17216

15162

3 Hours / 100 Marks Seat No. | | | | | | | | |

Instructions – (1) All Questions are *Compulsory.*

(2) Illustrate your answers with neat sketches wherever necessary.

(3) Figures to the right indicate full marks.

(4) Assume suitable data, if necessary.

(5) Use of Non-programmable Electronic Pocket Calculator is permissible.

Marks

1. **Attempt any TEN of the following:** **20**

a) If $3a - 7 + 2bi = 5i + ia - 5b$ find a, b.

b) If $z = 1 + i\sqrt{3}$ show that $z^2 + 4 = 2z$.

c) Define even and odd function.

d) If $f(x) = \sin x$ show that $f(3x) = 3f(x) - 4f^3(x)$

e) Evaluate $\lim\limits_{x \to 1} \left(\frac{1}{x-1} - \frac{1}{x^2 - x} \right)$

f) Evaluate $\lim\limits_{x \to 0} \frac{\cos 5x - \cos 3x}{x^2}$

g) Evaluate $\lim\limits_{x \to 0} \left(\frac{1+x}{1-x} \right)^{\frac{1}{x}}$

h) Find $\frac{dy}{dx}$ if $y = \frac{\sin x}{1 - \cos x}$

i) If $y = \log(\sec x + \tan x)$ find $\frac{dy}{dx}$

j) If $\tan^{-1}(x^2 + y^2) = a^2$ Find $\frac{dy}{dx}$

Marks

k) Using Bisection method find the root of $x^3 - x - 1 = 0$ (two iteration only)

l) Find by Jacobis method, the first iteration only, for the following equation $5x - y = 9, x - 5y + z = -4, y - 5z = 6$.

2. Attempt any <u>FOUR</u> of the following: **16**

a) Find the complex conjugate of $\frac{(2+i)^2}{2+3i}$

b) Simplify using De-moiver's theorem

$$\frac{(\cos 3\theta + i \sin 3\theta)^4 (\cos 5\theta - i \sin 5\theta)^{\frac{4}{5}}}{\left(\cos \frac{9}{2}\theta + i \sin \frac{9}{2}\theta\right)^{\frac{2}{3}} \left(\cos \frac{4}{5}\theta - i \sin \frac{4}{5}\theta\right)^{10}}$$

c) Using Euler's Exponential formula prove that:

(i) $\sin^2\theta + \cos^2\theta = 1$

(ii) $\cos h^2\theta - \sin h^2\theta = 1$

d) Use De-moivre's theorem to solve the equation $x^3 - 1 = 0$

e) If $f(x) = \frac{x+3}{4x-5}$ and $t = \frac{3+5x}{4x-1}$ then show that $f(t) = x$

f) If $f(t) = 50 \sin(50\pi t + 0.04)$ show that $f\left(\frac{2}{100} + t\right) = f(t)$

3. Attempt any <u>FOUR</u> of the following: **16**

a) If $f(x) = x^2 + 3$ then find the value of x for which $f(x) = f(2x+1)$

b) If $f(x) = 16^x + \log_2^x$ then find the value of $f\left(\frac{1}{4}\right)^2, f\left(\frac{1}{2}\right)$

c) Evaluate $\lim_{x \to 2} \frac{x^2 - 4}{\sqrt{x+2} - \sqrt{3x-2}}$

d) Evaluate $\lim_{x \to 3} \frac{\log x - \log 3}{x - 3}$

e) Evaluate $\lim_{x \to 0} \frac{4^x + 4^{-x} - 2}{x \cdot \sin x}$

f) Evaluate $\lim_{\theta \to \frac{\pi}{4}} \frac{\sin\theta - \cos\theta}{\theta - \frac{\pi}{4}}$

Marks

4. Attempt any FOUR of the following: 16

a) Using first principle find the derivative of $f(x) = x^n, x \in R$.

b) If u and v are differentiable functions of x and if $y = uv$ then prove that: $\frac{dy}{dx} = u \cdot \frac{dv}{dx} + v \cdot \frac{dy}{dx}$

c) Find $\frac{dy}{dx}$ if $y = \tan^{-1}\left(\frac{2x}{1 + 35x^2}\right)$

d) If $x^3y^2 = (x + y)^5$ show that $\frac{dy}{dx} = \frac{y}{x}$

e) If $y = \frac{(1 - x)^{\frac{1}{2}}}{(x - 1)^{\frac{5}{7}}(2x + 1)^{\frac{1}{3}}}$ find $\frac{dy}{dx}$

f) Find $\frac{dy}{dx}$ if $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$

5. Attempt any FOUR of the following: 16

a) Evaluate $\lim_{x \to \infty} \sqrt{x}\left(\sqrt{x + 1} - \sqrt{x}\right)$

b) Evaluate $\lim_{\theta \to \frac{\pi}{4}} \frac{2 - \sec^2\theta}{1 - \tan\theta}$

c) Using bisection method find the approximate value of $\sqrt{10}$ by performing three iterations.

d) Using Regula Falsi method find the root of $x^2 - \log_{10}^{x} = 12$ (upto three iterations only)

e) Find the approximate root of the equation $x^3 - 20 = 0$ by Newton-Raphson method (three iterations)

f) Obtain the approximate root value of equation $x^3 - 4x + 1 = 0$ using Regula-Falsi method upto 4 decimal places.

Marks

6. Attempt any FOUR of the following: 16

a) Differentiate $\cos^{-1}\left(2x\sqrt{1-x^2}\right)$ with respect to $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

b) If $y = e^{\tan^{-1}x}$ show that $\left(1+x^2\right)\frac{d^2y}{dx^2} + \left(2x-1\right)\frac{dy}{dx} = 0$

c) Solve using Gauss-Elimination method

$x + 2y + 3z = 14$

$3x + y + 2z = 11$

$2x + 3y + z = 11$

d) Solve by Gauss Seidal method (upto the iterations any)

$x + 7y - 3z = -22$

$5x - 2y + 3z = 18$

$2x - y + 6z = 22$

e) Solve the equations using Jacobi's method (upto three iterations)

$10x - 2y - 2z = 6$

$-x - y + 10z = 8$

$-x + 10y - 2z = 7$

f) Use Gauss-Seidal method to solve following equations (use two iterations)

$10x + 2y + z = 9$

$x + 10y - z = -22$

$-2x + 3y + 10z = 22$