13141

3 Hours / 100 Marks

Seat No.

Instructions - (1) All Questions are Compulsory.

- (2) Answer each next main Questions on a new page.
- (3) Illustrate your answers with neat sketches wherever necessary.
- (4) Figures to the right indicate full marks.
- (5) Assume suitable data, if necessary.
- (6) Use of Non-programmable Electronic Pocket Calculator is permissible.
- (7) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Attempt any <u>TEN</u> of the following:

20

a) Find
$$x$$
 if $\begin{vmatrix} x & 5 \\ 2 & x \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2 & 3 \end{vmatrix}$

b) Find
$$x$$
 and y if $\begin{vmatrix} 3 & -1 \\ 2 & 5 \end{vmatrix} \begin{vmatrix} -7 \\ 4 \end{vmatrix} = 2 \begin{vmatrix} x \\ y \end{vmatrix}$

c) Prove that
$$A + A^{T} = 0$$
 if $A = \begin{vmatrix} 0 & 4 & -3 \\ -4 & 0 & -2 \\ 3 & 2 & 0 \end{vmatrix}$ where 0 is

zero matrix of order 3.

Marks

d) If
$$A = \begin{vmatrix} 1 & -5 \\ 6 & 4 \end{vmatrix}$$
 and $B = \begin{vmatrix} 2 & 3 \\ 4 & -5 \end{vmatrix}$ find the matrix AB – 2I where I is the unit matrix of order 2.

e) Find A and B if
$$\frac{x+4}{x^2+x} = \frac{A}{x} + \frac{B}{x+1}$$

- f) Prove that $\cos 2\theta = \frac{1 \tan^2 \theta}{1 + \tan^2 \theta}$
- g) Prove that $\sin\left(\frac{\pi}{2} + \theta\right) = \cos\theta$
- h) Find $\sin \alpha$ if $\tan \left(\frac{\alpha}{2}\right) = \frac{1}{\sqrt{3}}$
- i) Find α and β if $\sin 80^{\circ} + \sin 50^{\circ} = 2\sin \alpha .\cos \beta$
- j) Prove that $\sin^{-1}(-x) = -\sin^{-1}(x)$
- k) Find the distance from the point (-2, 1) to the line 2x 3y + 5 = 0
- 1) The coefficient of range for the data is 0.5. Find x if the data is 120, 100, 130, 150, x where x is the smallest observation in the data.

17914

[3]

Marks

2. Attempt any **FOUR** of the following:

16

a) Using Cramer's rule to solve the equations:

$$x+3y-z=5$$
; $2x+y-3z=4$; $y+z=7+x$

- b) If $A = \begin{bmatrix} 2 & 7 \\ 1 & 0 \end{bmatrix}$ find $A^2 6A + I$ where I is unit matrix.
- c) Given: $\left\{ 3 \begin{bmatrix} 4 & 1 & 3 \\ 0 & -1 & -3 \end{bmatrix} 2 \begin{bmatrix} 3 & 2 & 4 \\ -6 & 1 & 3 \end{bmatrix} \right\} \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

Find x and y.

d) Verify that $(AB)^T = B^T A^T$ if $A = \begin{bmatrix} 3 & 4 \\ -2 & 1 \\ 1 & 0 \end{bmatrix}$ and

$$\mathbf{B} = \left[\begin{array}{rrr} -1 & 2 & -4 \\ 3 & 5 & -3 \end{array} \right]$$

- e) Resolve into Partial fraction : $\frac{2x+5}{3x^3-7x^2+4x}$
- f) Resolve into Partial fraction : $\frac{x^2 + 1}{(x-2)(x^2 + 1)}$

17914 [4]

1	M	โด	r	ks
		_		ю. Т

16

16

3. Attempt any <u>FOUR</u> of the following:

Use matrix inversion method to solve the system of equations: x+3y+2z=6; 3x-2y+5z=5; 2x-3y+6z=7

- b) Resolve into Partial fraction : $\frac{3x-5}{x^2(x-1)}$
- c) Resolve into partial fraction : $\frac{x^4}{x^3-1}$
- d) Prove that $\tan \left(\frac{\pi}{4} + \frac{A}{2}\right) = \sec A + \tan A$
- e) Prove that $2\cos\frac{\pi}{13}.\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} = 0$
- f) Prove that $\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{27}{11}\right)$

4. Attempt any **FOUR** of the following:

- a) Prove that sin(A B) = sin A.cos B cos A.sin B
- b) Prove that $\sin 3A = 3\sin A 4\sin^3 A$
- c) Prove that $\frac{\cos 2A + 2\cos 4A + \cos 6A}{\cos A + 2\cos 3A + \cos 5A} = \cos A \sin A \cdot \tan 3A$
- d) In any \triangle ABC, prove that

$$\cos A + \cos B - \cos C = 4\cos\left(\frac{A}{2}\right)\cos\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right) - 1$$

- e) Prove that $\sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{8}{17}\right) = \sin^{-1}\left(\frac{77}{85}\right)$
- f) Prove that $\tan^{-1}\left(\frac{1}{8}\right) + \cot^{-1}\left(5\right) = \tan^{-1}\left(\frac{1}{3}\right)$

5. Attempt any <u>FOUR</u> of the following:

16

- a) Find the value of $3\cos 2\theta + 2\sin 2\theta$ if $\tan \theta = \frac{2}{3}$.
- b) Prove that $\sin A \sin(60^\circ A) \sin(60^\circ + A) = \frac{1}{4} \sin 3A$
- c) Prove that $\csc^{-1}x + \sec^{-1}x = \frac{\pi}{2}$.
- d) If θ is the acute angle between the lines with slope m_1 and $m_2 \text{ then } \tan\theta = \left \lfloor \frac{m_1 m_2}{1 + m_1 m_2} \right \rfloor. \text{ Prove it.}$
- e) Find the perpendicular distance between the parallel lines 5x-12y+1=0 and 10x=24y+1
- f) Find the equation of a straight line passing through the point (2, -4) and the midpoint of the line segment joining the points A(6, -1) and B(5, 7).

6. Attempt any **FOUR** of the following:

16

a) If a line makes 'a' and 'b' as x-intercept and y-intercept respectively then prove that the equation of line can be

[6]

written as
$$\frac{x}{a} + \frac{y}{b} = 1$$
.

- b) Find the equation of straight line passing through the point of intersection of the lines 2x+5y+3=0 and x+7y+6=0 and parallel to the line 4x+y-7=0.
- c) Find the range and coefficient of range for the data:

Maxmimum Temperature	25-26	27-28	29-30	31-32	33-34	35-36
No. of days	2	11	12	10	4	1

d) Find the mean deviation from the mean for the following data:

Wt. in grams	10-15	15-20	20-25	25-30	30-35	35-40	40-45
No. of items	7	12	16	25	19	15	6

e) Find the variance and coefficient of variance for the data:

Rainfall	70-80	80-90	90-100	100-110	110-120	120-130	130-140	140-150
No.of places	6	7	12	19	21	18	11	6

f) In two factories A and B, engaged in the same area of the industry, the average weekly wages (in Rs.) and standard deviations are as follows:

Factory	Average wages	S.D.
A	34.5	5.0
В	28.5	4.5

Which of the factory is more consistent?

3 Hours / 100 Marks