SUMMER – 2016 EXAMINATION

MODEL ANSWER

Subject: ENGINEERING MATHEMATICS (EMS)

Subject Code: 17216

Important Instructions to examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.

2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.

3) The language errors such as grammatical, spelling errors should not be given more importance. (Not applicable for subject English and Communication Skills.)

4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by the candidate and those in the model answer may vary. The examiner may give credit for any equivalent figure drawn.

5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and the model answer.

6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidate's understanding.

7) For programming language papers, credit may be given to any other program based on equivalent concept.

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1.		**Attempt any <u>TEN</u> of the following:**		20
	a)	If $3a-7+2bi=5i+ia-5b$, find a,b		
	Ans	$(3a-7+5b)+(2b-5-a)i=0$	½	
		$\therefore\ 3a-7+5b=0$		
		$2b-5-a=0$		
		$3a+5b=7$	½	
		$-a+2b=5$		
		$\therefore\ 3a+5b=7$		
		$-3a+6b=15$		

		$11b=22$		
		$b=2$	½	
		$a=-1$	½	02
		--		
	b)	If $z=1+i\sqrt{3}$ show that $z^2+4=2z$		
	Ans	z^2+4		
		$=\left(1+i\sqrt{3}\right)^2+4$	½	
		$=1+2i\sqrt{3}+3i^2+4$	½	
		$=1+2i\sqrt{3}-3+4$	½	
		$=2+2i\sqrt{3}$		
		$=2\left(1+i\sqrt{3}\right)$		02
		$=2z$	½	
		OR		
		$L.H.S.=z^2+4$		
		$=\left(1+i\sqrt{3}\right)^2+4$	½	
		$=1+2i\sqrt{3}+3i^2+4$	½	
		$=1+2i\sqrt{3}-3+4$		
		$=2+2i\sqrt{3}$	½	
		$R.H.S.=2z$		
		$=2\left(1+i\sqrt{3}\right)$		
		$=2+2i\sqrt{3}=L.H.S.$	½	02

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1.	c)	Define even and odd function.		
	Ans	Even function:-		
		If $f(-x)=f(x)$ then function is even function.	1	
		Odd function:-		
		If $f(-x)=-f(x)$ then function is odd function	1	02
		--		
	d)	If $f(x)=\sin x$ show that $f(3x)=3f(x)-4f^{3}(x)$		
	Ans	$L.H.S.=f(3x)$		
		$=\sin 3x$	½	
		$=3\sin x-4\sin^{3}x$	1	
		$=3f(x)-4f^{3}(x)$	½	02
		$=R.H.S.$		
		OR		
		$R.H.S.=3f(x)-4f^{3}(x)$	½	
		$=3\sin x-4\sin^{3}x$	1	02
		$=\sin 3x$		
		$=f(3x)$	½	
		$=L.H.S.$		
		--		
	e)	Evaluate: $\lim_{x\to 1}\left(\frac{1}{x-1}-\frac{1}{x^{2}-x}\right)$		
	Ans	$\lim_{x\to 1}\left(\frac{1}{x-1}-\frac{1}{x^{2}-x}\right)$		
		$=\lim_{x\to 1}\left(\frac{1}{x-1}-\frac{1}{x(x-1)}\right)$	½	
		$=\lim_{x\to 1}\left(\frac{x-1}{x(x-1)}\right)$	½	
		$=\lim_{x\to 1}\frac{1}{x}$	½	
		$=1$	½	02
		OR $\lim_{x\to 1}\left(\frac{1}{x-1}-\frac{1}{x^{2}-x}\right)$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1.		$= \lim_{x\to 1}\left(\frac{1}{x-1}-\frac{1}{x(x-1)}\right)$	½	
		$= \lim_{x\to 1}\frac{1}{x-1}\left(1-\frac{1}{x}\right)$		
		$= \lim_{x\to 1}\frac{1}{x-1}\left(\frac{x-1}{x}\right)$	½	
		$= \lim_{x\to 1}\frac{1}{x}$	½	02
		$= 1$	½	
		---		
	f)	Evaluate $\lim_{x\to 0}\frac{\cos 5x-\cos 3x}{x^2}$		
	Ans	$\lim_{x\to 0}\frac{\cos 5x-\cos 3x}{x^2}$		
		$= \lim_{x\to 0}\frac{-2\sin\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)}{x^2}$	½	
		$= \lim_{x\to 0}\frac{-2\sin 4x\sin x}{x^2}$		
		$= \lim_{x\to 0}\frac{-2\sin 4x}{x}\frac{\sin x}{x}$	½	
		$= -2\left(\lim_{x\to 0}\frac{\sin 4x}{4x}\right)4\left(\lim_{x\to 0}\frac{\sin x}{x}\right)$		
		$= -2(1)4(1)$	½	
		$= -8$	½	02
		---		
	g)	Evaluate $\lim_{x\to 0}\left(\frac{1+x}{1-x}\right)^{\frac{1}{x}}$		
	Ans	$\lim_{x\to 0}\left(\frac{1+x}{1-x}\right)^{\frac{1}{x}}$		
		$= \lim_{x\to 0}\frac{(1+x)^{\frac{1}{x}}}{(1-x)^{\frac{1}{x}}}$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1.		$= \frac{\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}}}{\left[\lim\limits_{x\to 0}(1-x)^{-\frac{1}{x}}\right]^{-1}}$	½	
		$= \frac{e}{e^{-1}}$	½	
		$= e^{2}$	½	02
		---		
	h)	Find $\frac{dy}{dx}$ if $y = \frac{\sin x}{1-\cos x}$		
	Ans	$y = \frac{\sin x}{1-\cos x}$		
		$\frac{dy}{dx} = \frac{(1-\cos x)\cos x - \sin x(0+\sin x)}{(1-\cos x)^{2}}$	1½	
		$\frac{dy}{dx} = \frac{\cos x - \cos^{2} x - \sin^{2} x}{(1-\cos x)^{2}}$		
		$\frac{dy}{dx} = \frac{\cos x - (\cos^{2} x + \sin^{2} x)}{(1-\cos x)^{2}}$		
		$\frac{dy}{dx} = \frac{\cos x - 1}{(1-\cos x)^{2}}$		
		$\frac{dy}{dx} = \frac{-(1-\cos x)}{(1-\cos x)^{2}}$		
		$\frac{dy}{dx} = \frac{-1}{1-\cos x}$	½	02
		---		
	i)	If $y = \log(\sec x + \tan x)$ find $\frac{dy}{dx}$		
	Ans	$y = \log(\sec x + \tan x)$		
		$\frac{dy}{dx} = \frac{1}{(\sec x + \tan x)}(\sec x \tan x + \sec^{2} x)$	1½	
		$\frac{dy}{dx} = \frac{\sec x(\sec x + \tan x)}{\sec x + \tan x}$		02
		$\frac{dy}{dx} = \sec x$	½	

