Applied Mathematics

Prelim Question Paper Solution

Q.1 Attempt any FIVE of the following : **[10]**

Q.1(a) Define Even and odd functions. **[2]**

Ans.: A function f(x) is said to be even function if $f(-x) = f(x)$ [1 mark]

A function f(x) is said to be odd function if $f(-x) = -f(x)$ [1 mark]

Q.1(b) If $f(x) = x^3 - 3x^2 + 5$, find f(o) + f(3) **[2]**

Ans.: Given $f(x) = x^3 - 3x^2 + 5$

$\therefore$ $f(0) = (0)^3 - 3(0)^2 + 5 = 5$ [$\frac{1}{2}$ mark]

$f(3) = (3)^3 - 3(3)^2 + 5 = 5$ [$\frac{1}{2}$ mark]

$\therefore$ $f(0) + f(3) = 5 + 5 = 10$ [$\frac{1}{2}$ mark]

$\therefore$ $f(0) + f(3) = 10$ [$\frac{1}{2}$ mark]

Q.1(c) Find $\frac{dy}{dx}$ if $y = e^x \tan^{-1} x$ **[2]**

Ans.: $y = e^x \tan^{-1} x$

Diff wrt x

$\frac{dy}{dx} = e^x \frac{d}{dx}(\tan^{-1} x) + \tan^{-1} x \frac{d}{dx}(x)$ [use product rule] [$\frac{1}{2}$ mark]

$= e^x \left(\frac{1}{1+x^2}\right) + \tan^{-1} x\ (1)$ [1 mark]

$= \frac{e^x}{1+x^2} + \tan^{-1} x$ [$\frac{1}{2}$ mark]

Q.1(d) Evaluate $\int x \sin x\, dx$ **[2]**

Ans.: $\int x \sin x\, dx$

$= x \int \sin x\, dx - \int \sin x\, dx \cdot \frac{d}{dx}(x)\, dx$ [$\frac{1}{2}$ mark]

$= x(-\cos x) - \int (-\cos x) . 1\, dx$ [$\frac{1}{2}$ mark]

$= -x \cos x + \int \cos x\, dx$ [$\frac{1}{2}$ mark]

$= -x \cos x + \sin x + c$ [$\frac{1}{2}$ mark]

Q.1(e) Evaluate $\int \frac{1}{1+\cos 2x}\, dx$ **[2]**

Ans.: $\int \frac{1}{1+\cos 2x}\, dx$

$= \int \frac{1}{2\cos^2 x}\, dx$ [1 mark]

$= \frac{1}{2} \int \sec^2 x\, dx$ [$\frac{1}{2}$ mark]

$= \frac{1}{2} \tan x + c$ [$\frac{1}{2}$ mark]

Q.1 (f) Find the area bounded by the curve $y = x^3$, x axis and the ordinates x = 1, x = 3 **[2]**

Ans.: $A = \int_a^b y\, dx$

$= \int_1^3 x^3\, dx$ [$\frac{1}{2}$ mark]

$= \left[\frac{x^4}{4}\right]_1^3$ [$\frac{1}{2}$ mark]

$= \frac{3^4}{4} - \frac{1^4}{4} = \frac{81}{4} - \frac{1}{4}$ [½ mark]

$= \frac{80}{4} = 20$ sq. units [½ mark]

Q.1 (g) Find a real root of the equation $x^3 - 4x - 9 = 0$ is the interval (2, 3) by using Bisection Method (Use 2 iterations) **[2]**

Ans.:

Bisection Method					
A	**f(a)**	**B**	**f(b)**	**$C = \frac{a+b}{2}$**	**f(c)**
2.000	−9.000	3.000	6.000	2.500	−3.375
2.500	−3.375	3.000	6.000	2.750	

$f(x) = x^3 - 4x - 9$, a = 2, b = 3

$f(a) = f(2) = (2)^3 - 4(2) - 9 = -9 < 0$

$f(b) = f(3) = (3)^3 - 4(3) - 9 = 6 > 0$

$\therefore$ Root lies in (2, 3) [1 mark]

$x_0 = \frac{a+b}{2} = \frac{2+3}{2} = 2.5$

$f(x_0) = (2.5)^3 - 4(2.5) - 9 = -3.375 < 0$

Root lies in (2.5, 3) [½ mark]

