17105

21415

3 Hours/100 Marks

Seat No. | | | | | | | |

Instructions : *(1)* ***All*** *questions are* ***compulsory****.*

(2) Figures to the ***right*** *indicate* ***full*** *marks.*

(3) Assume suitable data, if necessary.

(4) ***Use*** *of Non-programmable Electronic Pocket Calculator is* ***permissible****.*

(5) Mobile phone, pager and ***any*** *other electronic communication devices are* ***not permissible*** *in Examination Hall.*

MARKS

1. Attempt **any ten** of the following : **20**

a) Solve $\begin{vmatrix} 2 & -3 \\ 4 & 3 \end{vmatrix} = \begin{vmatrix} x & 1 \\ -2 & x \end{vmatrix}$.

b) Find x if $\begin{vmatrix} 4 & 3 & 9 \\ 3 & -2 & 7 \\ 11 & 4 & x \end{vmatrix} = 0$.

c) If $A = \begin{bmatrix} 5 & 3 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ find 3A – 2B.

d) If $A = \begin{bmatrix} 3 & 9 \\ -1 & -3 \end{bmatrix}$ then show that A^2 is a null matrix.

e) Define singular and non-singular matrix.

f) Resolve into partial fraction : $\dfrac{1}{x^2 + 3x + 2}$.

g) If A = 30° verify the result sin 3A = 3 sinA – 4 $\sin^3$ A.

h) Without using calculator prove that, $\dfrac{\cos 21^\circ + \sin 21^\circ}{\cos 21^\circ - \sin 21^\circ} = \cot 24^\circ$.

P.T.O.

MARKS

i) If 2 sin 50° cos 70° = sin A – sin B. Find A and B.

j) Find the principal value of $\cos\left[\frac{\pi}{2} - \sin^{-1}\frac{1}{2}\right]$.

k) Prove that $\sin^{-1}\left(\frac{-1}{2}\right) + \cos^{-1}\left(\frac{-1}{2}\right) = \tan^{-1}(\infty)$.

l) Find the acute angle between the lines 3x – 2y + 4 = 0, and 2x – 3y – 7 = 0.

2. Attempt **any four** of the following : **16**

a) Solve by Crammer's Rule, $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1,\ \frac{3}{x}+\frac{1}{y}+\frac{2}{z}=4,\ \frac{9}{x}+\frac{1}{y}+\frac{4}{z}=16$.

b) If $\left\{3\begin{bmatrix}3 & 1\\ 4 & 0\\ 3 & -3\end{bmatrix} - 2\begin{bmatrix}0 & 2\\ -2 & 3\\ -5 & 4\end{bmatrix}\right\}\begin{bmatrix}-1\\ 2\end{bmatrix} = \begin{bmatrix}x\\ y\\ z\end{bmatrix}$ then find x, y and z.

c) Find inverse of $\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 3\\ 1 & 4 & 9\end{bmatrix}$ by adjoint method.

d) Resolve into partial fractions $\frac{x-5}{x^3+x^2-6x}$.

e) If $A=\begin{bmatrix}2 & -3\\ 1 & 5\end{bmatrix}$ $B=\begin{bmatrix}3 & -1 & 2\\ 1 & 0 & 1\end{bmatrix}$ then verify that $(AB)' = B'A'$.

f) If $A=\begin{bmatrix}2 & 4\\ 1 & 1\end{bmatrix}$ then show that $A^2 - 3A = 2I$. Where I is unit matrix of order 2.

3. Attempt **any four** of the following : **16**

a) Find AB if $A=\begin{bmatrix}3 & 2 & 1\\ -4 & 0 & 2\end{bmatrix}$ $B=\begin{bmatrix}3 & -1 & 2\\ 4 & 2 & 0\\ 5 & -7 & 6\end{bmatrix}$.

MARKS

b) If $A = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 2 & -5 \\ 1 & 0 & 3 \end{bmatrix}$ $C = \begin{bmatrix} 6 & -7 & 0 \\ -1 & 2 & 5 \\ 1 & 0 & 3 \end{bmatrix}$ then prove that (AB) C = A (BC).

c) Solve the equations by matrix method
$x + y + z = 3$; $2x - y + 3z = 4$; $3x + 4y + z = 8$.

d) Resolve into partial fraction $\dfrac{x^2 + 23x}{(x+3)(x^2+1)}$.

e) Resolve into partial fractions $\dfrac{x^4}{x^3+1}$.

f) Resolve into partial fractions $\dfrac{\sin\theta + 1}{(\sin\theta - 1)(\sin\theta + 2)}$.

4. Attempt **any four** of the following : **16**

a) Prove that $\sin A \sin(60^\circ - A) \sin(60^\circ + A) = \dfrac{\sin 3A}{4}$.

b) Prove that $\tan^{-1}\left(\dfrac{1}{2}\right) + \tan^{-1}\left(\dfrac{1}{3}\right) = \dfrac{\pi}{4}$.

c) Prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

d) Show that $\dfrac{\sin A + \sin 2A + \sin 3A + \sin 4A}{\cos A + \cos 2A + \cos 3A + \cos 4A} = \tan\left(\dfrac{5A}{2}\right)$.

e) Prove that $\sin 20^\circ \cdot \sin 40^\circ \cdot \sin 60^\circ \cdot \sin 80^\circ = \dfrac{3}{16}$.

f) Show that $\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B$.

5. Attempt **any four** of the following : **16**

a) Prove that
$\cos(A + B) = \cos A \cos B - \sin A \sin B$.

b) Prove that, $\tan 15^\circ + \tan 75^\circ = 4$.

MARKS

c) Show that $\cos^{-1}\left(\frac{4}{5}\right)-\cos^{-1}\left(\frac{12}{13}\right)-\cos^{-1}\left(\frac{63}{65}\right)$.

d) Prove that $\frac{\cos 2A + 2\cos 4A + \cos 6A}{\cos A + 2\cos 3A + \cos 5A} = \cos A - \sin A \tan 3A$.

e) If x and y are positive then prove that

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right) \text{ if } 1-xy \geq 0.$$

f) Prove that

$$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right).$$

6. Attempt **any four** of the following : **16**

a) Find the equation of line passing through the point of intersection of the lines $x + y = 0$; $2x - y = 9$ and parallel to the line $3x + 2y - 1 = 0$.

b) If m_1 and m_2 are slopes of two lines, then prove that the acute angle between two lines is $\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$.

c) Find perpendicular distance between (3, 2) and the line $4x - 6y = 5$.

d) Find the equation of line passing through the point of intersection of lines $x + y = 0$ and $2x - y = 9$ and through the point (2, 5).

e) Find the length of perpendicular from (–3, – 4) on the line $4(x + 2) = 3(y - 4)$.

f) Find the distance between the lines $5x - 12y + 1 = 0$ and $10x - 24y - 1 = 0$. Also prove that these lines are parallel to each other.