1. j) If $\tan^{-1}\left(x^2+y^2\right)=a^2$ find $\frac{dy}{dx}$

Ans $\tan^{-1}\left(x^2+y^2\right)=a^2$

$x^2+y^2=\tan\left(a^2\right)$

$2x+2y\frac{dy}{dx}=0$ 1½

$\frac{dy}{dx}=\frac{-x}{y}$ ½ 02

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k) Using Bisection method find the root of $x^3-x-1=0$ (two iterations only)

Ans $Let\ f\left(x\right)=x^3-x-1=0$

$f\left(1\right)=-1$ ½

$f\left(2\right)=5$ ½

$\therefore$ the root is in $\left(1,2\right)$

$x_1=\frac{a+b}{2}=\frac{1+2}{2}=1.5$ ½

$f\left(1.5\right)=0.875>0$

$\therefore$ the root is in (1,1.5) 02

$x_2=\frac{x_1+b}{2}=\frac{1+1.5}{2}=1.25$ ½

OR

$Let\ f\left(x\right)=x^3-x-1$

$f\left(1\right)=-1$ ½

$f\left(2\right)=5$ ½

$\therefore$ the root is in $\left(1,2\right)$

Iteration	a	b	$x=\frac{a+b}{2}$	$f\left(x\right)$
I	1	2	1.5	0.875
II	1	1.5	1.25	---

½ ½ 02

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l) Find by Jacobis method , the first iteration only, for the following equation $5x-y=9$, $x-5y+z=-4$, $y-5z=6$

Ans $x=\frac{9+y}{5}$

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1.		$y=\frac{-4-x-z}{-5}$ $z=\frac{6-y}{-5}$	1	
		Initial approximations : $x_0=y_0=z_0=0$ $x=\frac{9}{5}=1.8$ $y=\frac{4}{5}=0.8$ $z=\frac{6}{-5}=-1.2$	1	02
2.		**Attempt any <u>FOUR</u> of the following:**		16
	a)	Find the complex conjugate of $\frac{(2+i)^2}{2+3i}$		
	Ans	$\frac{(2+i)^2}{2+3i}$		
		$=\frac{4+4i+i^2}{2+3i}$	½	
		$=\frac{4+4i-1}{2+3i}=\frac{3+4i}{2+3i}$	½	
		$=\frac{3+4i}{2+3i}\times\frac{2-3i}{2-3i}$	½	
		$=\frac{6-9i+8i-12i^2}{4-9i^2}$	½	
		$=\frac{6-9i+8i+12}{4+9}$ $=\frac{18-i}{13}$	1	
		$=\frac{18}{13}-\frac{1}{13}i$	½	
		Conjugate is $\frac{18}{13}+\frac{1}{13}i$	½	04
	b)	Simplify using De-Moiver's theorem $\frac{(\cos 3\theta+i\sin 3\theta)^4(\cos 5\theta-i\sin 5\theta)^{\frac{4}{5}}}{\left(\cos\frac{9}{2}\theta+i\sin\frac{9}{2}\theta\right)^{\frac{2}{3}}\left(\cos\frac{4}{5}\theta-i\sin\frac{4}{5}\theta\right)^{10}}$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
2.	Ans	$\frac{(\cos 3\theta+i\sin 3\theta)^{4}(\cos 5\theta-i\sin 5\theta)^{\frac{4}{5}}}{\left(\cos\frac{9}{2}\theta+i\sin\frac{9}{2}\theta\right)^{\frac{2}{3}}\left(\cos\frac{4}{5}\theta-i\sin\frac{4}{5}\theta\right)^{10}}$		
		$=\frac{(\cos\theta+i\sin\theta)^{12}(\cos\theta+i\sin\theta)^{-4}}{(\cos\theta+i\sin\theta)^{3}(\cos\theta+i\sin\theta)^{-8}}$	2	
		$=(\cos\theta+i\sin\theta)^{12-4-3+8}$	1	
		$=(\cos\theta+i\sin\theta)^{13}$	½	04
		$=\cos 13\theta+\sin 13\theta$	½	
		--		
	c)	Using Euler's exponential formula prove that		
		$i)\ \sin^2\theta+\cos^2\theta=1$		
		$ii)\ \cosh^2\theta-\sinh^2\theta=1$		
	Ans	$i)\sin^2\theta+\cos^2\theta$		
		$=\left(\frac{e^{i\theta}-e^{-i\theta}}{2i}\right)^2+\left(\frac{e^{i\theta}+e^{-i\theta}}{2}\right)^2$	½	
		$=\frac{1}{4i^2}\left(e^{i\theta}-e^{-i\theta}\right)^2+\frac{1}{4}\left(e^{i\theta}+e^{-i\theta}\right)^2$		
		$=\frac{-1}{4}\left(e^{2i\theta}-2e^{i\theta}e^{-i\theta}+e^{-2i\theta}\right)+\frac{1}{4}\left(e^{2i\theta}+2e^{i\theta}e^{-i\theta}+e^{-2i\theta}\right)$	½	
		$=\frac{1}{4}\left(4e^{i\theta}e^{-i\theta}\right)=\frac{1}{4}\left(4e^{0}\right)$	½	
		$=1$	½	
		$ii)\cosh^2\theta-\sinh^2\theta$		
		$=\left(\frac{e^{\theta}+e^{-\theta}}{2}\right)^2-\left(\frac{e^{\theta}-e^{-\theta}}{2}\right)^2$	½	
		$=\frac{1}{4}\left(e^{\theta}+e^{-\theta}\right)^2-\frac{1}{4}\left(e^{\theta}-e^{-\theta}\right)^2$		
		$=\frac{1}{4}\left(e^{2\theta}+2e^{\theta}e^{-\theta}+e^{-2\theta}\right)-\frac{1}{4}\left(e^{2\theta}-2e^{\theta}e^{-\theta}+e^{-2\theta}\right)$	½	
		$=\frac{1}{4}\left(4e^{\theta}e^{-\theta}\right)=\frac{1}{4}\left(4e^{0}\right)$	½	04
		$=1$	½	
		--		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2.	d)	Use De-Moiver's theorem to solve the equation $x^3-1=0$		
	Ans	$x^3-1=0$		
		$\therefore\ x^3=1$		
		Put $x^3=z$		
		$\therefore\ x=z^{\frac{1}{3}}$		
		$\therefore\ z=1+0i$		
		$\operatorname{Re}(z)=1, \operatorname{Im}(z)=0$		
		$r=\lvert z\rvert=\sqrt{1+0}=1$	½	
		$\theta=\tan^{-1}\left(\frac{0}{1}\right)=0$	½	
		$z=r(\cos\theta+i\sin\theta)$	½	
		$z=1(\cos 0+i\sin 0)$		
		In general polar form , $z=r\left(\cos(2\pi k+\theta)+i\sin(2\pi k+\theta)\right)$		
		$z=1(\cos 2\pi k+i\sin 2\pi k)$	½	
		$z^{\frac{1}{3}}=(\cos 2\pi k+i\sin 2\pi k)^{\frac{1}{3}}$		
		$z=\cos\left(\frac{2\pi k}{3}\right)+i\sin\left(\frac{2\pi k}{3}\right)\quad ;\quad k=0,1,2$	½	
		when $k=0$		
		$z_1=\cos 0+i\sin 0=1$	½	
		when $k=1$		
		$z_2=\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)=-\frac{1}{2}+i\frac{\sqrt{3}}{2}$	½	
		when $k=2$		
		$z_3=\cos\left(\frac{4\pi}{3}\right)+i\sin\left(\frac{4\pi}{3}\right)=-\frac{1}{2}-i\frac{\sqrt{3}}{2}$	½	04
		--		
	e)	If $f(x)=\frac{x+3}{4x-5}$ and $t=\frac{3+5x}{4x-1}$ then show that $f(t)=x$		
	Ans	$f(t)=\frac{t+3}{4t-5}$	½	
		$=\frac{\frac{3+5x}{4x-1}+3}{4\left(\frac{3+5x}{4x-1}\right)-5}$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2.		$= \dfrac{\dfrac{3+5x+3(4x-1)}{4x-1}}{\dfrac{4(3+5x)-5(4x-1)}{4x-1}}$	1	
		$= \dfrac{3+5x+12x-3}{12+20x-20x+5}$	1	
		$= \dfrac{17x}{17}$	½	
		$= x$	½	04
		--		
	f)	If $f(t) = 50\sin(50\pi t + 0.04)$, show that $f\left(\dfrac{2}{100}+t\right) = f(t)$		
	Ans	$f\left(\dfrac{2}{100}+t\right) = 50\sin\left(50\pi\left(\dfrac{2}{100}+t\right)+0.04\right)$	1	
		$= 50\sin(\pi + 50\pi t + 0.04)$	1	
		$= -50\sin(50\pi t + 0.04)$	1	
		$= -f(t)$	1	04
		--		
3.		**Attempt any <u>FOUR</u> of the following:**		16
	a)	If $f(x) = x^2 + 3$ then find the value of x for which $f(x) = f(2x+1)$		
	Ans	$f(2x+1)$		
		$= (2x+1)^2 + 3$		
		$= 4x^2 + 4x + 1 + 3$	1	
		$= 4x^2 + 4x + 4$	1	
		Given $f(x) = f(2x+1)$		
		$\therefore\ x^2 + 3 = 4x^2 + 4x + 4$	½	
		$\therefore\ -3x^2 - 4x - 1 = 0$	½	
		$\therefore\ 3x^2 + 4x + 1 = 0$		
		$(x+1)(3x+1) = 0$		
		$\therefore\ x = -1, \dfrac{-1}{3}$	½,½	
		--		04

