

# 17216

**21415**

**3 Hours / 100 Marks**

Seat No.

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- Instructions* – (1) All Questions are *Compulsory*.  
(2) Answer each next main Question on a new page.  
(3) Figures to the right indicate full marks.  
(4) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

**Marks**

**1. Attempt any TEN of the following:**

**20**

- a) If  $\frac{10}{3+4i} = a+ib$  find  $a$  and  $b$ .
- b) If  $z = 3+4i$  Find  $z^2 - 6z + 25$
- c) If  $f(x) = x^2 + 6x + 10$  Find  $f(2) + f(-2)$
- d) If  $f(x) = \frac{a^x + a^{-x}}{2}$  prove that the function is even function.
- e) Evaluate  $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$
- f) Evaluate  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$
- g) Evaluate  $\lim_{x \rightarrow \infty} \left(\frac{x}{x+1}\right)^x$
- h) If  $y = e^x \sin x$  find  $\frac{dy}{dx}$ .

P.T.O.

- i) If  $y = \tan^{-1}\left(\frac{a+x}{1-ax}\right)$  Find  $\frac{dy}{dx}$ .
- j) If  $x = a \sec t$  and  $y = b \tan t$  then find  $\frac{dy}{dx}$ .
- k) Prove that the root of equation  $x^3 - x - 4 = 0$  lies between 0 and 2.
- l) Find the first iteration by using Jacobi's method for the following equation.  
 $4x + y + 3z = 17$ ,  $x + 5y + z = 14$  and  $2x - y + 8z = 12$ .

2. **Attempt any FOUR of the following:**

16

- a) If  $f(x) = \tan x$ , prove that  $f(2x) = \frac{2f(x)}{1-f^2(x)}$
- b) Simplify using De-moiver's theorem  

$$\frac{(\cos 3\theta + i \sin 3\theta)^4 (\cos 5\theta - i \sin 5\theta)^{4/5}}{(\cos^{3/5}\theta + i \sin^{3/5}\theta)^5 (\cos^{4/5}\theta + i \sin^{4/5}\theta)^{10}}$$
- c) Separate into real and imaginary part of  $\sin(x + iy)$
- d) Express in Polar form  $1 - \sqrt{3}i$
- e) Show that  $(1 + i)^{12} + (1 - i)^{12} = -128$
- f) If  $f(x) = \log\left(\frac{x+1}{x-1}\right)$  prove that  $f\left(\frac{1+x^2}{2x}\right) = 2f(x)$

3. Attempt any FOUR of the following:

16

- a) Find  $f(t)$ , if  $f(x) = \frac{2x+5}{3x-4}$  and  $t = \frac{5+4x}{3x-2}$ .
- b) Evaluate  $\lim_{x \rightarrow 0} \frac{3^x + 3^{-x} - 2}{x^2}$
- c) If  $f(x) = x^2 - 3x + 4$  solve  $f(1-x) = f(2x+1)$ .
- d) Evaluate  $\lim_{x \rightarrow 3} \frac{x^3 - 7x^2 + 15x - 9}{x^3 - 4x^2 - 3x + 18}$ .
- e) Evaluate  $\lim_{x \rightarrow \infty} \sqrt{x^2 + x + 1} - x$
- f) Evaluate  $\lim_{x \rightarrow 0} \frac{2 \sin x - \sin 2x}{x^3}$

4. Attempt any FOUR of the following:

16

- a) Find  $\frac{dy}{dx}$  if  $y = \cos^{-1}(2x^2 - 1)$
- b) If  $x^2 + y^2 - xy = 0$  find  $\frac{dy}{dx}$ .
- c) If  $x = a(1 + \cos \theta)$   $y = a(1 - \cos \theta)$  find  $\frac{dy}{dx}$ .
- d) Using first principle find derivate of  $f(x) = \tan x$ .
- e) If  $u$  and  $v$  are differentiable functions of  $x$  and  $y = u + v$  than prove that  $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ .
- f) If  $x^y = e^{x-y}$  prove that  $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$ .

5. Attempt any FOUR of the following:

16

- a) Evaluate  $\lim_{x \rightarrow 3} \frac{\log x - \log 3}{x-3}$
- b) Evaluate  $\lim_{x \rightarrow 0} \frac{(5^x - 1)\tan x}{\sqrt{x^2 + 16} - 4}$
- c) Find the root of the equation  $x^3 - 9x + 1 = 0$  which lies between 2 and 3 using Regula falsi method.
- d) Find a root of  $x^3 - 9x^2 - 18 = 0$  by Newton-Raphson method. (carry out 3 iterations)
- e) Using Newton-Raphson method, find approximate value of  $\sqrt{10}$  (carry out 3 iterations)
- f) Find the root of equation  $x^3 - 4x + 1 = 0$  using bisection method (carry out 3 iterations)

6. Attempt any FOUR of the following:

16

- a) Find  $\frac{d^2y}{dx^2}$  if  $x = a \cos \theta$   $y = a \sin \theta$ .
- b) Solve the following equation by Gauss elimination method  
 $2x + y + z = 10$ ,  $3x + 2y + 3z = 18$  and  $x + 4y + 9z = 16$ .
- c) Solve the following equation by Gauss-Sedial method taking two iterations.  
 $10x + y + z = 12$ ,  $2x + 10y + z = 13$  and  $2x + 2y + 10z = 14$ .
- d) Solve the following equation by Jacobi's method by performing two iteration's only  
 $15x + 2y + z = 18$ ,  $2x + 20y - 3z = 19$  and  $3x - 6y + 25z = 22$ .
- e) Solve by Jacobi method, carry out two iterations  
 $10x + y + 2z = 13$ ;  $3x + 10y + z = 14$ ;  $2x + 3y + 10z = 15$ .
- f) If  $y = e^{m \sin^{-1} x}$  prove that  $(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = m^2 y = 0$
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