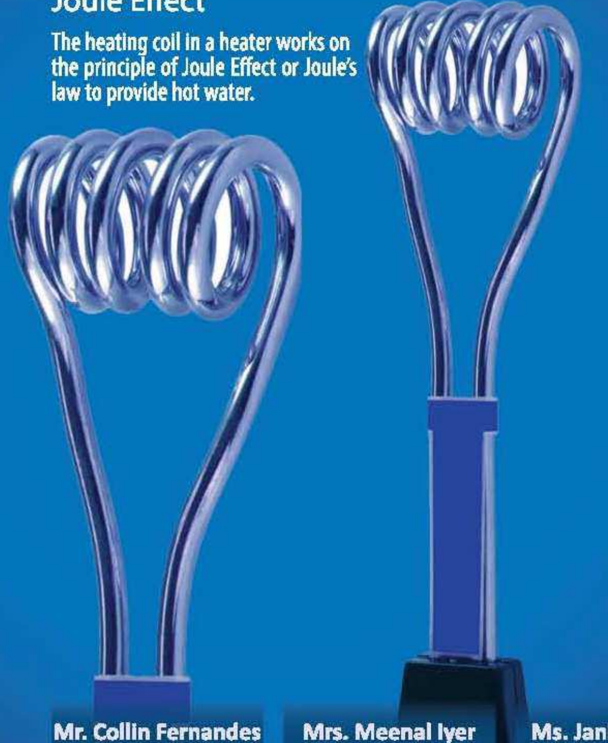
APPLIED SCIENCE
MECHANICAL ENGINEERING GROUP

Phy. Code: 17202 | Chem. Code: 17203

FIRST YEAR DIPLOMA
SEMESTER II
MSBTE 'G' SCHEME

Joule Effect

The heating coil in a heater works on the principle of Joule Effect or Joule's law to provide hot water.



Asphalt

Asphalt, a sticky, black residue obtained from petroleum, is widely used in road construction.

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APPLIED SCIENCE

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FIRST YEAR DIPLOMA

SEMESTER - II

First Edition: December 2015

Salient Features

- Concise content with complete coverage of revised G-scheme syllabus.
- Simple and Lucid language.
- Neat, Labelled and Authentic diagrams.
- Illustrative examples showing detailed solution of numericals.
- MSBTE Theory Questions and Numericals from Summer-2007 to Winter-2015.
- MSBTE Question Papers of Summer, Winter - 2014 and 2015.
- Three Model Question Papers for practice.
- Important Inclusions: Additional Theory Questions, Practice Problems, Knowledge Bank, Physics Behind.

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PREFACE

In the case of good books, the point is not how many of them you can get through, but rather how many can get through to you.

Target's "Applied Science: Mechanical Engineering Group" is a complete and thorough book critically analysed and extensively drafted to boost the students' confidence. Applied Science is divided into two parts: **Applied Science - Physics** and **Applied Science - Chemistry**. The book is prepared as per the revised scheme [G-scheme] of MSBTE curriculum effective from June 2012.

The **Topic-wise** classified format for each chapter of this book helps the students in easy comprehension. Each chapter includes the following features:

- **Theory** is provided in the form of pointers. Neat labelled diagrams have been provided wherever required. Definitions, statements and laws are specified with italic representation.
- **Illustrative Examples** are provided in relevant chapters in order to understand the application of different concepts and formulae.
- **Formulae** provided for quick recap.
- **MSBTE Theory Questions** covered in separate section to give a clear idea of the type of questions asked. *(Reference of answer to each question is provided.)*
- **MSBTE Numericals** till latest year are included.
- **Additional Theory Questions** to help the student gain insight on the various levels of theory-based questions.
- **Problems for Practice** *(With final answers)* in relevant chapters which covers an array of questions from simple to complex.
- **Knowledge Bank, Note,** etc. to enrich students' knowledge required to understand different concepts.
- **"Physics behind…."** is an effort to make students aware of real life engineering situations where physics plays prominent role or day-to-day experiences ruled by physics.

Three **Model Question Papers,** designed as per MSBTE Paper Pattern, are a unique tool to enable self-assessment for the students.

MSBTE Question Papers of years 2014 and 2015 are added at the end to make students familiar with the examination pattern.

The journey to create a complete book is strewn with triumphs, failures and near misses. If you think we've nearly missed something or want to applaud us for our triumphs, we'd love to hear from you.

Please write to us on : mail@targetpublications.org

A book affects eternity; one can never tell where its influence stops.

Best of luck to all the aspirants!

From,
Publisher

SYLLABUS

Applied Science : Physics

Topic and Contents	Hours	Marks
Topic 1 - Motion **Specific objectives:** ➢ State equations of motion. ➢ Apply laws of motion to solve problems. ➢ Differentiate between linear and circular motion. ➢ State meaning of centripetal acceleration, centripetal force. **1.1 Rectilinear and Angular Motion: [6 Marks]** • Equations of motion: $V = u + at$, $S = ut + 1/2at^2$, $V^2 = u^2 + 2as$ (no derivation), distance travelled by particle in n^{nt} second, (only equation), Uniform velocity, uniform acceleration and uniform retardation, equations of motion for motion under gravity. • Definition of angular displacement, angular velocity, angular acceleration, relation between angular velocity and linear velocity, three equations of angular motion (no derivation) angular distance travelled by particle in n^{nt} second (only equation). **1.2 Kinetics and Work Power Energy: [6 Marks]** • Definitions of momentum, impulse, impulsive force with formulae, statements of Newton's laws of motion with equations, applications of laws of motion—recoil of gun. • Definition of work, power and energy, equations for potential energy. Kinetic energy, work -energy principle. **1.3 Projectile Motion and circular motion: [4 Marks]** • Definition of a projectile motion, angle of projection, trajectory, time of flight and range with formulae. • Definition of a circular motion, centripetal acceleration, centripetal force, definition of centrifugal force, and its applications.	**10**	**16**
Topic 2 - Nondestructive testing of materials: **Specific objectives:** ➢ Describe the method of production of ultrasonic waves. ➢ Use NDT methods for quality testing of materials in industry. **2.1 Ultrasonic: [4 Marks]** • Ultrasonic waves-properties, production of ultrasonic waves by piezoelectric method **2.2 Non-destructive testing methods: [6 Marks]** • Destructive and Nondestructive testing, advantages of NDT, limitations of N.D.T., different N.D.T. Methods used in industries, criteria for selection of NDT method, Liquid penetration Testing (LPT): principle, procedure and applications, Ultrasonic testing methods: principle, procedure and applications.	**06**	**10**
Topic 3 - Thermocouple: **Specific objectives:** ➢ State meaning of thermoelectricity. ➢ State characteristics of thermocouple. • Concept of EMF, thermoelectricity, Seebeck effect; measurement of thermo emf, Peltier effect, Seebeck series; examples with different pairs of metals. • Variation of thermo emf with temperature, graph; neutral temperature, inversion temperature, Joule effect , comparison of Seebeck effect, Peltier effect and Joule effect.	**04**	**08**

Topic 4 - Modern physics: **Specific objectives:** ➢ State the concept of photocell ➢ State applications of X-ray ➢ State properties of LASER **4.1 Photo electricity: [6 Marks]** • Photon (quantum), Plank's hypothesis, energy of photon, properties of photons. • Photo electric effect: Circuit diagram, process of photoelectric emission, definitions: threshold frequency, threshold wavelength, stopping potential, characteristics of photoelectric effect. • Work function, Einstein's photoelectric equation, photo resistor (LDR) – symbol, principle, applications, photoelectric cell:- principle, applications. **4.2 X-rays: [6 Marks]** • Origin of X-rays, production of X-rays using Coolidge's X-ray tube, minimum wavelength of X-ray, properties of X-rays, applications of X-rays: engineering, medical and scientific. **4.3 Laser: [4 Marks]** • Laser, properties of laser, spontaneous and stimulated emission, population inversion, optical pumping, engineering applications of Laser.	**12**	**16**
TOTAL	**32**	**50**