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3.	b)	If $f(x)=16^x+\log_2 x$ then find the value of $f\left(\frac{1}{4}\right)^2,\ f\left(\frac{1}{2}\right)$		
	Ans	$f(x)=16^x+\log_2 x$		
		$\therefore\ f\left(\frac{1}{4}\right)^2=f\left(\frac{1}{16}\right)$		
		$=(16)^{\frac{1}{16}}+\log_2\left(\frac{1}{16}\right)$	1	
		$=-2.811$	1	
		$f\left(\frac{1}{2}\right)=(16)^{\frac{1}{2}}+\log_2\left(\frac{1}{2}\right)$	1	04
		$=3$	1	
		OR		
		$f(x)=16^x+\log_2 x$		
		$\therefore\ f\left(\frac{1}{4}\right)=(16)^{\frac{1}{4}}+\log_2\left(\frac{1}{4}\right)$	1	
		$=0$	½	
		$\therefore\left[f\left(\frac{1}{4}\right)\right]^2=0$	½	
		$f\left(\frac{1}{2}\right)=(16)^{\frac{1}{2}}+\log_2\left(\frac{1}{2}\right)$	1	
		$=3$	1	04
		--		
	c)	Evaluate: $\lim\limits_{x\to 2}\dfrac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}$		
	Ans	$=\lim\limits_{x\to 2}\dfrac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}\times\dfrac{\sqrt{x+2}+\sqrt{3x-2}}{\sqrt{x+2}+\sqrt{3x-2}}$	½	
		$=\lim\limits_{x\to 2}\dfrac{\left(x^2-4\right)\left(\sqrt{x+2}+\sqrt{3x-2}\right)}{x+2-(3x-2)}$	½	
			1	
		$=\lim\limits_{x\to 2}\dfrac{\left(x^2-4\right)\left(\sqrt{x+2}+\sqrt{3x-2}\right)}{-2x+4}$		
		$=\lim\limits_{x\to 2}\dfrac{(x-2)(x+2)\left(\sqrt{x+2}+\sqrt{3x-2}\right)}{-2(x-2)}$		
		$=\lim\limits_{x\to 2}\dfrac{(x+2)\left(\sqrt{x+2}+\sqrt{3x-2}\right)}{-2}$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3.		$= \dfrac{(2+2)\left(\sqrt{2+2}+\sqrt{6-2}\right)}{-2}$	½	
		$= -8$	½	04
		---		
	d) Ans	Evaluate: $\lim\limits_{x \to 3} \dfrac{\log x - \log 3}{x-3}$ Put $x = 3 + h$ as $x \to 3, h \to 0$ $= \lim\limits_{h \to 0} \dfrac{\log(3+h) - \log 3}{3+h-3}$	1	
		$= \lim\limits_{h \to 0} \dfrac{\log\left(\dfrac{3+h}{3}\right)}{h}$ $= \lim\limits_{h \to 0} \dfrac{1}{h}\log\left(1+\dfrac{h}{3}\right)$	1	
		$= \lim\limits_{h \to 0} \log\left(1+\dfrac{h}{3}\right)^{\frac{1}{h}}$	½	
		$= \log\left[\lim\limits_{h \to 0}\left(1+\dfrac{h}{3}\right)^{\frac{3}{h}}\right]^{\frac{1}{3}}$	½	
		$= \log e^{\frac{1}{3}}$	½	
		$= \dfrac{1}{3}\log e$ $= \dfrac{1}{3}$	½	04
		---		
	e) Ans	Evaluate: $\lim\limits_{x \to 0} \dfrac{4^x + 4^{-x} - 2}{x \sin x}$ $= \lim\limits_{x \to 0} \dfrac{4^x + \dfrac{1}{4^x} - 2}{x \sin x}$	½	
		$= \lim\limits_{x \to 0} \dfrac{\dfrac{\left(4^x\right)^2 + 1 - 2.4^x}{4^x}}{x \sin x}$	½	
		$= \lim\limits_{x \to 0} \dfrac{\left(4^x - 1\right)^2}{x^2} \times \dfrac{x}{\sin x} \times \dfrac{1}{4^x}$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3.		$= \left(\lim_{x\to 0}\frac{4^x-1}{x}\right)^2\left(\lim_{x\to 0}\frac{x}{\sin x}\right)\left(\frac{1}{4^0}\right)$	½	
		$= (\log 4)^2 (1)$	1	04
		$= (\log 4)^2$	½	
		--		
	f)	Evaluate: $\lim_{\theta\to\frac{\pi}{4}}\frac{\sin\theta-\cos\theta}{\theta-\frac{\pi}{4}}$		
	Ans	Put $\theta=\frac{\pi}{4}+h$, as $\theta\to\frac{\pi}{4}$, $h\to 0$		
		$= \lim_{h\to 0}\frac{\sin\left(\frac{\pi}{4}+h\right)-\cos\left(\frac{\pi}{4}+h\right)}{\frac{\pi}{4}+h-\frac{\pi}{4}}$	1	
		$= \lim_{h\to 0}\frac{\sin\frac{\pi}{4}\cos h+\cos\frac{\pi}{4}\sin h-\left(\cos\frac{\pi}{4}\cos h-\sin\frac{\pi}{4}\sin h\right)}{h}$	1	
		$= \lim_{h\to 0}\frac{\frac{1}{\sqrt{2}}\cos h+\frac{1}{\sqrt{2}}\sin h-\left(\frac{1}{\sqrt{2}}\cos h-\frac{1}{\sqrt{2}}\sin h\right)}{h}$	½	
		$= \lim_{h\to 0}\frac{\frac{1}{\sqrt{2}}\cos h+\frac{1}{\sqrt{2}}\sin h-\frac{1}{\sqrt{2}}\cos h+\frac{1}{\sqrt{2}}\sin h}{h}$	½	
		$= \lim_{h\to 0}\frac{\frac{2}{\sqrt{2}}\sin h}{h}$	½	
		$= \sqrt{2}\left(\lim_{h\to 0}\frac{\sin h}{h}\right)$		04
		$= \sqrt{2}$	½	
		--		
4.		**Attempt any <u>FOUR</u> of the following:**		16
	a)	Using first principal find the derivative of $f(x)=x^n, x\varepsilon R$		
	Ans	$\frac{dy}{dx}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$		