$x_1 = \frac{2.5+3}{2} = 2.750$

$\therefore$ After two iterations, solution is x = 2.75 [½ mark]

Q.2 Attempt any THREE of the following : **[12]**

Q.2(a) Find $\frac{dy}{dx}$ if $x^2 + y^2 - xy - 2x + 5y - 6 = 0$ at (1, 2) **[4]**

Ans.: Find $\frac{dy}{dx}$ if $x^2 + y^2 - xy - 2x + 5y - 6 = 0$ at (1, 2)

Given $x^2 + y^2 - xy - 2x + 5y - 6 = 0$

Diff w.r.t. x

$\therefore\ 2x + 2y\frac{dy}{dx} - \left(x\frac{dy}{dx} + y(1)\right) - 2 + 5\frac{dy}{dx} - 0 = 0$ [1 mark]

$\therefore\ 2x + 2y\frac{dy}{dx} - x\frac{dy}{dx} - y - 2 + 5\frac{dy}{dx} = 0$ [1 mark]

$\therefore\ (2y - x + 5)\frac{dy}{dx} = y - 2x + 2$

$\therefore\ \frac{dy}{dx} = \frac{y - 2x + 2}{2y - x + 5}$ [1 mark]

At (1, 2)

$\left.\frac{dy}{dx}\right|_{(1,2)} = \frac{(2) - 2(1) + 2}{2(2) - (1) + 5}$ [½ mark]

$= \frac{2}{8}$

$\left.\frac{dy}{dx}\right|_{(1,2)} = \frac{1}{4}$ [½ mark]

Q.2(b) If $x = a\cos^3\theta$, $y = b\sin^3\theta$, find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$ **[4]**

Ans.: $x = a\cos^3\theta$
diff. w.r.t. θ

$$\frac{dx}{d\theta} = \frac{d}{d\theta}(a\cos^3\theta)$$
$$= a\frac{d}{d\theta}(\cos^3\theta)$$
$$= a\,3(\cos\theta)^2 \times (-\sin\theta) \quad \text{(Chain Rule)}$$
$$= -3a(\cos\theta)^2\sin\theta$$

$y = b\sin^3\theta$
diff. w.r.t. θ

$$\frac{dy}{d\theta} = \frac{d}{d\theta}(b\sin^3\theta)$$
$$= b\frac{d}{d\theta}(\sin\theta)^3$$
$$= b\,3(\sin\theta)^2 \times (\cos\theta) \quad \text{(Chain Rule)}$$
$$= 3b(\sin\theta)^2 \times \cos\theta$$

[1 + 1 mark]

By parametric function

$$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{3b(\sin\theta)^2\cos\theta}{-3a(\cos\theta)^2\sin\theta} = -\frac{-b\sin\theta}{a\cos\theta}$$ [1 mark]

$$\therefore \left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{4}} = \frac{-b\sin\frac{\pi}{4}}{a\cos\frac{\pi}{4}} = \frac{-b(1/\sqrt{2})}{a(1/\sqrt{2})} = \frac{-b}{a}$$ [1 mark]

Q.2(c) If I_1 and I_2 be the currents and R_1 and R_2 be two resistances in parallel to the total current $I = I_1 + I_2$ which is constant. Then the heat developed in a circuit is given by $H = \frac{1}{J}\left(I_1^2R_1t + I_2^2R_2t\right)$. Show that heat developed in a circuit is minimum if $I_1R_1 = I_2R_1$ where R_1, R_2, t, J are constants. **[4]**

Ans.: Given $H = \frac{1}{J}(I_1^2R_1t + I_2^2R_2t)$

$$H = \frac{1}{J}\left(I_1^2R_1t + (I - I_1)^2R_2t\right) \qquad (\because I = I_1 + I_2 \;\; \therefore I_2 = I - I_1)$$

Diff. w.r.t. I_1

$$\frac{dH}{dI_1} = \frac{1}{J}(2I_1R_1t + 2(I - I_1)(-1)R_2t)$$
$$= \frac{1}{J}(2I_1R_1t - 2IR_2t + 2I_1R_2t)$$ [1 mark]

Diff. w.r.t. I_1

$$\frac{d^2H}{dI_1^2} = \frac{1}{J}(2R_1t + 2R_2t)$$ [1 mark]

