\frac{\sin 19° + \cos 11°}{\cos 19° - \sin 11°} = \sqrt{3}$

(e) Prove that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$

(f) Prove that $\sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{8}{17}\right) = \sin^{-1}\left(\frac{77}{85}\right)$

5. Attempt any FOUR of the following : **16**

(a) Prove that $\cos\left(\frac{\pi}{2} + \theta\right) = -\cos\theta$

(b) Prove that $\frac{\sin A + 2\sin 2A + \sin 3A}{\cos A + 2\cos 2A + \cos 3A} = \tan 2A$

(c) Prove that $\frac{\sin 7x + \sin x}{\cos 5x - \cos 3x} = \sin 2x - \cos 2x \cot x$

(d) Prove that $\frac{\sin 9\theta}{\sin 3\theta} - \frac{\cos 9\theta}{\cos 3\theta} = 2$

(e) Prove that $\cos C + \cos D = 2\cos\left(\frac{C + D}{2}\right)\cos\left(\frac{C - D}{2}\right)$

(f) If $x > 0$, $y > 0$, then prove that $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left[\frac{x - y}{1 + xy}\right]$

6. Attempt any FOUR of the following : **16**

(a) If $P(x_1, y_1)$ be any point outside the line $ax + by + c = 0$, then prove that perpendicular distance from the point to the line is $d = \left| \frac{ax_1 + by_1 + c}{\sqrt{A^2 + B^2}} \right|$

(b) If m_1 and m_2 are the slope of two lines then prove that angle between two lines is $\theta = \tan^{-1} \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

(c) Find the length of perpendicular on the line $3x + 4y - 6 = 0$ from the point (3, 4).

(d) Find the equation of straight line passing through the point of intersection of lines $4x + 3y = 8$ and $x + y = 1$ and parallel to the line $5x - 7y = 3$.

(e) Find the equation of line passing through the point of intersection of the lines $2x + 3y = 13$, $5x - y = 7$ and passing through the point $(1, -1)$.

(f) Find the acute angle between the lines $3x - 2y + 4 = 0$ and $2x - 3y - 7 = 0$.