		$\frac{dy}{dx}=\lim_{h\to 0}\frac{(x+h)^n-x^n}{h}$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4.		$\frac{dy}{dx}=\lim_{h\to 0}\frac{x^{n}+nx^{n-1}h+\frac{n\left(n-1\right)}{2!}x^{n-2}h^{2}+\ldots+h^{n}-x^{n}}{h}$	1	
		$\frac{dy}{dx}=\lim_{h\to 0}\frac{nx^{n-1}h+\frac{n\left(n-1\right)}{2!}x^{n-2}h^{2}+\ldots+h^{n}}{h}$		
		$\frac{dy}{dx}=\lim_{h\to 0}\frac{h\left(nx^{n-1}+\frac{n\left(n-1\right)}{2!}x^{n-2}h+\ldots+h^{n-1}\right)}{h}$	½	
		$\frac{dy}{dx}=\lim_{h\to 0}\left(nx^{n-1}+\frac{n\left(n-1\right)}{2!}x^{n-2}h+\ldots+h^{n-1}\right)$	½	
		$\frac{dy}{dx}=nx^{n-1}+0+\ldots+0$	½	
		$\frac{dy}{dx}=nx^{n-1}$	½	04
		--		
	b)	If u and v are differentiable functions of x and if $y=uv$, then prove that $\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{dy}{dx}$		
	Ans	Given $y=uv$ Let $\delta u,\delta v,\delta y$ are small increments in u,v,y respectively corresponding to increment δx in x. $\therefore\ y+\delta y=\left(u+\delta u\right)\left(v+\delta v\right)$		
		$y+\delta y=uv+u\delta v+v\delta u+\delta u\delta v$	1	
		$\delta y=uv+u\delta v+v\delta u+\delta u\delta v-y$ $\delta y=uv+u\delta v+v\delta u+\delta u\delta v-uv$		
		$\delta y=u\delta v+v\delta u+\delta u\delta v$	½	
		$\because\ \delta u,\delta v$ are very small. $\therefore\ \delta u\delta v$ is negligible. $\therefore\ \delta y=u\delta v+v\delta u$		
		$\therefore\ \frac{\delta y}{\delta x}=\frac{u\delta v+v\delta u}{\delta x}$	½	
		$\lim_{\delta x\to 0}\frac{\delta y}{\delta x}=u\lim_{\delta x\to 0}\frac{\delta v}{\delta x}+v\lim_{\delta x\to 0}\frac{\delta u}{\delta x}$	1	
		$\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}$	1	04
		--		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4.	c)	Find $\frac{dy}{dx}$ if $y=\tan^{-1}\left(\frac{2x}{1+35x^2}\right)$		
	Ans	$y=\tan^{-1}\left(\frac{2x}{1+35x^2}\right)$		
		$\therefore\ y=\tan^{-1}\left(\frac{7x-5x}{1+(7x)(5x)}\right)$	1	
		$\therefore\ y=\tan^{-1}(7x)-\tan^{-1}(5x)$	1	
		$\therefore\ \frac{dy}{dx}=\frac{1}{1+(7x)^2}\times 7-\frac{1}{1+(5x)^2}\times 5$	1½	
		$\therefore\ \frac{dy}{dx}=\frac{7}{1+49x^2}-\frac{5}{1+25x^2}$	½	04
		--		
	d)	If $x^3y^2=(x+y)^5$ show that $\frac{dy}{dx}=\frac{y}{x}$		
	Ans	$x^3y^2=(x+y)^5$	½	
		$\therefore\ \log\left(x^3y^2\right)=\log(x+y)^5$	½	
		$\therefore\ \log x^3+\log y^2=5\log(x+y)$	½	
		$\therefore\ 3\log x+2\log y=5\log(x+y)$	1	
		$\therefore\ 3\frac{1}{x}+2\frac{1}{y}\frac{dy}{dx}=5\left[\frac{1}{x+y}\left(1+\frac{dy}{dx}\right)\right]$		
		$\therefore\ \frac{3}{x}+\frac{2}{y}\frac{dy}{dx}=\frac{5}{x+y}+\left(\frac{5}{x+y}\right)\frac{dy}{dx}$		
		$\therefore\ \left(\frac{2}{y}-\frac{5}{x+y}\right)\frac{dy}{dx}=\frac{5}{x+y}-\frac{3}{x}$	½	
		$\therefore\ \left(\frac{2x+2y-5y}{y(x+y)}\right)\frac{dy}{dx}=\frac{5x-3x-3y}{x(x+y)}$		
		$\therefore\ \left(\frac{2x-3y}{y}\right)\frac{dy}{dx}=\frac{2x-3y}{x}$	½	
		$\therefore\ \frac{dy}{dx}=\frac{y}{x}$	½	04
		--		
	e)	If $y=\frac{(1-x)^{\frac{1}{2}}}{(x+1)^{\frac{5}{7}}(2x+1)^{\frac{1}{3}}}$ find $\frac{dy}{dx}$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4.		$y=\frac{(1-x)^{\frac{1}{2}}}{(x+1)^{\frac{5}{7}}(2x+1)^{\frac{1}{3}}}$		
		$\therefore \log y=\log\left[\frac{(1-x)^{\frac{1}{2}}}{(x-1)^{\frac{5}{7}}(2x+1)^{\frac{1}{3}}}\right]$	½	
		$\therefore \log y=\log(1-x)^{\frac{1}{2}}-\log(x-1)^{\frac{5}{7}}-\log(2x+1)^{\frac{1}{3}}$	1	
		$\therefore \log y=\frac{1}{2}\log(1-x)-\frac{5}{7}\log(x-1)-\frac{1}{3}\log(2x+1)$	½	
		$\therefore \frac{1}{y}\frac{dy}{dx}=\frac{1}{2(1-x)}(-1)-\frac{5}{7(x-1)}-\frac{1}{3(2x+1)}(2)$	1½	
		$\therefore \frac{dy}{dx}=y\left[\frac{-1}{2(1-x)}-\frac{5}{7(x-1)}-\frac{2}{3(2x+1)}\right]$	½	04
		OR		
		$y=\frac{(1-x)^{\frac{1}{2}}}{(x-1)^{\frac{5}{7}}(2x+1)^{\frac{1}{3}}}$		
		$\therefore \log y=\log\left[\frac{(1-x)^{\frac{1}{2}}}{(x-1)^{\frac{5}{7}}(2x+1)^{\frac{1}{3}}}\right]$	½	
		$\therefore \log y=\log(1-x)^{\frac{1}{2}}-\log(x-1)^{\frac{5}{7}}-\log(2x+1)^{\frac{1}{3}}$	1	