Put $\frac{dH}{dI_1} = 0$

$$\rightarrow \frac{1}{J}(2I_1R_1t - 2IR_2t + 2I_1R_2t) = 0$$
$$2I_1R_1t - 2IR_2t + 2I_1R_2t = 0$$
$$I_1R_1 + I_1R_2 = IR_2$$
$$I_1R_1 = IR_2 - I_1R_2$$
$$= (I - I_1)R_2$$
$$I_1R_1 = I_2R_2$$ [1 mark]

Put $I_1R_1 = I_2R_2$ in $\frac{d^2H}{dI_1^2}$: $\frac{1}{J}(2R_1t + 2R_2t) > 0$

$\therefore$ Heat is minimum if $I_1R_1 = I_2R_2$ [1 mark]

Q.2(d) A beam is bent in the form of the curve y = 2 sin x – sin 2x. find the radius of curvature at x = $\frac{\pi}{2}$ [4]

Ans.: $y = 2\sin x - \sin 2x$

$$\frac{dy}{dx} = 2\cos x - 2\cos 2x \qquad \text{[1 mark]}$$

$$\frac{d^2y}{dx^2} = -2\sin x - 4\sin 2x \qquad \text{[1 mark]}$$

Put $x = \frac{\pi}{2}$

$$\frac{dy}{dx} = 2\cos\left(\frac{\pi}{2}\right) - 2\cos 2\left(\frac{\pi}{2}\right) = 0 - 2(-1) = 2 \qquad [\tfrac{1}{2}\text{ mark}]$$

$$\frac{d^2y}{dx^2} = -2\sin\left(\frac{\pi}{2}\right) - 4\sin 2\left(\frac{\pi}{2}\right) = -2 \qquad [\tfrac{1}{2}\text{ mark}]$$

$$\text{Radius of curvature} = \rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left[1+(2)^2\right]^{3/2}}{-2}$$

$$= \left|\frac{5\sqrt{5}}{-2}\right| = \frac{5}{2}\sqrt{5} \text{ units} \qquad \text{[1 mark]}$$

Q.3 Attempt any THREE of the following : [12]

Q.3(a) Find the equation of tangent & normal to the curve $x^2 + 3xy + y^2 = 5$ at (1, 1) [4]

Ans.: $x^2 + 3xy + y^2 = 5$

$$2x + 3\left(x\frac{dy}{dx} + y\cdot 1\right) + 2y\frac{dy}{dx} = 0$$

$$2x + 3x\frac{dy}{dx} + 3y + 2y\frac{dy}{dx} = 0$$

$$3x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - 3y$$

$$(3x + 2y)\frac{dy}{dx} = -2x - 3y$$

$$\frac{dy}{dx} = \frac{-2x-3y}{3x+2y} \qquad \text{[1 mark]}$$

At x = 1 and y = 1

$$m = \text{slope} = \frac{dy}{dx} = \frac{-2(1)-3(1)}{3(1)+2(1)} = \frac{-5}{5} = -1 \qquad \text{[1 mark]}$$

Equation of tangent is

$y - y_1 = m(x - x_1)$

$y - 1 = -1\,(x - 1)$

$y - 1 = -x + 1$

$x + y - 2 = 0$ [1 mark]

Equation of normal is

$$y - y_1 = -\frac{1}{m}(x - x_1)$$

$$y - 1 = \frac{-1}{-1}(x - 1)$$

$y - 1 = x - 1$

$x = y$

$x - y = 0$ [1 mark]

Q.3(b) If $e^y = y^x$ prove that $\frac{dy}{dx} = \frac{(\log y)^2}{\log y - 1}$. **[4]**

Ans.: Given : $e^y = y^x$

Taking logarithm

$\therefore \quad y = x \log y$ … (1)

Diff. w.r.t. 'x'

$\therefore \quad \frac{dy}{dx} = x\left(\frac{1}{y}\frac{dy}{dx}\right) + \log y\ (1)$

$\therefore \quad \frac{dy}{dx} = \frac{x}{y}\ \frac{dy}{dx} + \log y$ [1 mark]

$\therefore \quad \frac{dy}{dx} - \frac{x}{y}\frac{dy}{dx} = \log y$

$\therefore \quad \left(1-\frac{x}{y}\right)\frac{dy}{dx} = \log y$

$\therefore \quad \left(\frac{y-x}{y}\right)\frac{dy}{dx} = \log y$

$\therefore \quad \frac{dy}{dx} = \log y \cdot \left(\frac{y}{y-x}\right)$ [1 mark]