Applied Science : Chemistry

Topic and Contents	Hours	Marks
Topic 1 - Metallurgy **Specific objectives:** ➢ Explain the process of extraction of iron from its ore. ➢ Explain different processes of heat treatment. ➢ State effects of alloying elements on properties of steels. **1.1 Metallurgy: [6 Marks]** • Definition of metallurgy, ores of iron. • Extraction of pig iron by smelting in blast furnace with chemical reactions in different zones, products of blast furnace - composition, properties and applications of pig iron, slag and flue gases. • Properties and applications of commercial forms of iron - pig iron, cast iron, wrought iron. **1.2 Steels: [8 Marks]** • Definition of steel, preparation of steel from pig iron using open hearth process, basic oxygen process. • Classification of plain carbon steel - low carbon, medium carbon, high carbon steels with their properties and applications. • Alloy steels: Effects of alloying elements C, Ni, Co, V, Mo, W, Cr on properties of steel, composition, properties and applications of heat resisting steel (nichrome), magnetic steel (alnico), 18-8 stainless steel, 18-4-1 high speed steel. • Heat treatment of steels: Definition and purposes of - hardening, tempering, annealing, normalising.	**08**	**14**
Topic 2 - Corrosion **Specific objectives:** ➢ Explain mechanism of atmospheric corrosion and immersed corrosion. ➢ Describe different methods of protection of metal from corrosion. **2.1 Corrosion: [6 Marks]** • Definition of corrosion, types of corrosion.	**10**	**14**

• Atmospheric corrosion: Definition, mechanism of oxidation corrosion, types of oxide films and their significance, factors affecting rate of atmospheric corrosion. • Immersed corrosion: Definition, mechanism of immersed corrosion by galvanic cell action - with evolution of hydrogen gas and absorption of oxygen gas, factors affecting immersed corrosion. **2.2 Protection of metals by: [8 Marks]** • Modification of environment, modification of properties of metal, electrochemical protection by sacrificial anodic protection and impressed current cathodic protection, use of protective coatings. • Application of metallic coatings: By galvanising, tinning, metal spraying, electroplating, metal cladding, cementation - sherardizing, chromising, colourising. • Application of non-metallic coatings: paint - definition, characteristics, constituents of paint and their functions.		
Topic 3 - Fuels **Specific objectives:** ➢ State characteristics of a good fuel. ➢ Write significance of proximate analysis of a fuel. ➢ Explain fractional distillation of crude petroleum. **3.1 Properties of fuels: [4 Marks]** • Definition of a fuel, calorific value and ignition temperature. Characteristics of a good fuel, classification of fuels with suitable examples, advantages and disadvantages of solid fuels, liquid fuels and gaseous fuels. **3.2 Classification of fuels: [8 Marks]** • Solid fuels: Analysis of solid fuel - proximate analysis for determination of moisture, volatile matter, ash and fixed carbon, significance of proximate analysis, determination of gross calorific value by using bomb calorimeter. • Liquid fuels: Origin, fractional distillation of crude petroleum, boiling range, composition, and applications of petroleum fractions obtained, composition, properties, applications of - Biodiesel. • Gaseous fuels: Composition, properties, applications of - Biogas, LPG, CNG.	**07**	**12**
Topic 4 - Lubricants **Specific objectives:** ➢ Write functions of lubricants. ➢ Describe the mechanism of lubrication. ➢ State characteristics of lubricants. **Lubricants:** • Definition of lubricant, functions of lubricants. • Classification of lubricants: Solid lubricants - characteristics and applications of graphite and molybdenum disulphide. Liquid lubricants - characteristics and applications of synthetic fluid (silicone oil), water as a lubricant (coolant). Semisolid lubricant - characteristics and applications of grease (plastic lubricant). • Mechanism of lubrication: Definition of lubrication, mechanism of fluid film lubrication, boundary lubrication, extreme pressure lubrication. • Characteristics: Physical characteristics of lubricants - viscosity, viscosity index, oiliness, volatility, flash and fire point, cloud and pour point. Chemical characteristics of lubricants - acid value or neutralisation number, emulsification, saponification value. • Selection of lubricants for road rollers, steam engines, sewing machine, concrete mixer, I.C. engine, cutting tools, gears.	**07**	**10**
TOTAL	**32**	**50**

Contents

Applied Science
Physics

01 Rectilinear and Angular Motion

1.0 Introduction

1.0.(a) Motion:

i. A body which does not change its position with time w.r.t. its surroundings is said to be at rest.

ii. ***Motion*** *of a body means the change in position of that body with time.*

e.g. Motion of person walking on the foot path, a running train or a car, crawling insect, movement of air particles etc.

1.0.(b) Rectilinear motion:

i. *Motion of a particle along a straight line is called* ***rectilinear motion*** *or* ***translational motion*** *or* ***linear motion****.*

ii. For a body in rectilinear motion, all the particles of the body have their displacements parallel to each other.

e.g. Motion of a car along a straight line, a body falling freely under gravity etc.

1.0.(c) Kinematics:

Kinematics *is that branch of mechanics which deals with the study of motion of material objects without taking into account the factors (i.e., nature of forces, nature of bodies etc.) which cause motion.*

1.0.(d) Displacement:

i. ***Displacement*** *of an object in motion in a given time is defined as the difference between the final and initial positions of the object w.r.t. a fixed point.* It is denoted by letter 's'.

ii. It is the shortest distance between the two positions of the object and its direction is from initial to final position of the object during the given interval of time.

1.1 Velocity and Uniform Velocity

1.1.(a) Velocity:

i. *Distance travelled by a moving body per unit time in a given direction is called* ***(linear) velocity****.*

OR

Velocity *means the rate at which displacement takes place.* It is denoted by letter 'v'.

ii. **Formula:** Velocity $= \dfrac{\text{displacement}}{\text{time}} = \dfrac{s}{t}$

iii. **Unit:** S.I. unit is m/s and C.G.S. unit is cm/s.

1.1.(b) Uniform velocity:

i. *A body is said to be moving with a* ***uniform velocity*** *if it covers equal displacements in equal intervals of time in a particular direction.*

ii. In such a case, acceleration is zero.

iii. Uniform velocity is independent of time interval.

iv. No net force acts on the body when it is moving with uniform velocity.

1.2 Acceleration and Retardation

1.2.(a) Acceleration:

i. ***Acceleration** is defined as the rate of change of velocity with respect to time.* It is denoted by letter 'a'.

ii. **Formula:** $a = \frac{\text{change in velocity}}{\text{time}} = \frac{v-u}{t}$

where, u = initial velocity and v = final velocity, t = time

iii. **Unit:** S.I. unit is m/s^2 and C.G.S. unit is cm/s^2.

iv. If a particle moves with constant velocity, its acceleration is zero.

Note:

i. *If the velocity of a body increases with time, then it is said to have a **positive acceleration** or **acceleration**.*

ii. *If the velocity of a body decreases with time, then it is said to have a **negative acceleration** or **retardation** or **deceleration.***

iii. Negative acceleration is also called retardation.

1.2.(b) Difference between positive acceleration and negative acceleration:

No.	Positive acceleration	Negative acceleration
i.	If the velocity of a body increases with time, then it is said to have a **positive acceleration** or **acceleration**.	If the velocity of a body decreases with time, then it is said to have a **negative acceleration** or **retardation** or **deceleration.**
ii.	Here, the applied force acts in the direction of motion.	Here, the applied force acts in the direction opposite to the direction of motion.
iii.	e.g. Acceleration of a speeding car.	e.g. Retardation of a car with the brakes applied to it.

Illustrative Example:

The speed of a vehicle is increased from 36 km/hr to 90 km/hr in 5 s. What is acceleration of the vehicle during this time interval?