		$\therefore \frac{1}{y}\frac{dy}{dx}=\frac{1}{(1-x)^{\frac{1}{2}}}\frac{1}{2}(1-x)^{\frac{-1}{2}}(-1)-\frac{1}{(x-1)^{\frac{5}{7}}}\frac{5}{7}(x-1)^{\frac{-2}{7}}-\frac{1}{(2x+1)^{\frac{1}{3}}}\frac{1}{3}(2x+1)^{\frac{-2}{3}}(2)$	1½	
		$\therefore \frac{dy}{dx}=y\left[\frac{-1}{2(1-x)}-\frac{5}{7(x-1)}-\frac{2}{3(2x+1)}\right]$	1	04
		--		
	f)	If $x=a(\theta-\sin\theta)$ and $y=a(1-\cos\theta)$ find $\frac{dy}{dx}$		
	Ans	$x=a(\theta-\sin\theta),\ y=a(1-\cos\theta)$		
		$\frac{dx}{d\theta}=a(1-\cos\theta)$	1½	
		$\therefore \frac{dy}{d\theta}=a\sin\theta$	1½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4.		$\therefore \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$		
		$\therefore \frac{dy}{dx}=\frac{a\sin\theta}{a(1-\cos\theta)}$	1	04
		$\therefore \frac{dy}{dx}=\frac{\sin\theta}{1-\cos\theta}$		
		--		
5.		**Attempt any FOUR of the following:**		16
	a)	Evaluate $\lim\limits_{x\to\infty}\sqrt{x}\left(\sqrt{x+1}-\sqrt{x}\right)$		
	Ans	$\lim\limits_{x\to\infty}\sqrt{x}\left(\sqrt{x+1}-\sqrt{x}\right)$		
		$=\lim\limits_{x\to\infty}\sqrt{x}\left(\sqrt{x+1}-\sqrt{x}\right)\times\frac{\left(\sqrt{x+1}+\sqrt{x}\right)}{\left(\sqrt{x+1}+\sqrt{x}\right)}$	½	
		$=\lim\limits_{x\to\infty}\frac{\sqrt{x}\,(x+1-x)}{\left(\sqrt{x+1}+\sqrt{x}\right)}$	½	
		$=\lim\limits_{x\to\infty}\frac{\sqrt{x}\,(1)}{\sqrt{x}\left(\sqrt{1+\frac{1}{x}}+1\right)}$	1	
		$=\lim\limits_{x\to\infty}\frac{1}{\left(\sqrt{1+\frac{1}{x}}+1\right)}$	1	
		$=\frac{1}{\sqrt{1+0}+1}$	½	04
		$=\frac{1}{2}$	½	
		OR		
		$\lim\limits_{x\to\infty}\sqrt{x}\left(\sqrt{x+1}-\sqrt{x}\right)$		
		$=\lim\limits_{x\to\infty}\sqrt{x}\left(\sqrt{x+1}-\sqrt{x}\right)\times\frac{\left(\sqrt{x+1}+\sqrt{x}\right)}{\left(\sqrt{x+1}+\sqrt{x}\right)}$	½	
		$=\lim\limits_{x\to\infty}\frac{\sqrt{x}\,(x+1-x)}{\left(\sqrt{x+1}+\sqrt{x}\right)}$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5.		$= \lim_{x\to\infty} \frac{\sqrt{x}}{\left(\sqrt{x+1}+\sqrt{x}\right)}$	1	
		Put $x=\frac{1}{t}$ as $x\to\infty, t\to 0$		
		$\lim_{t\to 0}\frac{\sqrt{\frac{1}{t}}}{\left(\sqrt{\frac{1}{t}+1}+\sqrt{\frac{1}{t}}\right)}$	½	
		$= \lim_{t\to 0}\frac{\sqrt{\frac{1}{t}}}{\sqrt{\frac{1}{t}}\left(\sqrt{1+t}+1\right)}$		
		$= \lim_{t\to 0}\frac{1}{\left(\sqrt{1+t}+1\right)}$	½	
		$= \frac{1}{\sqrt{1+0}+1}$	½	
		$= \frac{1}{2}$	½	04
		--		
	b)	Evaluate $\lim_{\theta\to\frac{\pi}{4}} \frac{2-\sec^2\theta}{1-\tan\theta}$		
	Ans	$\lim_{\theta\to\frac{\pi}{4}} \frac{2-\sec^2\theta}{1-\tan\theta}$		
		$= \lim_{\theta\to\frac{\pi}{4}} \frac{2-\left(1+\tan^2\theta\right)}{1-\tan\theta}$	½	
		$= \lim_{\theta\to\frac{\pi}{4}} \frac{2-1-\tan^2\theta}{1-\tan\theta}$		
		$= \lim_{\theta\to\frac{\pi}{4}} \frac{1-\tan^2\theta}{1-\tan\theta}$	½	
		$= \lim_{\theta\to\frac{\pi}{4}} \frac{\left(1-\tan\theta\right)\left(1+\tan\theta\right)}{1-\tan\theta}$	1	
		$= \lim_{\theta\to\frac{\pi}{4}} \left(1+\tan\theta\right)$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5.		$= 1 + \tan\frac{\pi}{4}$	½	
		$= 1 + 1$	½	04
		$= 2$	½	
		--		
	c)	Using bisection method find the approximate value of $\sqrt{10}$ by performing three iterations.		
	Ans	Let $x = \sqrt{10}$		
		$\therefore\ x^2 = 10$		
		$\therefore\ x^2 - 10 = 0$		
		Let $f(x) = x^2 - 10$		
		$f(3) = -1 < 0$	½	
		$f(4) = 16 > 0$	½	
		$\therefore$ the root is in $(3,4)$		
		$x_1 = \frac{a+b}{2} = \frac{3+4}{2} = 3.5$	1	
		$f(3.5) = 2.25 > 0$		
		$\therefore$ the root is in $(3,3.5)$		
		$x_2 = \frac{x_1+b}{2} = \frac{3+3.5}{2} = 3.25$	1	
		$f(x_2) = 0.563 > 0$		
		$\therefore$ the root is in $(3,3.25)$		
		$x_3 = \frac{x_2+b}{2} = \frac{3+3.25}{2} = 3.125$	1	04
		OR		
		Let $x = \sqrt{10}$		
		$\therefore\ x^2 = 10$		
		$\therefore\ x^2 - 10 = 0$		
		Let $f(x) = x^2 - 10$		
		$f(3) = -1 < 0$	½	
		$f(4) = 16 > 0$	½	
		$\therefore$ the root is in $(3,4)$		