From (1) $y = x \log y$

$\frac{dy}{dx} = \log y\left(\frac{x\log y}{x\log y - x}\right)$ [1 mark]

$\quad = \log y\left(\frac{x\log y}{x(\log y - 1)}\right)$

$\frac{dy}{dx} = \frac{(\log y)^2}{\log y - 1}$ [1 mark]

Q.3(c) Find $\frac{dy}{dx}$ if $y = x^{\sin x} + (\tan x)^x$ **[4]**

Ans.: Let $u = x^{\sin x}$, $v = (\tan x)^x$

$\therefore \quad y = u + v$ … (1)

Diff. w.r.t. 'x'

$\therefore \quad \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ … (2)

$u = x^{\sin x}$

Taking logarithm

$\log u = \sin x \cdot \log x$ [1 mark]

Diff. w.r.t. 'x'

$\therefore \frac{1}{u}\ \frac{du}{dx} = \sin x\left(\frac{1}{x}\right) + (\log x)(\cos x)$

$\therefore \quad \frac{du}{dx} = u\left(\frac{\sin x}{x} + \log x \cdot \cos x\right)$

$\quad \frac{du}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + \log x \cdot \cos x\right)$ … (3) [1 mark]

$v = (\tan x)^x$

taking logarithm

$\log v = x \log(\tan x)$

diff. w.r.t. x

$\therefore \quad \frac{1}{V}\frac{dV}{dx} = x \cdot \frac{1}{\tan x} \cdot \sec^2 x + \log(\tan x)\ \cdot(1)$ [1 mark]

$\therefore \ \dfrac{dv}{dx} = v\left(\dfrac{x\sec^2 x}{\tan x} + \log(\tan x)\right)$

$\therefore \ \dfrac{dv}{dx} = (\tan x)^x \left(\dfrac{x\sec^2 x}{\tan x} + \log(\tan x)\right)$ ...(4)

From (2), (3), (4)

$\dfrac{dy}{dx} = x^{\sin x}\left(\dfrac{\sin x}{x} + (\log x)(\cos x)\right) + (\tan x)^x \left(\dfrac{x\sec^2 x}{\tan x} + \log(\tan x)\right)$ [1 mark]

Q.3(d) Evaluate : $\int \dfrac{(\tan^{-1}x)}{1+x^2}\, dx$ **[4]**

***Ans.*:** $I = \int \dfrac{(\tan^{-1}x)^3}{1+x^2}dx$

Put $t = \tan^{-1} x$

$\dfrac{dt}{dx} = \dfrac{1}{1+x^2}$

$dt = \dfrac{1}{1+x^2}dx$ [1 mark]

$I = \int t^3\, dt$ [1 mark]

$= \dfrac{t^4}{4} + c$ [1 mark]

$= \dfrac{(\tan^{-1}x)^4}{4} + c$ [1 mark]

Q.4 Attempt any THREE of the following : **[12]**

Q.4(a) Evaluate : $\int \dfrac{e^x(x+1)}{\cos^2(x\,e^x)}\, dx$ **[4]**

***Ans.*:** $I = \int \dfrac{e^x(x+1)}{\cos^2(x e^x)}dx$

Put $t = xe^x$ [½ mark]

$\dfrac{dt}{dx} = xe^x + e^x$ [½ mark]

$dt = (xe^x + e^x)\, dx$ [½ mark]

$dt = e^x(x+1)\, dx$ [½ mark]

$I = \int \dfrac{1}{\cos^2 t}dt$ [½ mark]

$= \int \sec^2 t\, dt$ [½ mark]

$= \tan t + c$ [½ mark]

$= \tan(xe^x) + c$ [½ mark]

Q.4(b) Evaluate : $\int \dfrac{dx}{5+4\cos x}$ **[4]**

***Ans.*:** Put $\tan\dfrac{x}{2} = t$ $\quad\therefore\ dx = \dfrac{2dt}{1+t^2}$ $\quad\therefore\ \cos x = \dfrac{1-t^2}{1+t^2}$ [1 mark]