Solution:

Given: $u = 36 \text{ km/hr} = 36 \times \frac{5}{18} \text{ m/s} = 10 \text{ m/s}$, $v = 90 \text{ km/hr} = 90 \times \frac{5}{18} \text{ m/s} = 25 \text{ m/s}$,

t = 5 s

To find: Acceleration (a)

Formula: $a = \frac{v-u}{t}$

Calculation: Using formula, we get

$$a = \frac{25-10}{5} = \frac{15}{5} = \mathbf{3\ m/s^2}$$

Ans: The acceleration of the car in 5 seconds is $\mathbf{3\ m/s^2}$.

Note: $1 \text{ km/hr} = 1 \text{ kmph} = \frac{1 \times 1000 \text{ m}}{1 \times 3600 \text{ s}}$ ....[$\because$ 1 km = 1000 m and 1 hr = 3600 s]

i.e., $1 \text{ km/hr} = 1 \text{ kmph} = \frac{5}{18} \text{ m/s}$

1.2.(c) Uniform acceleration:

i. *A particle is said to have* ***uniform acceleration*** *if its velocity increases by equal amounts in equal intervals of time, how-so-ever small these intervals may be.*

ii. When a particle moves with uniform acceleration, its average acceleration measured for different time intervals is constant.

e.g. Acceleration of a freely falling body.

1.2.(d) Uniform Retardation:

A body is said to have ***uniform retardation or deceleration*** *if its velocity decreases by equal amounts in equal intervals of time, how-so-ever small these time intervals may be.*

e.g. Retardation of a car in motion with the brakes applied, a ball thrown vertically upwards etc.

1.3 Kinematical equations of motion; Distance covered by a particle in n^{th} second

1.3.(a) Kinematical equations for rectilinear motion with uniform acceleration:

i. First equation of motion: $v = u + at$

ii. Second equation of motion: $s = ut + \frac{1}{2}at^2$

iii. Third equation of motion: $v^2 = u^2 + 2as$

where, u = initial velocity, v = final velocity

a = uniform acceleration, s = distance travelled

t = time in second

iv. Distance travelled in n^{th} second by a particle (or body) moving with uniform acceleration:

$$s_{n^{th}} = s_n - s_{n-1} = u + \frac{a}{2}(2n - 1)$$

where, u = initial velocity of the body,

s_n = distance travelled in n seccond,

s_{n-1} = distance travelled in (n – 1) second,

$s_{n^{th}}$ = distance travelled in n^{th} second,

a = uniform acceleration

Illustrative Examples:

Example 1

A car changes its speed uniformly from 50 km/hr to 140 km/hr in a distance of 400 m. What is the acceleration?

Solution:

Given: $u = 50\text{ km/hr} = 50 \times \frac{5}{18}\text{ m/s} = 13.89\text{ m/s}$, s = 400 m,

$v = 140\text{ km/hr} = 140 \times \frac{5}{18}\text{ m/s} = 38.89\text{ m/s}$

To find: Acceleration (a)

Formula: $v^2 = u^2 + 2as$

Calculation: Using formula,

$$v^2 - u^2 = 2as$$

$$\therefore \quad a = \frac{v^2 - u^2}{2s}$$

$$= \frac{(38.89)^2 - (13.89)^2}{2 \times 400}$$

$$= \frac{1512.4 - 192.4}{800} = 1.649 \text{ m/s}^2$$

$$\approx \mathbf{1.65\ m/s^2}$$

Ans: The acceleration of the car is **1.65 m/s²**.

Example 2

A body starts from rest and moves with a uniform acceleration of 2.5 m/s². What will be the velocity when it has covered a distance of 500 m? How much time will it require to cover this distance?

Solution:

Given: $a = 2.5$ m/s², $s = 500$ m, $u = 0$ ($\because$ the body starts from rest)

To find:
i. Velocity after covering 500 m (v)
ii. Time required to cover 500 m (t)

Formulae: i. $v^2 = u^2 + 2as$ ii. $v = u + at$

Calculation: Using formula (i),

$$v^2 = (0)^2 + 2 \times 2.5 \times 500 = \sqrt{2500}$$

$$\therefore \quad v = \mathbf{50\ m/s}$$

Using formula (ii),

$$t = \frac{v - u}{t} = \frac{50 - 0}{2.5} = \frac{50}{2.5}$$

$$\therefore \quad t = \mathbf{20\ s}$$

Ans: i. The velocity of the body after covering 500 m is **50 m/s.**
ii. The time required to cover 500 m is **20 s.**

Example 3

A train crosses a tunnel in 20 second. At the entry of tunnel, its velocity is 60 km/hr and at the exit of tunnel, its velocity becomes 30 km/hr. Find length of the tunnel.

Solution:

Given: $u = 60 \text{ km/hr} = 60 \times \frac{5}{18} \text{ m/s} = 16.67 \text{ m/s}$,

$v = 30 \text{ km/hr} = 30 \times \frac{5}{18} \text{ m/s} = 8.33 \text{ m/s}, \ t = 20 \text{ s}$

To find: Length of the tunnel (s)

Formulae: i. $a = \frac{v - u}{t}$ ii. $v^2 = u^2 + 2as$

Calculation: Using formula (i),

$$a = \frac{8.33 - 16.67}{20} = -\,0.417 \text{ m/s}^2$$ (negative sign indicates retardation)

Using formula (ii),

$(8.33)^2 = (16.67)^2 + 2(-0.417) \times s$

$\therefore \quad s = \dfrac{(8.33)^2 - (16.67)^2}{-2(0.417)}$

$= \dfrac{69.39 - 277.89}{-0.834} = \dfrac{-208.5}{-0.834}$

$\therefore \quad s = \mathbf{250\ m}$

Ans: The length of the tunnel is **250 m.**

Example 4

A vehicle has initial velocity of 3 m/s. It accelerates for 12 seconds at the rate of 3 m/s^2. Calculate the final velocity and the distance travelled during this time.

Solution:

Given: $u = 3$ m/s, $t = 12$ s, $a = 3$ m/s^2

To find: i. Final velocity (v)
ii. Distance travelled during 12 second (s)

Formulae: i. $v = u + at$ ii. $s = ut + \dfrac{1}{2}at^2$

Calculation: Using formula (i),

$v = 3 + (3)(12)$

$\therefore \quad v = \mathbf{39\ m/s}$

Using formula (ii),

$s = (3)(12) + \dfrac{1}{2}(3)(12)^2 = 36 + 216$

$\therefore \quad s = \mathbf{252\ m}$

Ans: The final velocity of the vehicle is **39 m/s** and the distance travelled by it in 12 seconds is **252 m**.

Example 5

The speed of truck is reduced from 120 kmph to 60 kmph over a distance of 300 m. Find uniform retardation and distance further travelled before coming to rest.

Solution:

Given: $u = 120 \text{ kmph} = 120 \times \dfrac{5}{18} \text{ m/s} = 33.33 \text{ m/s}$,

$v = 60 \text{ kmph} = 60 \times \dfrac{5}{18} = 16.67 \text{ m/s}$,

$s = 300$ m

To find: i. Uniform retardation ii. Distance travelled further (s)

Formula: $v^2 = u^2 + 2as$

Calculation: Using formula,

$$a = \frac{v^2 - u^2}{2s} = \frac{(16.67)^2 - (33.33)^2}{2(300)} = \frac{-833}{600} \text{ m/s}^2$$

$\therefore \quad a = \mathbf{-1.388\ m/s^2}$

(The –ve sign indicates retardation of truck.)

For calculating the further distance travelled by truck before coming to rest,

u = 16.67 m/s and v = 0 ....($\because$ truck comes to rest)

$\therefore$ Using formula,

$$s = \frac{v^2 - u^2}{2a} = \frac{(0)^2 - (16.67)^2}{2(-1.388)}$$

$= \mathbf{100.1\ m}$

Ans: The truck has a uniform retardation of **1.388 m/s²** and it covers **100.1 m** further before coming to rest.

Example 6

A bus starts from Thane. The bus attains a uniform velocity of 72 km/hr in 2 min and travels with this velocity for 10 min; then it retards for 3 min and comes to rest at Turbhe. Calculate distance between Thane and Turbhe.

