21314

17105

3 Hours/ 100 Marks Seat No.

Instructions : *(1)* ***All*** *questions are* ***compulsory****.*

(2) Answer ***each*** *next main question on a* ***new page****.*

(3) Illustrate your answers with ***neat*** *sketches* ***wherever*** *necessary.*

(4) Figures to the ***right*** *indicate* ***full*** *marks.*

(5) ***Assume*** *suitable data, if* ***necessary****.*

(6) ***Use*** *of non-programmable Electronic Pocket Calculator is* ***permissible****.*

MARKS

1. Attempt **any ten** of the following : **20**

a) Find 'x' if $\begin{vmatrix} 1 & 2x & 4x^2 \\ 1 & 4 & 16 \\ 1 & 1 & 1 \end{vmatrix} = 0.$

b) Find the value of $\begin{vmatrix} -1 & 2 \\ 3 & 2 \end{vmatrix} + \begin{vmatrix} -3 & -2 \\ -1 & 2 \end{vmatrix}$.

c) Find 'X' such that $2\left\{X + \begin{bmatrix} 2 & -1 & 3 \\ 4 & 2 & 0 \end{bmatrix}\right\} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix}$.

d) If $A = \begin{bmatrix} 1 & -1 \\ 3 & -4 \end{bmatrix}$ find $|A^T|$.

e) If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ show that A is orthogonal matrix.

f) Resolve into partial fraction $\frac{1}{x^2 - x}$.

g) Express the following as product of trigonometric function : $\sin 7\theta - \sin 5\theta$.

h) Express as sum or difference of trigonometric function $2\cos 117° \sin 53°$.

MARKS

i) Prove that $\sin^{-1}(-x) = -\sin^{-1}x$.

j) Prove that $\cos\left[\pi/2 - \sin^{-1}\left(\frac{1}{2}\right)\right] = \frac{1}{2}$.

k) Find the intercept made by the line $5x - 3y = 15$ on co-ordinate axes.

l) Show that the lines $2x + 3y - 1 = 0$ and $3x - 2y + 6 = 0$ are perpendicular.

2. Attempt **any four** of the following : **16**

a) Solve by determinant method :
$2x - 4y + 3z = 1$, $x - 2y + 4z = 3$; $3x - y + 5z = 2$

b) Find 'r' by using Cramer's rule $4r + 2t = 4 + 7s$, $3r - 6s - 7t = 5$, $2r - 2t = -3 - 4s$.

c) If $A = \begin{bmatrix} 1 & 2 & 6 \\ 7 & 4 & 10 \\ 1 & 3 & 5 \end{bmatrix}$ find $A^2 - 3A + I$ where I is unit matrix of order 2.

d) If $A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$, $C = \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$ verify $(AB)C = A(BC)$.

e) If $A = \begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 3 \end{bmatrix}$ Prove that $(AB)^T = B^T A^T$.

f) Resolve into partial fraction $\frac{x^2 + 1}{x^2 - 1}$.

3. Attempt **any four** of the following : **16**

a) Express the matrix A as the sum of a sum of symmetric and skew symmetric matrix where $A = \begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & -7 \end{bmatrix}$.

b) Find A^{-1} by adjoint method, if $A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 0 & 4 \\ 1 & -1 & 1 \end{bmatrix}$.

MARKS

c) Solve by matrix method

$x + y + z = 3,\ 3x - 2y + 3z = 4,\ 5x + 5y + z = 11.$

d) Resolve into partial fraction

$\frac{3x^2 + 17x + 14}{x^3 - 8}.$

e) Resolve into partial fraction $\frac{x^2 - 2x + 3}{x^3 + x}.$

f) Resolve into partial fraction $\frac{2x^4 + x^2 + 4}{\left(x^2 + 1\right)\left(x^2 - 2\right)\left(2x^2 + 3\right)}.$

4. Attempt **any four** of the following : **16**

a) Prove that $\frac{\cot\theta - \cot 2\theta}{\cot\theta + \cot 2\theta} = \frac{\sin\theta}{\sin 3\theta}.$

b) If A and B are obtuse angles and $\sin A = \frac{5}{13}$, $\cos B = {}^{-4}\!/_{5}$ evaluate cos (A+B).

c) In any triangle ABC, prove that $\tan A + \tan B + \tan C = \tan A \tan B \tan C.$

d) Find the value of :
$\sin(-690°)\cos(-330°) + \cos(-750°)\sin(-240°).$

e) Prove that

$\frac{\cos 3A + 2\cos 5A + \cos 7A}{\cos A + 2\cos 3A + \cos 5A} = \cos 2A - \sin 2A \tan 3A.$

f) Prove that $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi.$

5. Attempt **any four** of the following : **16**

a) Prove that $\cos(A - B) = \cos A \cos B + \sin A \sin B.$

b) Prove that $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$

MARKS

c) Prove that $\frac{\sin x \ \sin 2x + \sin 3x \ \sin 6x}{\sin x \ \cos 2x + \sin 3x \ \cos 6x}$ = tan 5x.

d) Prove that sin 20° sin 40° sin 60° sin 80° = $\frac{3}{16}$.

e) Prove that $\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$.

f) Prove that $\tan^{-1}\left(\frac{1}{11}\right) + \cot^{-1}\left(\frac{6}{5}\right) = \sec^{-1}\sqrt{2}$.

6. Attempt **any four** of the following : **16**

a) If m_1 and m_2 are slopes of two lines then prove that acute angle between two lines is $\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$.

b) Show that perpendicular distance of a point (x_1, y_1) from the line $ax + by + c = 0$ is $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$.

c) Find the equation of line which passes through the point (–3, 10) and the sum of whose X and Y intercept is 8.

d) Find the equation of line passing through the point of intersection of the lines $3x + y - 10 = 0$ and $x + 7y + 40 = 0$ and perpendicular to the lines $3x = 4y$.

e) Find the distance between parallel lines $3x + 4y + 5 = 0$ and $6x + 8y = 25$.

f) For what value of ‘k’ the lines $x - ky = 14$ and $4x + (k - 3)y + 3 = 0$ are perpendicular to each other.