$I = \int \dfrac{\frac{2\,dt}{1+t^2}}{5+4\left(\frac{1-t^2}{1+t^2}\right)}$

$= \int \dfrac{\frac{2\,dt}{1+t^2}}{\frac{5+5t^2+4-4t^2}{1+t^2}}$ [1 mark]

$= 2 \int \frac{dt}{t^2 + 9}$

$= 2 \int \frac{dt}{t^2 + 3^2}$ [1 mark]

$= 2 \left(\frac{1}{3}\right) \tan^{-1}\left(\frac{t}{3}\right) + c$ [½ mark]

$= \frac{2}{3} \tan^{-1}\left(\frac{\tan\frac{x}{2}}{3}\right) + c$ [½ mark]

Q.4(c) Evaluate : $\int x \cdot \tan^{-1}x \; dx$ **[4]**

Ans.: Let $I = \int x \cdot \tan^{-1} x \, dx$

Integrating by Parts

$I \rightarrow \tan^{-1}x \rightarrow (u)$
$A \rightarrow x \rightarrow (v)$

$I = \tan^{-1} x \int x \, dx - \int \left[\frac{d}{dx}(\tan^{-1} x) \int x dx\right] dx$

$= \tan^{-1} \left(\frac{x}{2}\right)^2 - \int \frac{1}{1+x^2} \cdot \frac{x^2}{2} dx$ [½ mark]

$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{x^2}{x^2+1} dx$ [½ mark]

$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{x^2+1-1}{x^2+1} dx$ [½ mark]

$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \left(1 - \frac{1}{1+x^2}\right) dx$ [½ mark]

$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} (x - \tan^{-1} x) + c$ [½ mark]

$= \frac{1}{2} (x^2 \tan^{-1}x - x + \tan^{-1} x) + c$ [½ mark]

$I = \frac{1}{2} (x^2 + 1) \tan^{-1} x - \frac{1}{2} x + c$ [1 mark]

Q.4(d) $\int \frac{\cos x}{(2 + \sin x)(3 + \sin x)} dx$ **[4]**

Ans.: Put t = sin x

$\frac{dt}{dx} = \cos x$ [1 mark]

$dt = \cos x \, dx$

$\int \frac{1}{(2+t)(3+t)} dt$

$\frac{1}{(2+t)(3+t)} = \frac{A}{2+t} + \frac{B}{3+t}$ [1 mark]

$1 = A(3 + t) + B(2 + t)$

Put $t = -2$

$1 = A(1)$

$1 = A$

Put $t = -3$

$1 = B(-1)$

$-1 = B$

$\therefore \int \left(\frac{1}{2+t} + \frac{-1}{3+t}\right) dt$ [1 mark]

$= \log(2 + t) - \log(3 + t) + c$ [½ mark]

$= \log(2 + \sin x) - \log(3 + \sin x) + c$ [½ mark]

Q.4(e) Evaluate: $\displaystyle\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt[n]{\cot x}}$ **[4]**

Ans.: Let $I = \displaystyle\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt[n]{\cot x}}$

$$= \int_{\pi/6}^{\pi/3} \frac{1}{1+\frac{\sqrt[n]{\cos x}}{\sqrt[n]{\sin x}}}\, dx$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt[n]{\sin x}}{\sqrt[n]{\sin x}+\sqrt[n]{\cos x}} \quad \dots (1)$$ [1 mark]

Using property $\displaystyle\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt[n]{\sin(\pi/2-x)}}{\sqrt[n]{\sin(\pi/2-x)}+\sqrt[n]{\cos(\pi/2-x)}}\, dx$$

$$\therefore\ I = \int_{\pi/6}^{\pi/3} \frac{\sqrt[n]{\cos x}}{\sqrt[n]{\cos x}+\sqrt[n]{\sin x}}\, dx \quad \dots (2)$$

Adding (1) and (2)

$$2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt[n]{\sin x}+\sqrt[n]{\cos x}}{\sqrt[n]{\cos x}+\sqrt[n]{\sin x}}\, dx$$ [1 mark]

$$\therefore\ 2I = \int_{\pi/6}^{\pi/3} 1\, dx$$

$$\therefore\ 2I = \left[x\right]_{\pi/6}^{\pi/3} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$$ [1 mark]

$$\therefore\ I = \frac{\pi}{12}$$ [1 mark]

Q.5 Attempt any TWO of the following : [12]