Solution:

Given: u = 0 m/s ….($\because$ Bus starts from rest),

$v = 72\text{ km/hr} = 72 \times \frac{5}{18}\text{ m/s} = 20\text{ m/s}$, $t_1 = 2\text{ min} = 2 \times 60\text{ s} = 120\text{ s}$,

$t_2 = 10\text{ min} = 10 \times 60\text{ s} = 600\text{ s}$, $t_3 = 3\text{ min} = 3 \times 60\text{ s} = 180\text{ s}$

To find: Distance between Thane and Turbhe

Formulae: i. $v = u + at$ ii. $v^2 = u^2 + 2as$ iii. $s = ut + \frac{1}{2}at^2$

Calculation: For the first part of the motion of bus, using formula (i),

$$a = \frac{v - u}{t_1} = \frac{20 - 0}{120} = 0.167\text{ m/s}^2$$

Using formula (ii),

$$s = \frac{v^2 - u^2}{2a} = \frac{(20)^2 - (0)^2}{2 \times 0.167}$$

$\therefore$ $s = 1197.6\text{ m} = s_1$ (say)…(1)

Now, as the bus continues to move with 20 m/s for 10 min.,

We have, u = 20 m/s, a = 0 ….($\because$ velocity is constant)

Using formula (iii),

$$s = ut_2 + \frac{1}{2}at_2^2$$

$$s = 20 \times 600 + \frac{1}{2} \times (0) \times (600)^2$$

$\therefore$ $s = 12000\text{ m} = s_2$ (say) …(2)

Considering the retarding motion of the bus,

$u = 20\text{ m/s}$, $t_3 = 180\text{ s}$, $v = 0$ ….($\because$ Bus comes to rest)

Using formula (i),

$$a = \frac{0 - 20}{180} = -0.11\text{ m/s}^2$$

Now, using formula (ii)

$$s = \frac{v^2 - u^2}{2a} = \frac{(0)^2 - (20)^2}{2 \times (-0.11)}$$

$\therefore$ $s = 1818.18 \text{ m} = s_3$ (say) …(3)

$\therefore$ The total distance covered by the bus in moving from Thane to Turbhe $= s_1 + s_2 + s_3$

$= 1197.6 + 12000 + 1818.18$ ….[From (1), (2) and (3)]

$=$ **15015.78 m ≈ 15 km**

Ans: The distance between stations Thane and Turbhe is **approximately 15 km.**

Example 7

A particle having initial velocity 5 m/s moves with a constant acceleration 2 ms^{-2} for 10 seconds along a straight line. Find the displacement of the particle in the last second and the total distance travelled in 10 seconds.

Solution:

Given: $u = 5 \text{ ms}^{-1}$, $a = 2 \text{ ms}^{-2}$, $t = 10$ s

To find:
i. Displacement of particle in last second $\left(s_{10^{th}}\right)$
ii. Total distance travelled in 10 second (s)

Formulae: i. $s_{n^{th}} = u + \frac{a}{2}(2n - 1)$ ii. $s = ut + \frac{1}{2}at^2$

Calculation: Using formula (i),

$$s_{10^{th}} = 5 + \frac{2}{2}(2 \times 10 - 1) = 5 + 19 = \mathbf{24\ m}$$

Using formula (ii),

$$s = 5 \times 10 + \frac{1}{2} \times 2 \times 10^2 = 50 + 100 = \mathbf{150\ m}$$

Ans: i. The displacement of particle in last second is **24 m**.
ii. The total distance travelled in 10 second is **150 m**.

Example 8

A van starting from rest is moving with uniform acceleration. If it gains a velocity of 54 km/hr in 6 seconds, find its acceleration and distance covered in 4th second.

Solution:

Given: $u = 0$ …($\because$ Van starts from rest), $t = 6$ s,

$$v = 54 \text{ km/hr} = 54 \times \frac{5}{18} \text{ m/s} = 15 \text{ m/s}$$

To find: i. Acceleration (a) ii. Distance covered in 4th second $\left(s_{4^{th}}\right)$

Formulae: i. $a = \frac{v - u}{t}$ ii. $s_{n^{th}} = u + \frac{a}{2}(2n - 1)$

Calculation: Using formula (i),

$$a = \frac{v - u}{t} = \frac{15 - 0}{6}$$

$\therefore$ $a = \mathbf{2.5\ m/s^2}$

Using formula (ii),

$$s_{4^{th}} = 0 + \frac{2.5}{2}(2 \times 4 - 1)$$

$$= 0 + \frac{2.5}{2} \times (8 - 1)$$

$$= \frac{2.5 \times 7}{2}$$

$$\therefore \quad s_{5^{th}} = \mathbf{8.75\ m}$$

Ans: Acceleration of the body is **2.5 m/s^2** and distance travelled by it in 4^{th} second is **8.75 m**.

Example 9

A body moving with a uniform acceleration covers 56 m in 4^{th} second and 90 m in 7^{th} second during its motion. Calculate its initial velocity.

Solution:

Given: $s_{4^{th}} = 56$ m, $s_{7^{th}} = 90$ m

To find: Initial velocity (u)

Formula: $s_{n^{th}} = u + \frac{a}{2}(2n - 1)$

Calculation: Using formula,

$$s_{4^{th}} = u + \frac{a}{2}(2 \times 4 - 1)$$

$$\therefore \quad 56 = u + \frac{a}{2}(8 - 1)$$

$$\therefore \quad 56 = u + \frac{7}{2}a \qquad \text{....(1)}$$

Now, using formula again,

$$\therefore \quad s_{7^{th}} = u + \frac{a}{2}(2 \times 7 - 1)$$

$$= u + \frac{a}{2}(13)$$

$$\therefore \quad 90 = u + \frac{13}{2}a \qquad \text{....(2)}$$

Subtracting equation (1) from equation (2), we get

$$(90 - 56) = 0 + \left(\frac{13}{2}a\right) - \left(\frac{7}{2}a\right)$$

$$\therefore \quad 34 = a\left(\frac{13}{2} - \frac{7}{2}\right) = a\left(\frac{6}{2}\right)$$

$$\therefore \quad 34 = 3a \Rightarrow a = \frac{34}{3}$$

$$\therefore \quad a = 11.33\ \text{m/s}^2$$

Substituting the value of 'a' in equation (1), we get

$$56 = u + \frac{7}{2}(11.33)$$

$\therefore$ $56 = u + 39.655$

$\therefore$ $u = 56 - 39.655$

$\therefore$ $u = \mathbf{16.345\ m/s}$

Ans: The initial velocity of the body was **16.345 m/s**.

1.3.(b) Equations of motion for motion under gravity:

i. *Motion of a body only under the influence of gravity is called* ***motion under gravity or free fall.***

ii. For a body which is falling freely **vertically downward** under gravity towards the earth, equations of motion are as follows:

a. $v = u + gt$ b. $s = ut + \frac{1}{2}gt^2$ c. $v^2 = u^2 + 2gs$

iii. When the body is **thrown vertically upwards** i.e., against gravity, equations of motion are as follows:

a. $v = u - gt$ b. $s = ut - \frac{1}{2}gt^2$ c. $v^2 = u^2 - 2gs$

iv. For a body falling freely, the equation for the distance travelled during n^{th} second after it is dropped is as follows:

$$s_{n^{th}} = u + \frac{1}{2}g\,(2n - 1) = \frac{1}{2}g\,(2n - 1) \qquad \ldots(\because u = 0 \text{ as body starts from rest})$$

v. When a body is **thrown upward** with a velocity u, then the distance travelled by it during n^{th} second is, $s_{n^{th}} = u - \frac{1}{2}g\,(2n - 1)$

where, u = initial velocity,
v = final velocity, t= time,
g = gravitational acceleration = 9.81 m/s^2,
s = distance travelled, t = time in second,
$s_{n^{th}}$ = distance travelled during n^{th} second

Physics behind sky diving…

At the instant when the sky diver jumps out of the plane, he is under free fall and the only force acting on him is his weight. The opposing force of air resistance (called drag) arising as he starts descending goes on increasing. At a certain stage, this force equals sky diver's weight so that the net force acting on him becomes zero and he now falls with a constant velocity called terminal velocity. Now, when he opens the parachute, the drag force increases and slows down the motion of the sky diver till it balances his weight again. Hence, he falls with a slower terminal velocity and can steer his way to a safe landing.

