17105

14115

3 Hours / 100 Marks Seat No. | | | | | | | | |

Instructions – (1) All Question are *Compulsory.*

(2) Answer each next main Question on a new page.

(3) Figures to the right indicate full marks.

(4) Assume suitable data, if necessary.

(5) Use of Non-programmable Electronic Pocket Calculator is permissible.

(6) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Attempt any <u>TEN</u> of the following: 20

a) Define :

(i) Transpose matrix

(ii) Orthogonal matrix

b) If $A = \begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ find $|A - B|$

c) If $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -3 \\ -2 & -1 \\ 3 & 1 \end{bmatrix}$ find A.B

d) Resolve into partial fractions : $\dfrac{x+1}{x^2 - x}$

Marks

e) Define :

(i) Compound angle

(ii) Allied angles

f) Show that $\frac{1}{1-\sin\theta}+\frac{1}{1+\sin\theta}=2\sec^2\theta$

g) If $\angle A=30°$ verify that $\sin 3A = 3\sin A - 4\sin^3 A$

h) If $2\sin 50° \cos 70° = \sin A - \sin B$ find A and B.

i) Express as product : $\cos 4\theta + \cos 8\theta$

j) Prove that : $2\tan^{-1}(1/3) = \tan^{-1}(3/4)$

k) Prove that the lines $5x-12y+1=0$ and $10x-24y-1=0$ are parallel to each other.

l) Prove that the lines $2x-3y+1=0$ and $3x+2y-5=0$ are perpendicular to each other.

m) Find x if $\tan^{-1} 1 + \tan^{-1} x = 0$

2. Attempt any FOUR of the following: **16**

a) Find x, using Crammer's Rule :

$x+z=4,\ y+z=2,\ x+y=0$

b) Find y using Crammer's Rule :

$$x-\frac{2}{y}+\frac{2}{z}=6,\ 3x+\frac{4}{y}-\frac{1}{z}=1,\ 4x+\frac{1}{y}-\frac{3}{z}=4$$

c) Solve by determinant method

$$\frac{2}{x-1}+\frac{3}{y-3}=5,\ \frac{3}{x-1}-\frac{4}{y-3}=-1$$

d) Show that $\sin 50° - \sin 70° + \sin 10° = 0$

e) Prove that $\tan 15° + \tan 75° = 4$

f) Show that $\frac{\sin 7x+\sin x}{\cos 5x-\cos 3x}=\sin 2x-\cos 2x\cdot\cot x$

Marks

3. Attempt any <u>FOUR</u> of the following: 16

a) Find the matrix X such that AX = B

where $A = \begin{bmatrix} -3 & -2 & -1 \\ 6 & 4 & 2 \\ 9 & 6 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

b) Find the value of x and y if

$$\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x & 5 & -3 \\ 2 & y & 5 \end{bmatrix} = \begin{bmatrix} 5 & -3 & 7 \\ 7 & 7 & 1 \end{bmatrix}$$

c) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ then show that $A^2 - 4A$ is a scalar matrix.

d) Verify that (AB)C = A(BC) if

$A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$ $C = \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$

e) Prove that $\dfrac{\operatorname{cosec}A}{\operatorname{cosec}A - 1} + \dfrac{\operatorname{cosec}A}{\operatorname{cosec}A + 1} = 2\sec^2 A$

f) In any $\Delta\,ABC$, show that $\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$

4. Attempt any <u>FOUR</u> of the following: 16

a) Verify that $AA^{-1} = I$ if $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

b) Express the following equations in the matrix form and solve them using matrix inversion method :
$x + 3y + 2z = 6,\ 3x - 2y + 5z = 5, 2x - 3y + 6z = 7$

Marks

c) Resolve into partial fractions : $\dfrac{x^3+1}{x^2+2x}$

d) Resolve into partial fractions : $\dfrac{x^2+2x+3}{(x^2+2x+2)\ (x^2+2x+5)}$

e) Resolve into partial fractions : $\dfrac{x^2}{(x+1)\ (x+2)^2}$

f) Resolve into partial fractions : $\dfrac{x^2+1}{x\,(x^2-1)}$

5. Attempt any <u>FOUR</u> of the following: 16

a) Show that $\dfrac{\sin A+\sin 2A+\sin 3A+\sin 4A}{\cos A+\cos 2A+\cos 3A+\cos 4A}=\tan\left(\dfrac{5A}{2}\right)$

b) Prove that $\dfrac{\cot\theta+\operatorname{cosec}\theta-1}{\cot\theta-\operatorname{cosec}\theta+1}=\cot\ (\theta/2)$

c) Show that $\tan^{-1}(1/2)+\tan^{-1}(1/5)+\tan^{-1}(1/8)=\pi/4$

d) Show that $\cos^{-1}(4/5)-\cos^{-1}(12/13)=\cos^{-1}(63/65)$

e) Find the principal value of

(i) $\cos\left(\pi/2-\sin^{-1}1/2\right)$

(ii) $\cos^{-1}(-1/2)-\sin^{-1}(1/2)$

f) $x>0,\ y>0$ prove that $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\dfrac{x+y}{1-xy}\right)$

Marks

6. Attempt any <u>FOUR</u> of the following: 16

a) Find the acute angle between the lines $2x+3y=13$ and $2x-5y+7=0$

b) Find the equation of line passing through the point of intersection of the lines $4x+3y=8$ and $x+y=1$ and parallel to the line $5x-7y=3$.

c) Find the perpendicular distance between the point (3, 2) and the line $4x-6y=5$.

d) If m_1 and m_2 are slopes of the two lines then prove that, the acute angle between two lines is,

$$\theta = \tan^{-1}\left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

e) Find the equation of a line passing through (2, 5) and the point of intersection of $x+y=0$ and $2x-y=9$

f) Find the equation of line which is perpendicular to the line $5x-2y=7$ and passes through the mid-point of the line joining the points (2, 7) and (– 4, 1).