Q.5(a) Find the area between the parabola $y^2 = 4x$ and $x^2 = 4y$ [6]

Ans.: $x^2 = 4y \quad \therefore\ y = \dfrac{x^2}{4}$

Put $y = \dfrac{x^2}{4}$ in $y^2 = 4x$

$\rightarrow \left(\dfrac{x^2}{4}\right)^2 = 4x$

$\rightarrow x^4 = 64\,x$ [1 mark]

$x\,(x^3 - 64) = 0 \rightarrow x = 0$ or $x = 4$

$\therefore\ y_1 = \dfrac{x^2}{2}$ [½ mark]

$y^2 = 4x$

$y = 2\sqrt{x}$ [½ mark]

$\therefore\ y_2 = 2\sqrt{x}$

$\therefore\ A = \displaystyle\int_a^b (y_1 - y_2)\, dx$ [½ mark]

$= \displaystyle\int_0^4 \left(\frac{x^2}{4} - 2\sqrt{x}\right) dx$ [½ mark]

$= \left[\dfrac{1}{4}\dfrac{x^3}{2} - 2\dfrac{x^{3/2}}{3/2}\right]_0^4$ [1 mark]

$= \left[\frac{1}{12}x^3 - \frac{4}{3}x\sqrt{x}\right]_0^4$ [½ mark]

$= \left[\frac{1}{12}4^2 - \frac{4}{3}4\sqrt{4}\right] - 0$ [½ mark]

$= \frac{16}{3}$ sq. units. [1 mark]

Q.5(b) (i) Form a Differential Equation for y = A sin mx + B cos mx **[3]**

Ans.: y = A sin m x + B cos m x

diff. w.r.t. 'x'

$\frac{dy}{dx} = A \cdot \cos mx \cdot m + B(-\sin mx) \cdot m$ [1 mark]

Diff w.r.t. 'x'

$\frac{d^2y}{dx^2} = A(-\sin mx)m^2 + B(-\cos mx)\, m^2$ [½ mark]

$= -m^2 (A \sin mx + B \text{ com } x)$ [½ mark]

$\frac{d^2y}{dx^2} = -m^2y$ [½ mark]

$\frac{d^2y}{dx^2} + m^2y = 0$ [½ mark]

Q.5(b) (ii) Solve $\frac{dy}{dx} + \frac{y}{x} = \sin x$ **[3]**

Ans.: Comparing with

$\frac{dy}{dx} + py = Q$

$p = \frac{1}{x}$, Q = sin x

Both are functions of only x.

∴ Given D.E. is Linear D.E.

I.F. = $e^{\int p dx}$

= $e^{\int \frac{1}{x} dx}$

= $e^{\log x}$

= x [1 mark]

Solution is

Y (I.F.) = ∫ Q (IF) dx + c [½ mark]

∴ y (x) = ∫ x. sinx dx + c

∴ xy = x (– cos x) – ∫ (1) (–cos x) dx + c [½ mark]

∴ xy = –x cos x + sin x + c [½ mark]

Q.5(c) The quantity of charge of coulombs passes through a conducting wire during small interval of time t sec is given by $\frac{dq}{dt}$ = i, where i is current in ampere. If i = 10 sin 100t and that q = 0, t = 0. Find the charge at time t **[6]**

Ans.: $\frac{dq}{dt} = i$

$\frac{dq}{dt} = 10 \sin 100\, t$ [1 mark]

dq = 10 sin 100t dt

Integrating

∫dq = 10 ∫ sin 100 t dt [1 mark]

$q = 10\, \frac{(-\cos 100t)}{100} + c$ [1 mark]

$q = -\frac{\cos 100t}{10} + c$

when q = 0, t = 0

$0 = -\frac{\cos 0}{10} + c \rightarrow 0 = -\frac{1}{10} + c \rightarrow 0 = -\frac{1}{10} + c$ [1 mark]

$c = \frac{1}{10}$

$\therefore\ q = -\frac{\cos 100\,t}{10} + \frac{1}{10}$ [1 mark]

$q = \frac{1}{10}(1 - \cos 100\ t)$ coulombs [1 mark]

Q.6 Attempt any TWO of the following : [12]

Q.6(a) (i) Solve the following system of equations by Jacobi–iteration Method (2 iterations) 10x + y + 2z = 13, 3x + 10y + z = 14, 2x + 3y + 10z = 15 [3]