Illustrative Examples:

Example 1

A ball is dropped from the top of a building 490 m high. How long will it take to reach the ground? What will be its velocity when it strikes the ground?

Solution:

Given: $h = 490$ m, $u = 0$ [$\because$ the ball is dropped], $g = 9.81$ m/s^2

To find:
i. Time taken to reach the ground (t)
ii. Velocity of ball when it strikes ground (v)

Formulae: i. $h = ut + \frac{1}{2}gt^2$ ii. $v = u + gt$

Calculation: Using formula (i),

$$490 = 0 + \frac{1}{2} \times 9.81\, t^2$$

$$\therefore \quad t^2 = \frac{490}{4.9} = 100$$

$$\therefore \quad t = \sqrt{100}$$

$\therefore$ t = **10 seconds**

Using formula (ii),

$$v = 0 + 9.81 \times 10$$

$\therefore$ v = **98.1 m/s**

Ans: i. The time taken by the ball to reach the ground is **10 s.**
ii. The velocity of the ball when it strikes the ground is **98.1 m/s**.

Example 2

A body is thrown up with a velocity of 20 m/s. Find the maximum height to which it will reach and time required to reach maximum height.

Solution:

Given: $u = 20$ m/s, $g = 9.81$ m/s^2,
$v = 0$ m/s ($\because$ The body comes to rest at max. height)

To find:
i. Time to reach maximum height (t)
ii. Maximum height reached (h)

Formulae: i. $v = u - gt$ ii. $v^2 = u^2 - 2gh$

Calculation: Using formula (i),

$$0 = 20 - 9.81\, t$$

$$\therefore \quad 9.81\, t = 20$$

$\therefore$ $t = \frac{20}{9.81} = 2.038$ s $\approx$ **2.04 s**

Using formula (ii),

$$0 = (20)^2 - 2 \times 9.81\, h$$

$$\therefore \quad (2 \times 9.81)\, h = (20)^2 \qquad \therefore \quad 19.62\, h = 400$$

$\therefore$ $h = \frac{400}{19.62} = 20.387$ m $\approx$ **20.39 m**

Ans: The maximum height reached by the body is **20.39 m** and the time taken by it to reach this height is **2.04 s**.

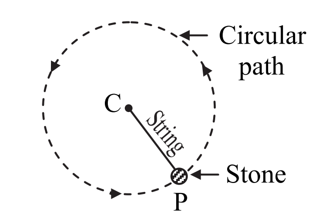
1.4 Circular motion or Angular motion

1.4.(a) Circular motion:

Motion of a particle along the circumference of a circle is called ***circular motion.***

e.g.

i. Motion of a cyclist along a circular path.
ii. Motion of the moon around the earth.
iii. Motion of the earth around the sun.
iv. Motion of the tips of hands of a clock.
v. Motion of the electrons around the nucleus in an atom.
vi. Consider a small piece of stone attached to the end of a string of length CP as shown in the figure. If stone is rotated in a circle, the motion of stone along the circular path is circular motion or angular motion.



Circular motion

1.4.(b) Angular displacement:

i. *Angle traced by the radius vector in a given time when the particle moves from one position to other along the circular path is called as* ***angular displacement.*** It is denoted by letter 'θ'.

ii. **Formula:** Angular displacement $= \dfrac{\text{Length of arc}}{\text{Radius of circle}} = \dfrac{\text{Linear displacement}}{\text{Radius of arc}}$

i.e., $\theta = \dfrac{s}{r}$

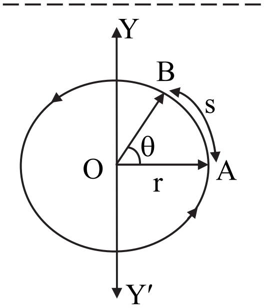
iii. Consider a particle performing circular motion in anticlockwise sense as shown in the figure.

Let, A = initial position of particle at t = 0
B = final position of particle after time t
r = radius of the circle
s = length of arc AB

then, angular displacement in time t $= \theta = \dfrac{AB}{OA} = \dfrac{s}{r}$

iv. **Unit:** S.I. unit is radian or rad.

v. **Direction:** Direction of θ is given by Right hand thumb rule.



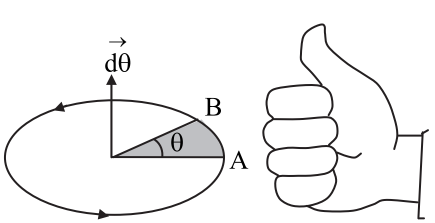
Angular displacement

Knowledge Bank

Right Hand Thumb Rule:

Imagine the axis of rotation to be held in right hand with the fingers curled around it and thumb out-stretched. If the curled fingers give the direction of motion of a particle performing circular motion, then the direction of out-stretched thumb gives the direction of angular displacement vector.

It is perpendicular to the plane of the circle.



Since, angular displacement $(\theta) = \frac{s}{r}$

∴ Linear displacement (s) = r × angular displacement (θ)

vi. If a particle performing circular motion completes one revolution, then angular displacement is given by, $\theta = 360° = 2\pi^c$

where, π^c represents angular displacement in radian.

Note: ***One radian*** *is the angle subtended at the centre of a circle by an arc of length equal to radius of the circle.*

vii. Thus, angular displacement of a rotating body in its N rotations, $\theta = 2\pi N$.

1.4.(c) Angular velocity:

i. ***Angular velocity*** *of a particle performing circular motion is defined as the time rate of change of limiting angular displacement.*

OR

The rate of change of angular displacement with respect to time is called ***angular velocity***. It is denoted by letter 'ω'.

ii. **Formula:** $\omega = \frac{d\theta}{dt}$

where, dθ = change in angular displacement in time dt.

iii. **Unit:** S.I. unit is rad s^{-1} or radian/second.

iv. **Uniform Angular velocity:**

When a body in circular motion undergoes equal angular displacements in equal intervals of time, then it is said that the body rotates with ***uniform angular velocity****. Such a body is said to be in* ***uniform circular motion (U.C.M.)****.*

1.4.(d) Frequency of revolution:

i. *The number of revolutions performed by a particle performing uniform circular motion in one second is called as* ***frequency of revolution***. It is denoted by letter 'f'.

ii. **Formula:** $f = \frac{1}{T} = \frac{1}{\left(\frac{2\pi}{\omega}\right)} = \frac{\omega}{2\pi} = \frac{v}{2\pi r}$

iii. **Unit:** S.I. unit is s^{-1} or hertz (Hz).

1.4.(e) Period of revolution:

i. *The time taken by a particle performing uniform circular motion to complete one revolution is called as* ***period of revolution***. It is denoted by letter 'T'.

ii. **Formula:** $T = \frac{2\pi}{\omega} = \frac{1}{f}$

iii. **Unit:** SI unit is second and C.G.S. unit is also second.

iv. The relations between quantities θ, ω, T and f are:

$$\theta = \omega t = \frac{2\pi t}{T} = 2\pi f t$$

1.4.(f) Relation between linear velocity and angular velocity:

For a particle undergoing uniform circular motion,
Angular displacement is given as,

angular displacement (θ) = $\frac{\text{Linear displacement(s)}}{\text{Radius(r)}} = \frac{s}{r}$ ....(1)

Angular velocity (ω) = $\frac{\text{Angular displacement}(\theta)}{\text{time}(t)} \Rightarrow \theta = \omega t$ ....(2)

From equations (1) and (2), we get,

$\frac{s}{r} = \omega t \Rightarrow \frac{s}{t} = r\omega$

But Linear velocity, $v = \frac{s}{t}$

$\therefore$ $v = r\omega$

Thus, linear velocity is radius times the angular velocity.