<table>
<tr><th>Que. No.</th><th>Sub. Que.</th><th>Model answers</th><th>Marks</th><th>Total Marks</th></tr>
<tr><td>5.</td><td></td><td>
<table>
<tr><th>Iteration</th><th>a</th><th>b</th><th>$x = \frac{a+b}{2}$</th><th>$f(x)$</th></tr>
<tr><td>I</td><td>3</td><td>4</td><td>3.5</td><td>2.25</td></tr>
<tr><td>II</td><td>3</td><td>3.5</td><td>3.25</td><td>0.563</td></tr>
<tr><td>III</td><td>3</td><td>3.25</td><td>3.125</td><td>---</td></tr>
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1</td><td>04</td></tr>
<tr><td></td><td></td><td>---</td><td></td><td></td></tr>
<tr><td></td><td>d)</td><td>Using Regula-Falsi method, find the root of $x^2 - \log_{10} x = 12$
(up to thtree iterations only)</td><td></td><td></td></tr>
<tr><td></td><td>Ans</td><td>$x^2 - \log_{10} x = 12$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$\therefore\ x^2 - \log_{10} x - 12 = 0$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>Let $f(x) = x^2 - \log_{10} x - 12$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$f(3) = -3.477 < 0$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$f(4) = 3.398 > 0$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$\therefore$ the root is in $(3,4)$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{3(3.398) - 4(-3.477)}{3.398 + 3.477} = 3.506$</td><td>1</td><td></td></tr>
<tr><td></td><td></td><td>$f(x_1) = -0.253 < 0$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$\therefore$ the root is in $(3.5,4)$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$x_2 = \frac{3.5(3.398) - 4(-0.253)}{3.398 + 0.253} = 3.54$</td><td>1</td><td></td></tr>
<tr><td></td><td></td><td>$f(x_2) = -0.017 < 0$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$\therefore$ the root is in $(3.54,4)$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$x_3 = \frac{3.53(3.398) - 4(-0.017)}{3.398 + 0.017} = 3.542$</td><td>1</td><td>04</td></tr>
<tr><td></td><td></td><td>OR</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$x^2 - \log_{10} x = 12$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$\therefore\ x^2 - \log_{10} x - 12 = 0$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>Let $f(x) = x^2 - \log_{10} x - 12$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$f(3) = -3.477 < 0$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$f(4) = 3.398 > 0$</td><td>½</td><td></td></tr>
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<tr><th>a</th><th>b</th><th>$f(a)$</th><th>$f(b)$</th><th>$x=\frac{af(b)-bf(a)}{f(b)-f(a)}$</th><th>$f(x)$</th></tr>
<tr><td>3</td><td>4</td><td>-3.477</td><td>3.398</td><td>3.506</td><td>-0.253</td></tr>
<tr><td>3.506</td><td>4</td><td>-0.253</td><td>3.398</td><td>3.54</td><td>-0.017</td></tr>
<tr><td>3.542</td><td>4</td><td>-0.017</td><td>3.398</td><td>3.542</td><td>---</td></tr>
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1</td><td>04</td></tr>
<tr><td></td><td></td><td>---</td><td></td><td></td></tr>
<tr><td></td><td>e)</td><td>Find the approximate root of the equation $x^3-20=0$ by Newton-Raphson method (three iterations)</td><td></td><td></td></tr>
<tr><td></td><td>Ans</td><td>Let $f(x)=x^3-20$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$f(2)=-12<0$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$f(3)=7>0$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$f'(x)=3x^2$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>Initial root $x_0=3$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$\therefore\ f'(3)=27$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$x_1=x_0-\frac{f(x_0)}{f'(x_0)}=3-\frac{f(3)}{f'(3)}=2.741$</td><td>1</td><td></td></tr>
<tr><td></td><td></td><td>$x_2=2.74-\frac{f(2.741)}{f'(2.741)}=2.715$</td><td>1</td><td></td></tr>
<tr><td></td><td></td><td>$x_2=2.71-\frac{f(2.715)}{f'(2.715)}=2.714$</td><td>½</td><td>04</td></tr>
<tr><td></td><td></td><td>OR</td><td></td><td></td></tr>
<tr><td></td><td></td><td>Let $(x)=x^3-20$</td><td></td><td></td></tr>
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<tr><td></td><td></td><td>$\therefore\ f'(3)=27$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$x_i=x-\frac{f(x)}{f'(x)}=x-\frac{x^3-20}{3x^2}$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$=\frac{3x^3-x^3+20}{3x^2}$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$=\frac{2x^3+20}{3x^2}$</td><td>1</td><td></td></tr>
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<tr><th>Que. No.</th><th>Sub. Que.</th><th>Model Answers</th><th>Marks</th><th>Total Marks</th></tr>
<tr><td>5.</td><td></td><td>$x_1 = 2.741$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$x_2 = 2.715$</td><td>½</td><td>04</td></tr>
<tr><td></td><td></td><td>$x_3 = 2.714$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>---</td><td></td><td></td></tr>
<tr><td></td><td>f)</td><td>Obtain the approximate root of the equation $x^3 - 4x + 1 = 0$ using Regula-Falsi method upto 4 decimal places.</td><td></td><td></td></tr>
<tr><td></td><td>Ans</td><td>$x^3 - 4x + 1 = 0$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>Let $f(x) = x^3 - 4x + 1$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$f(0) = 1 > 0$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$f(1) = -2 < 0$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$\therefore$ the root is in $(0,1)$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$x_1 = \dfrac{af(b) - bf(a)}{f(b) - f(a)} = \dfrac{1(1) - 0(-2)}{1+2} = 0.3333$</td><td>1</td><td></td></tr>
<tr><td></td><td></td><td>$f(x_1) = -0.296 < 0$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$\therefore$ the root is in $(0.3333,0)$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$x_2 = \dfrac{0.3333(1) - 0(-0.296)}{1+0.296} = 0.2572$</td><td>1</td><td></td></tr>
<tr><td></td><td></td><td>$f(x_2) = -0.012 < 0$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$\therefore$ the root is in $(0.2572,0)$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$x_3 = \dfrac{0.2571(1) - 0(-0.011)}{1+0.011} = 0.2542$</td><td>1</td><td>04</td></tr>
<tr><td></td><td></td><td>OR</td><td></td><td></td></tr>
<tr><td></td><td></td><td>Let $f(x) = x^3 - 4x + 1$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$f(0) = 1 > 0$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$f(1) = -2 < 0$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$\therefore$ the root is in $(0,1)$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>
<table>
<tr><th>a</th><th>b</th><th>$f(a)$</th><th>$f(b)$</th><th>$x = \dfrac{af(b) - bf(a)}{f(b) - f(a)}$</th><th>$f(x)$</th></tr>
<tr><td>1</td><td>0</td><td>-2</td><td>1</td><td>0.3333</td><td>-0.296</td></tr>
<tr><td>0.3333</td><td>0</td><td>-0.296</td><td>1</td><td>0.2572</td><td>-0.012</td></tr>
<tr><td>0.2572</td><td>0</td><td>-0.012</td><td>1</td><td>0.2542</td><td>---</td></tr>
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</table>