Ans.: Rearranging equation

x = (13 – y – 2z)/10 … (1)

y = (14 – 3x – z)/10 … (2)

z = (15 – 2x – 3y)/10 … (3) [1 mark]

Initially taking $x_0 = y_0 = z_0 = 0$

$x_1 = \frac{13}{10} = 1.300$

$y_1 = \frac{14}{10} = 1.400$

$z_1 = \frac{15}{10} = 1.500$ [1 mark]

$x_2 = (13 - 1.4 - 2\ (1.5))/10 = 0.860$

$y_2 = (14 - 3\ (1.3) - (1.5))/10 = 0.860$

$z_2 = (15 - 2(1.3) - 3\ (1.4))/10 = 0.820$ [1 mark]

$\therefore$ After two iterations the solution is

x = 0.860

y = 0.860

z = 0.820

Q.6(a) (ii) Solve the following system of equations by Gauss Seidal Method 2x + y – z = 3, 3x + y + 2z = 13; x + y – z = 1 [3]

Ans.: Rewriting equation,

x = (3 – y + x)/2

y = (1 – x + z)

z = (13 – 3x – y)/2

Initially $x_0 = y_0 = z_0 = 0$

$x_1 = (3 - 0 + 0)/2 = 1.500$ [1 mark]

$y_1 = (1 - 1.5 + 0) = -0.5$

$z_1 = (13 - 3\ (1.5) - (-0.5))/2 = 4.5000$

$x_2 = (3 - (-0.5) + (4.5))/2 = 4$

$y_2 = (1 - 4 + 4.5) = 1.500$ [1 mark]

$z_2 = (13 - 3(4) - 1.5)/2 = -0.25$

$\therefore$ after two iteration the solution is

X = 4, y = 1.5, z = –0.25 [1 mark]

Q.6(b) Solve the following system of equations by Gauss Elimination Method 4x – y – 3z = 1, 3x – 2y + 4z = 7; x + 2y + z = 2 [6]

Ans.: 4x – y – 3z = 1 …(1)

3x – 2y + 4z = 7 …(2)

x + 2y + z = 2 …(3)

2x eq. (1) – eq. (2)

$$\begin{array}{rl} & 8x - 2y - 6z = 2 \\ - & 3x - 2y + 4z = 7 \\ & (-)\ (+)\ (-)\ (-) \\ \hline & 5x - 10z = -5 \end{array}$$

$\therefore$ $x - 2z = -1$ ...(4) [1½ marks]

Adding eq. (2) and (3)

$$\begin{array}{rl} & 3x - 2y + 4z = 7 \\ + & x + 2y + z = 2 \\ \hline & 4x + 5z = 9 \end{array}$$

...(5) [1½ marks]

4x eq. (4) – eq (5)

$$\begin{array}{rl} & 4x - 8z = -4 \\ - & 4x + 5z = 9 \\ & (-)\ (+)\ (-) \\ \hline & -13z = -13 \end{array}$$

$\therefore$ $z = 1$, putting in eq. (5) [1 mark]

$\therefore$ $4x + 5 = 9$

$\therefore$ $x = 1$ putting in eq. (3) [1 mark]

$1 + 2y + 1 = 2$

$\therefore$ $y = 0$

$\therefore$ Solution is $x = 1, y = 0, z = 1$ [1 mark]

Q.6(c) Using Newton–Raphson method find approximate value of $\sqrt[3]{100}$ (perform three iterations) [6]

Ans.: $\therefore$ $f(x) = x^3 - 100$

$\therefore$ $f'(x) = 3x^2$

$\therefore$ $f(4) = -36$ [1 mark]

$f(5) = 25$

$\therefore$ the root is in (4, 5)

$\therefore$ start with $x_0 = 5$ [½ mark]

$$\therefore\ x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 5 - \frac{f(5)}{f'(5)} = 5 - \frac{25}{75} = 4.667$$

[1½ marks]

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 4.667 - \frac{f(4.667)}{f'(4.667)} = 4.667 - \frac{1.651}{65.343} = 4.642$$

[1½ marks]

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 4.642 - \frac{f(4.642)}{f'(4.642)} = 4.642 - \frac{0.027}{64.644} = 4.642$$

[1½ marks]

Solution is $\sqrt[3]{100} = 4.642$

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