1.4.(g) Angular acceleration:

i. *The rate of change of angular velocity with respect to time is called* ***angular acceleration****.* It is denoted by α.

ii. **Formula:** $\alpha = \frac{\omega - \omega_0}{t - t_0}$

where, ω_0 and ω are the angular velocities of a particle performing circular motion at instants t_0 and t respectively.

When $t_0 = 0$, $\alpha = \frac{\omega - \omega_0}{t}$

iii. **Unit:** S.I. unit is rad /s^2

iv. **Uniform angular acceleration:**
If angular velocity of body in circular motion changes by equal amounts in equal intervals of time, then it is said to have ***uniform angular acceleration****.*

Illustrative Examples:

Example 1
Calculate the angular displacement of second hand of a clock in 5 seconds.
Solution:

Given: For second hand of clock, period T = 60 s, t = 5 s

To find: Angular displacement (θ)

Formula: $\theta = \frac{2\pi t}{T}$

Calculation: Using formula,

$\theta = \frac{2 \times 3.142 \times 5}{60}$

$\therefore$ θ = **0.5237 rad**

Ans: The angular displacement of second hand in 5 seconds is **0.5237 rad.**

Example 2

Calculate the angular velocity of the tip of a minute hand.

Solution:

Given: $T = 60$ mins $= 60 \times 60$s $= 3600$s.

To find: Angular velocity (ω)

Formula: $\omega = \dfrac{2\pi}{T}$

Calculation: Using formula,

$$\omega = \frac{2\pi}{3600} = \frac{2(3.142)}{3600}$$

$\therefore$ $\omega = \mathbf{1.744 \times 10^{-3}}$ **rad/s**

Ans: The angular velocity of the tip of the minute hand is $\mathbf{1.744 \times 10^{-3}}$ **rad/s.**

Example 3

A pointer of a clock of length 12 mm is moving with angular velocity of 0.124 rad/s. What is the linear velocity of the tip of the pointer?

Solution:

Given: r = length of the pointer = 12 mm = 12×10^{-3} m, $\omega = 0.124$ rad/s

To find: Linear velocity (v)

Formula: $v = r\omega$

Calculation: Using formula,

$v = 12 \times 10^{-3} \times 0.124 = 0.00148$ m/s

$= \mathbf{1.488 \times 10^{-3}}$ **m/s**

Ans: The linear velocity of tip of pointer is $\mathbf{1.488 \times 10^{-3}}$ **m/s.**

Example 4

In case of uniform circular motion, if radius vector of 60 cm subtends an angle of $\dfrac{\pi}{6}$ radian in 2 s, calculate angular velocity and linear velocity.

Solution:

Given: $r = 60$ cm $= 0.6$ m, $\theta = \dfrac{\pi}{6}$ rad, $t = 2$ second

To find:
i. Angular velocity (ω)
ii. Linear velocity (v)

Formulae: i. $\omega = \dfrac{\theta}{t}$ ii. $v = r\omega$

Calculation: Using formula (i),

$$\omega = \frac{\pi/6}{2} = \frac{\pi}{12} = \frac{3.142}{12}$$

$\therefore$ $\omega = \mathbf{0.262}$ **rad/s**

Using formula (ii),

$v = 0.6 \times 0.262$

$\therefore$ $v = \mathbf{0.1572}$ **m/s**

Ans: i. The angular velocity for the given uniform motion is **0.262 rad/s.**

ii. The linear velocity is **0.1572 m/s.**

Example 5

A motorcycle with 10 cm wheel diameter has an angular velocity of 200 rad/s. Calculate its linear velocity.

Solution:

Given: D = 10 cm $\Rightarrow$ $r = \frac{10}{2} = 5$ cm = 0.05 m, $\omega = 200$ rad/s

To find: Linear velocity (v)

Formula: $v = r\omega$

Calculation: Using formula,

$v = 0.05 \times 200$

$\therefore$ $v = \mathbf{10\ m/s}$

Ans: The linear velocity of the motorcycle is **10 m/s**.

Example 6

What is the angular speed of the minute hand of a clock? If the minute hand is 4 cm long, what is the linear speed of its tip? Also find its frequency of rotation.

Solution:

Given: Length of minute hand, r = 4 cm, T = 60 min = 60 × 60 = 3600s

To find: i. Angular speed (ω) ii. Linear speed (v) iii. Frequency (f)

Formulae: i. $\omega = \frac{2\pi}{T}$ ii. $v = r\omega$ iii. $f = \frac{1}{T}$

Calculation: Using formula (i),

$$\omega = \frac{2 \times 3.142}{3600}$$

$\therefore$ $\omega = \mathbf{1.746 \times 10^{-3}\ rad/s}$

Using formula (ii),

$v = 4 \times 10^{-2} \times 1.746 \times 10^{-3}$

$\therefore$ $v = \mathbf{6.984 \times 10^{-5}\ m/s}$

Using formula (iii),

$$f = \frac{1}{3600} = \mathbf{2.78 \times 10^{-4}\ Hz}$$

Ans: The minute hand of the clock has angular speed **1.746×10^{-3} rad/s,** a linear speed of **6.984×10^{-5} m/s** and a frequency of **2.78×10^{-4} Hz.**

Example 7

A car travelling at 36 km/hr has wheels of 86 cm in diameter. What is the angular speed of wheels about the axis?

Solution:

Given: $v = 36\text{ km/hr} = 36 \times \frac{5}{18}\text{ m/s} = 10\text{ m/s}$,

$D = 86\text{ cm} \Rightarrow r = \frac{D}{2} = \frac{86}{2} = 43\text{ cm} = 0.43\text{ m}$

To find: Angular speed of wheels about the axis (ω)

Formula: $v = r\omega$

Calculation: Using formula,

$$\omega = \frac{v}{r} = \frac{10}{0.43} = 23.2558 \text{ m/s}$$

$\therefore$ $\omega \approx \mathbf{23.26\ rad/s}$

Ans: The angular speed of the wheels about the axis is **23.26 rad/s**.

Example 8

The frequency of a particle performing circular motion changes from 60 r.p.m to 180 r.p.m in 10 seconds. Calculate the angular acceleration.

Solution:

Given: $f_0 = 60 \text{ r.p.m} = \frac{60}{60} \text{r.p.s.} = 1 \text{ rev/s}$, $f = 180 \text{ r.p.m} = \frac{180}{60} \text{ r.p.s.} = 3 \text{ rev/s}$, $t = 10$ s

To find: Angular acceleration (α)

Formula: $\alpha = \dfrac{\omega - \omega_0}{t}$

Calculation: Using formula,

$$\alpha = \frac{2\pi f - 2\pi f_0}{t} = \frac{2\pi(3-1)}{10} \quad \text{....}[\because \omega = 2\pi f]$$

$$= \frac{2 \times 3.142 \times 2}{10}$$

$\therefore$ $\alpha = \mathbf{1.26\ rad/s^2}$

Ans: The angular acceleration of the particle is **1.26 rad/s²**.

1.4.(h) Relation between linear acceleration and angular acceleration:

For a particle undergoing uniform circular motion, the angular acceleration is given as,

$$\alpha = \frac{\omega - \omega_0}{t} \quad \text{....(1)}$$

where, α = Angular acceleration,

ω = final angular velocity = $\frac{v}{r}$, ω_0 = initial angular velocity = $\frac{u}{r}$

Substituting these values in equation (1), we get

$$\alpha = \frac{\left(\frac{v}{r} - \frac{u}{r}\right)}{t} = \frac{v-u}{rt}$$

On rearranging, we get

$$\alpha = \left(\frac{v-u}{t}\right) \times \frac{1}{r}$$

$\therefore$ $\alpha = a \times \dfrac{1}{r}$ $\quad \left[\because a = \text{linear acceleration} = \dfrac{v-u}{t}\right]$

$\therefore$ $a = r \times \alpha$ ....[Relation between linear and angular acceleration.]

where, a = linear acceleration, r = radius, α = angular acceleration

Illustrative Examples:

Example 1

A particle is revolving in a circular path of radius 0.2 m with an angular acceleration of π rad/s^2. Calculate its linear acceleration.