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6.		**Attempt any <u>FOUR</u> of the following:**		16
	a)	Differentiate $\cos^{-1}\left(2x\sqrt{1-x^2}\right)$ with respect to $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$		
	Ans	Let $u=\cos^{-1}\left(2x\sqrt{1-x^2}\right)$ and $v=\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$		
		Put $x=\sin\theta \Rightarrow \sin^{-1}x=\theta$		
		$u=\cos^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$	½	
		$u=\cos^{-1}\left(2\sin\theta\cos\theta\right)$		
		$u=\cos^{-1}\left(\sin 2\theta\right)$		
		$u=\cos^{-1}\left(\cos\left(\frac{\pi}{2}-2\theta\right)\right)$	½	
		$u=\frac{\pi}{2}-2\theta$		
		$u=\frac{\pi}{2}-2\sin^{-1}x$		
		$\frac{du}{dx}=0-2\frac{1}{\sqrt{1-x^2}}$	½	
		$\frac{du}{dx}=-\frac{2}{\sqrt{1-x^2}}$		
		$v=\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$		
		$v=\sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right)$	½	
		$v=\sec^{-1}\left(\frac{1}{\sqrt{\cos^2\theta}}\right)$		
		$v=\sec^{-1}\left(\frac{1}{\cos\theta}\right)$		
		$v=\sec^{-1}\left(\sec\theta\right)$	½	
		$v=\theta$		
		$v=\sin^{-1}x$		
		$\frac{dv}{dx}=\frac{1}{\sqrt{1-x^2}}$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6.		$\therefore \frac{dy}{dz}=\frac{\frac{dy}{dx}}{\frac{dz}{dx}}=\frac{-\frac{2}{\sqrt{1-x^2}}}{\frac{1}{\sqrt{1-x^2}}}$ $\therefore \frac{dy}{dz}=-2$ **OR** Let $u=\cos^{-1}\left(2x\sqrt{1-x^2}\right)$ and $v=\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ Put $x=\cos\theta \Rightarrow \cos^{-1}x=\theta$ $u=\cos^{-1}\left(2\cos\theta\sqrt{1-\cos^2\theta}\right)$ $u=\cos^{-1}\left(2\cos\theta\sin\theta\right)$ $u=\cos^{-1}\left(\sin 2\theta\right)$ $u=\cos^{-1}\left(\cos\left(\frac{\pi}{2}-2\theta\right)\right)$ $u=\frac{\pi}{2}-2\theta$ $u=\frac{\pi}{2}-2\cos^{-1}x$ $\frac{du}{dx}=0-2\left(\frac{-1}{\sqrt{1-x^2}}\right)$ $\frac{du}{dx}=\frac{2}{\sqrt{1-x^2}}$ $v=\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ $v=\sec^{-1}\left(\frac{1}{\sqrt{1-\cos^2\theta}}\right)$ $v=\sec^{-1}\left(\frac{1}{\sqrt{\sin^2\theta}}\right)$ $v=\sec^{-1}\left(\frac{1}{\sin\theta}\right)$ $v=\sec^{-1}\left(\cos ec\theta\right)$	½ ½ ½ ½ ½ ½	04