Solution:

Given: $r = 0.2$ m, $\alpha = \pi$ rad/s^2

To find: Linear acceleration (a)

Formula: $a = r\,\alpha$

Calculation: Using formula, we get

$a = 0.2 \times \pi = 0.2 \times 3.142$

$\therefore$ $a = \mathbf{0.6284\ m/s^2}$

Ans: The linear acceleration of the particle is **0.6284 m/s^2**.

Example 2

A vehicle is moving with a velocity of 75 km/hr. The diameter of wheel is 400 mm. Find the angular velocity of the wheel. Also find the angular retardation if the vehicle comes to rest over a distance of 500 m under constant retardation.

Solution:

Given: $v = 75\ \text{km/hr} = 75 \times \dfrac{5}{18}\ \text{m/s} = 20.83\ \text{m/s}$

$d = 400\ \text{mm} = 400 \times 10^{-3}\ \text{m} \Rightarrow r = \dfrac{400 \times 10^{-3}\ \text{m}}{2} = 200 \times 10^{-3}\ \text{m} = 0.2\ \text{m}$

$s = 500$ m, $v = 0$ m/s … ($\because$ vehicle comes to rest)

To find: i. Angular velocity (ω)

ii. Angular retardation

Formulae: i. $v = r\omega$ ii. $v^2 = u^2 + 2as$ iii. $a = r\,\alpha$

Calculation: Using formula (i),

$$\omega = \frac{v}{r} = \frac{20.83}{0.2} = \mathbf{104.15\ rad/s}$$

Using formula (ii),

$$a = \frac{v^2 - u^2}{2s} = \frac{(0)^2 - (20.83)^2}{2 \times 500} = -\,0.433\ \text{m/s}^2$$

The –ve sign indicates retardation.

Using formula (iii),

$$\alpha = \frac{a}{r} = \frac{-0.433}{0.2} = -\,2.165\ \text{rad/s}^2$$

The –ve sign indicates angular retardation of wheel.

$\therefore$ Retardation $= \mathbf{2.165\ rad/s^2}$

Ans: The angular velocity of the wheel is **104.15 rad/s** and its angular retardation is **2.165 rad/s^2**.

1.5 Equations of angular motion with constant angular velocity; Angular distance travelled by a particle in nth second

i. **First equation of angular motion:** $\omega = \omega_0 + \alpha t$

ii. **Second equation of angular motion:** $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

iii. **Third equation of angular motion:** $\omega^2 = \omega_0^2 + 2\alpha\theta$

iv. **Angular distance travelled by a particle in nth second:** $\theta_{n^{th}} = \omega_0 + \frac{\alpha}{2}(2n - 1)$

where, α = uniform angular acceleration
ω_0 = initial angular velocity
ω = final angular velocity
θ_n = angular distance travelled in n second
θ_{n-1} = angular distance travelled in (n – 1) second
$\theta_{n^{th}} = \theta_n - \theta_{n-1}$ = distance travelled in nth second.

Knowledge Bank

A flywheel is a rotating mechanical device that is used to store rotational energy. Flywheels resist changes in rotational speed. Flywheels are used in systems in which continuous supply of energy is lacking. Use of flywheel provides a continuous supply of energy to the system.

Illustrative Examples:

Example 1

A particle, initially at rest, performs circular motion with uniform angular acceleration 0.2 rad/s^2. What speed will it attain in 5 seconds?

Solution:

Given: $\alpha = 0.2$ rad/s^2, t = 5 s, $\omega_0 = 0$ ….($\because$ Particle starts from rest)

To find: Speed attained in 5 s (ω)

Formula: $\omega = \omega_0 + \alpha t$

Calculation: Using formula,

$\omega = 0 + (0.2) \times 5 =$ **1 rad/s**

Ans: The speed attained by the particle in 5 seconds is **1 rad/s.**

Example 2

A flywheel is rotating at 120 r.p.m. It is brought to rest in 10 revolutions. Calculate the uniform retardation.

Solution:

Given: $f_0 = 120$ r.p.m. $= \frac{120}{60} = 2$ r.p.s, N = 10,

f = 0 ($\because$ Flywheel is brought to rest)

To find: Retardation

Formulae: i. $\theta = 2\pi N$ ii. $\omega^2 = \omega_0^2 + 2\alpha\theta$

Calculation: Using formula (i),

$\theta = 2 \times 3.142 \times 10 = 62.84$ rad

Using formula (ii), we get

$$\alpha = \frac{\omega^2 - \omega_0^2}{2\theta}$$

$$\therefore \quad \alpha = \frac{(2\pi f)^2 - (2\pi f_0)^2}{2\theta} \quad \text{....}[\because \omega = 2\pi f]$$

$$= \frac{(2\pi \times 0)^2 - (2 \times 3.142 \times 2)^2}{2 \times 62.84} = \frac{-157.95}{125.68}$$

$= -1.257$ rad/s^2 ....(The –ve sign indicates retardation)

$\therefore$ Retardation = **1.257 rad/s^2**

Ans: The retardation of the flywheel is **1.257 rad/s^2**.

Example 3

A body, starting from rest, is subjected to an acceleration of 0.2 rad/s^2. Calculate its angular displacement in 5th second.

Solution:

Given: $\alpha = 0.2$ rad/s^2, $\omega_0 = 0$ ....[$\because$ The body starts from rest]

To find: Angular displacement in 5th second $\left(\theta_{5^{th}}\right)$

Formula: $\theta_{n^{th}} = \omega_0 + \frac{\alpha}{2}(2n - 1)$

Calculation: Using formula,

$$\theta_{5^{th}} = 0 + \frac{0.2}{2} \times (2 \times 5 - 1) = 0 + \frac{0.2}{2}(9)$$

$\therefore$ $\theta_{5^{th}}$ = **0.9 radian**

Ans: The angular displacement of the body in 5th second is **0.9 radian**.

Formulae

1. **Velocity:** $v = \frac{s}{t}$

   where, s = displacement, t = time

2. **Acceleration:** $a = \frac{v - u}{t}$

   where, u = initial velocity,
   v = final velocity, t = time

3. **Equations of rectilinear motion:**

   i. $v = u + at$

   ii. $s = ut + \frac{1}{2}at^2$

   iii. $v^2 = u^2 + 2as$

   iv. $s_{n^{th}} = u + \frac{a}{2}(2n - 1)$

   where, a = acceleration

   $s_{n^{th}} = s_n - s_{n-1}$ = distance covered by the body in nth second.

4. **Equations of motion for motion under gravity:**

   i. $v = u \pm gt$

   where, g = acceleration due to gravity

   ii. $h = ut \pm \frac{1}{2}gt^2$

   where, h = vertical distance covered by the body

   iii. $v^2 = u^2 \pm 2gh$

20. A particle is rotating at 300 r.p.m. If the radius of rotation is 1.5 m, Calculate linear velocity. [W-10] [2 M]

Solution:

Given: $f = 300 \text{ r.p.m.} = \frac{300}{60} \text{ rev/s} = 5 \text{ r.p.s.}$,

$r = 1.5$ m

To find: Linear velocity (v)

Formulae: i. $\omega = 2\pi f$ ii. $v = r\omega$

Calculation: Using formula (i),

$\omega = 2\pi \times 5$

$= 31.42$ rad/s

Using formula (ii),

$v = 31.42 \times 1.5$

$\therefore$ $v = \mathbf{47.13\ m/s}$

Ans: The linear velocity of the particle is **47.13 m/s**.

---

21. A flywheel rotating at 800 r.p.m. accelerates to 2000 r.p.m. in 10 minute. Calculate the uniform acceleration and angular displacement with given period. [W-07; S-14] [4 M]

Solution:

Given: $f_0 = 800 \text{ r.p.m.} = \frac{800}{60}$ r.p.s.

$\therefore$ $\omega_0 = 2\pi f_0$

$= \frac{800 \times 2 \times 3.142}{60}$ radian/s

$= 83.79$ rad/s,

$f = 2000 \text{ r.p.m.} = \frac{2000}{60}$ r.p.s.

$\therefore$ $\omega = 2\pi f = \frac{2000 \times 2 \times 3.142}{60}$

$= 209.47$ rad/s,

$t = 10 \text{ minute} = 10 \times 60 = 600$ s

To find: i. Uniform acceleration (a)

ii. Angular displacement (θ)

Formulae: i. $\alpha = \frac{\omega - \omega_0}{t}$

ii. $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

Calculation: Using formula (i),

$\alpha = \frac{209.47 - 83.79}{600}$

$\therefore$ $\alpha = \mathbf{0.209\ rad/s^2}$

Using formula (ii),

$\theta = 83.79 \times 600 + \frac{1}{2}(0.209) \times (600)^2$

$= 50274 + 37620$

$\therefore$ $\theta = \mathbf{87{,}894\ rad}$

Ans: The uniform acceleration of the flywheel is **0.209 rad/s²** and its angular displacement is **87,894 rad**.

---

22. A wheel of diameter 3 m increases its speed uniformly from 150 r.p.m. to 300 r.p.m. in 30 seconds. Calculate angular acceleration and linear acceleration. [W-14] [4 M]

Solution:

Given: $d = 3$ m

$\therefore$ $r = \frac{3}{2} = 1.5$ m, $t = 30$ s,

$f_0 = 150 \text{ r.p.m.} = \frac{150}{60}$ r.p.s.

$= 2.5$ r.p.s.

$f = 300 \text{ r.p.m.} = \frac{300}{60}$ r.p.s.

$= 5$ r.p.s.

To find: i. Angular acceleration (α)

ii. Linear acceleration (a)

Formulae: i. $\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{2\pi(f - f_0)}{t}$

ii. $a = r\alpha$

Calculation: Using formula (i),

$\alpha = \frac{2 \times 3.142(5 - 2.5)}{30}$

$= \frac{6.284 \times 2.5}{30}$

$\therefore$ $\alpha = \mathbf{0.524\ rad/s^2}$

Using formula (ii),

$a = 1.5 \times 0.524$

$\therefore$ $a = \mathbf{0.786\ m/s^2}$

Ans: The angular acceleration of the wheel is **0.524 rad/s²** and its linear acceleration is **0.786 m/s²**.

Additional Theory Questions

1. Distinguish between: positive acceleration and negative acceleration.

Ans: *Refer 1.2.(b)*

2. What is meant by motion under gravity?

Ans: *Refer 1.3.(b)*

3. State the relation between linear displacement and angular displacement.

Ans: *Refer 1.4.(b)*

4. State the relation between linear acceleration and angular acceleration.

Ans: *Refer 1.4.(h)*

Problems for Practice

1. A body starts from rest. It accelerates for 8 second at the rate 0.2 m/s^2. Determine its final velocity.

2. A car starts from rest with uniform acceleration of 0.5 m/s^2. Find the distance travelled in 2 min.

3. A railway wagon moving at 36 km/hr attains a velocity of 72 km/hr and covers a distance of 500 m. Find the uniform acceleration of the wagon.

4. A scooter has initial velocity of 2 m/s. It accelerates for 10 s at the rate of 0.2 m/s^2. Determine the final velocity and the distance travelled during this time.

5. A cyclist starts from rest and travels with a uniform acceleration 0.4 m/s^2. Find his displacement during the 5^{th} second.

6. A car moving with constant acceleration travels 40 m and 80 m during 5^{th} second and 10^{th} second of travel respectively. Calculate initial velocity of the car.

7. A stone is dropped from the top of a building 98 m high. Find the time required for the stone to reach the ground.

8. A body is allowed to fall from the top of a building 150 m high. After what time will it hit the ground? What will be its velocity at that time?

9. A motorcycle with 18 cm wheel diameter has an angular velocity of 50 rad/s. Calculate its linear velocity.

10. A wheel 60 cm in diameter turns at 120 r.p.m.:
    i. What is the angular velocity in rad/s?
    ii. What is the linear velocity of a point on the rim of the wheel?

11. A flywheel rotating at 180 r.p.m. slows down to 120 r.p.m. in 10 min. Find
    i. the uniform retardation in rad/s^2,
    ii. the angular displacement within the given period of time.

12. What will be the angular velocity of a pulley starting from rest with an acceleration 0.2 rad/s^2 at the end of 2 min? If the pulley is retarded and comes to rest in 2 min., find the retardation of the pulley.

13. A wheel of diameter 2 m increases its speed uniformly from 60 r.p.m. to 120 r.p.m. in 10 s. Calculate angular acceleration.

14. A wheel starts from rest under uniform acceleration and completes one revolution in 4^{th} second. Find the angular acceleration and angular velocity after 11 s from start.

15. A train crosses a tunnel in 40 seconds. At the entry of tunnel its velocity is 45 km/hr and at the exit of tunnel its velocity becomes 30 km/hr. Find length of the tunnel.

16. A car starts from a station and moves with a constant acceleration. It covers a distance of 19 m during 10^{th} s of its motion. Find the acceleration and velocities after 9^{th} and 10^{th} second from the start.

17. A bus starts from rest from a station P and attains a velocity of 36 km/hr in 4 min. It continues to travel with this velocity for 12 min. and finally comes to rest at station Q after retarding for 2 min. Calculate the distance PQ.

18. A body starting from rest is moving with uniform acceleration. If it gains a velocity of 90 km/hr in 5 second, find its acceleration and distance covered in 7^{th} second.

19. A body is thrown up with a velocity of 30 m/s. Find the maximum height to which it will reach and time required to reach maximum height.

20. A pointer of a clock of length 8 mm is moving with angular velocity of 0.112 rad/s. What is the linear velocity of the tip of the pointer?

21. An automobile travelling at 45 km/hr has wheels of 80 cm in diameter. What is the angular speed of wheels about the axis?

22. A flywheel is rotating at 420 revolution per minute. It is brought to rest in 40 revolutions. Calculate uniform retardation.

23. In case of uniform circular motion, if radius vector of 75 cm subtends an angle of $\frac{\pi}{4}$ radian in 4 s, calculate angular velocity and linear velocity.

24. A car is moving with a velocity of 60 km/hr. The diameter of wheels is 600 mm. Find the angular velocity of the wheel. Also find the angular retardation if the car comes to rest over a distance of 800 m under constant retardation.

25. A motorcycle with 15 cm wheel diameter has an angular velocity of 30 rad/s. Calculate its linear velocity.

26. The speed of truck is reduced from 100 kmph to 60 kmph over a distance of 200 m. Find uniform retardation and distance further travelled before coming to rest.

Answers to Practice Problems

1. 1.6 m/s
2. 3600 m
3. 0.3 m/s^2
4. 4 m/s, 30 m
5. 1.8 m
6. 4 m/s
7. 4.47 s
8. 5.53 s, 54.25 m/s
9. 4.5 m/s
10. 12.57 rad/s, 3.771 m/s
11. 0.0104 rad/s^2, 11291.28 rad
12. 24 rad/s, 0.2 rad/s^2
13. 0.628 rad/s^2
14. 1.795 rad/s^2, 19.745 rad/s
15. 416.8 m
16. 2 m/s^2, 18 m/s, 20 m/s
17. 9600 m
18. 5 m/s^2, 32.5 m
19. 45.87 m, 3.06 s
20. 8.96×10^{-4} m/s
21. 31.25 rad/s
22. 2.985 rad/s^2
23. 0.196 rad/s, 0.147 m/s
24. 5.57 rad/s, 0.579 rad/s^2
25. 4.5 m/s
26. 1.235 m/s^2, 112.5 m