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6.		$v = \sec^{-1}\left(\sec\left(\frac{\pi}{2} - \theta\right)\right)$	½	04
		$v = \frac{\pi}{2} - \theta$		
		$v = \frac{\pi}{2} - \cos^{-1} x$		
		$\frac{dv}{dx} = \frac{1}{\sqrt{1 - x^2}}$	½	
		$\therefore \frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{\frac{2}{\sqrt{1 - x^2}}}{\frac{1}{\sqrt{1 - x^2}}}$	½	
		$\therefore \frac{du}{dv} = 2$	½	
		OR		
		$\text{Let } u = \cos^{-1}\left(2x\sqrt{1 - x^2}\right) \text{ and } v = \sec^{-1}\left(\frac{1}{\sqrt{1 - x^2}}\right)$		
		$\therefore \frac{du}{dx} = \frac{-1}{\sqrt{1 - \left(2x\sqrt{1 - x^2}\right)^2}} \times \left[\left(2x \frac{1}{2\sqrt{1 - x^2}}\right)(-2x) + 2\sqrt{1 - x^2}\right]$	1	
		$= \frac{-1}{\sqrt{1 - \left(2x\sqrt{1 - x^2}\right)^2}}\left[\frac{-2x^2}{\sqrt{1 - x^2}} + 2\sqrt{1 - x^2}\right]$		
		$= \frac{-1}{\sqrt{1 - 4x^2\left(1 - x^2\right)}}\left[\frac{-2x^2 + 2\left(1 - x^2\right)}{\sqrt{1 - x^2}}\right]$		
		$= \frac{-1}{\sqrt{1 - 4x^2 + 4x^4}}\left[\frac{-2x^2 + 2 - 2x^2}{\sqrt{1 - x^2}}\right]$		
		$= \frac{-1}{\sqrt{\left(2x^2 - 1\right)^2}}\left[\frac{2 - 4x^2}{\sqrt{1 - x^2}}\right]$		
		$= \pm \frac{-1}{\left(2x^2 - 1\right)}\left[\frac{-2\left(2x^2 - 1\right)}{\sqrt{1 - x^2}}\right]$		
		$\therefore \frac{du}{dx} = \pm \frac{2}{\sqrt{1 - x^2}}$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6.		$v=\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$		
		$\therefore v=\cos^{-1}\left(\sqrt{1-x^2}\right)$		
		$\therefore \frac{dv}{dx}=\frac{-1}{\sqrt{1-\left(1-x^2\right)}}\times\left[\frac{1}{2\sqrt{1-x^2}}\left(-2x\right)\right]$	1	
		$=\frac{-1}{x}\times\left[\frac{-x}{\sqrt{1-x^2}}\right]$		
		$\therefore \frac{dv}{dx}=\frac{1}{\sqrt{1-x^2}}$	½	
		$\therefore \frac{du}{dv}=\frac{\frac{du}{dx}}{\frac{dv}{dx}}=\frac{\frac{\pm 2}{\sqrt{1-x^2}}}{\frac{1}{\sqrt{1-x^2}}}$	½	
		$\therefore \frac{du}{dv}=\pm 2$	½	04
		---		
	b)	If $y=e^{\tan^{-1}x}$ show that $\left(1+x^2\right)\frac{d^2y}{dx^2}+\left(2x-1\right)\frac{dy}{dx}=0$		
	Ans	$y=e^{\tan^{-1}x}$		
		$\therefore \frac{dy}{dx}=e^{\tan^{-1}x}\frac{1}{1+x^2}$	1	
		$\therefore \left(1+x^2\right)\frac{dy}{dx}=y$		
		$\therefore \left(1+x^2\right)\frac{d^2y}{dx^2}+\frac{dy}{dx}\left(2x\right)=\frac{dy}{dx}$	2	
		$\therefore \left(1+x^2\right)\frac{d^2y}{dx^2}+\frac{dy}{dx}\left(2x\right)-\frac{dy}{dx}=0$	½	
		$\therefore \left(1+x^2\right)\frac{d^2y}{dx^2}+\left(2x-1\right)\frac{dy}{dx}=0$	½	04
		OR		
		$y=e^{\tan^{-1}x}$		
		$\therefore \frac{dy}{dx}=e^{\tan^{-1}x}\frac{1}{1+x^2}$	1	
		$\therefore \frac{dy}{dx}=\frac{y}{1+x^2}$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6.		$\therefore \frac{d^2y}{dx^2} = \frac{\left(1+x^2\right)\frac{y}{1+x^2} - y\left(2x\right)}{\left(1+x^2\right)^2}$	1	
		$\therefore \frac{d^2y}{dx^2} = \frac{y - y\left(2x\right)}{\left(1+x^2\right)^2}$		
		$\therefore \frac{d^2y}{dx^2} = \frac{y\left(1-2x\right)}{\left(1+x^2\right)^2}$	1	
		$L.H.S. = \left(1+x^2\right)\frac{d^2y}{dx^2} + \left(2x-1\right)\frac{dy}{dx}$		
		$= \left(1+x^2\right)\frac{y\left(1-2x\right)}{\left(1+x^2\right)^2} + \left(2x-1\right)\frac{y}{1+x^2}$	½	
		$= -\left(2x-1\right)\frac{y}{1+x^2} + \left(2x-1\right)\frac{y}{1+x^2}$		
		$= 0$	½	04
		$= R.H.S.$		
		---		
	c)	Solve using Gauss elimination method: $x+2y+3z=14$, $3x+y+2z=11$, $2x+3y+z=11$		
	Ans	$x+2y+3z=14$ $3x+y+2z=11$ $2x+3y+z=11$		
		$x+2y+3z=14$ $9x+3y+6z=33$ $6x+2y+4z=22$ and $2x+3y+z=11$ $-$ ____________ $-$ ____________ $-5x-z=-8$ $7x+5z=22$	1	
		$-25x-5z=-40$ $7x+5z=22$ $+$ ____________ $-18x=-18$		
		$\therefore x=1$	1	
		$y=2$	1	04
		$z=3$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6.		***Note:*** *In the above solution, first y is eliminated and then z is eliminated to find the value of x first. If in case the problem is solved by elimination of another unknown i. e., either first x or z, appropriate marks to be given as per above scheme of marking.*		
		--		
	d)	Solve the following equations by Gauss-Seidel method:		
		$x+7y-3z=-22\ ,\ 5x-2y+3z=18\ ,\ 2x-y+6z=22$		
	Ans	$x=\frac{1}{5}(18+2y-3z)$		
		$y=\frac{1}{7}(-22-x+3z)$		
		$z=\frac{1}{6}(22-2x+y)$	1	
		Starting with $y_0=z_0=0$		
		$x_1=3.6$		
		$y_1=-3.657$		
		$z_1=1.857$	1	
		$x_2=1.023$		
		$y_2=-2.493$	1	
		$z_2=2.91$		
		$x_3=0.857$		04
		$y_3=-2.018$		
		$z_3=3.045$	1	
		--		
	e)	Solve the equtions using Jacobi's method (upto three iterations)		
		$10x-2y-2z=6\ ,\ -x-y+10z=8\ ,\ -x+10y-2z=7$		
	Ans	$x=\frac{1}{10}(6+2y+2z)$		
		$y=\frac{1}{10}(7+x+2z)$		
		$z=\frac{1}{10}(8+x+y)$	1	
		Starting with $x_0=y_0=z_0=0$		
		$x_1=0.6$		
		$y_1=0.7$		
		$z_1=0.8$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6.		$x_2 = 0.9$ $y_2 = 0.92$ $z_2 = 0.93$	1	
		$x_3 = 0.97$ $y_3 = 0.976$ $z_3 = 0.982$	1	04
		--		
	f)	Use Gauss-Seidel method to solve following equations (use two iterations)		
	Ans	$10x + 2y + z = 9\ ,\ x + 10y - z = -22\ ,\ -2x + 3y + 10z = 22$ $x = \frac{1}{10}(9 - 2y - z)$ $y = \frac{1}{10}(-22 - x + z)$ $z = \frac{1}{10}(22 + 2x - 3y)$	1	
		Starting with $y_0 = z_0 = 0$ $x_1 = 0.9$ $y_1 = -2.29$ $z_1 = 3.067$	1½	
		$x_2 = 1.051$ $y_2 = -1.998$ $z_2 = 3.009$	1½	04
		--		
		Important Note *In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.* -- --		