WINTER – 2015 EXAMINATION

MODEL ANSWER

Subject: BASIC MATHEMATICS

Subject Code: 17104

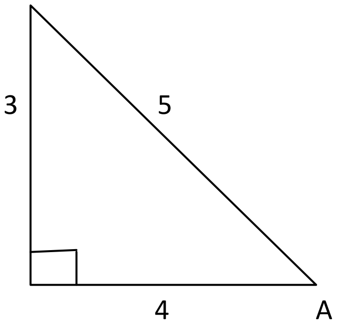
Important Instructions to examiners:

- The model answer shall be the complete solution for each and every question on the question paper.
- Numerical shall be completely solved in a step by step manner along with step marking.
- All alternative solutions shall be offered by the expert along with self-explanatory comments from the expert.
- In case of theoretical answers, the expert has to write the most acceptable answer and offer comments regarding marking scheme to the assessors.
- In should offer the most convincing figures / sketches / circuit diagrams / block diagrams / flow diagrams and offer comments for step marking to the assessors.
- In case of any missing data, the expert shall offer possible assumptions / options and the ensuing solutions along with comments to the assessors for effective assessment.
- In case of questions which are out of the scope of curricular requirement, the expert examiner shall solve the question and mention the marking scheme in the model answer. However, the experts are requested to submit their clear cut opinion about the scope of such question in the paper separately to the coordinator.
- Experts shall cross check the DTP of the final draft of the model answer prepared by them.

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1.		**Attempt any TEN of the following:**		20
	(a)	Find missing term , if $\begin{vmatrix}4&3&9\\3&-2&7\\11&4&--\end{vmatrix}=0$		
	Ans.	Let the missing term be x		
		$\therefore\begin{vmatrix}4&3&9\\3&-2&7\\11&4&x\end{vmatrix}=0$		
		$\therefore 4(-2x-28)-3(3x-77)+9(12+22)=0$	1	
		$\therefore -8x-112-9x+231+306=0$		
		$\therefore -17x+425=0$		
		$\therefore -17x=-425$	½	
		$\therefore x=25$	½	2
	(b)	If $\begin{bmatrix}3&-6\\4&2\end{bmatrix}+\begin{bmatrix}2&3\\-2&1\end{bmatrix}=\begin{bmatrix}a&b\\c&d\end{bmatrix}$, find a,b,c,d		
	Ans.	$\begin{bmatrix}3&-6\\4&2\end{bmatrix}+\begin{bmatrix}2&3\\-2&1\end{bmatrix}=\begin{bmatrix}a&b\\c&d\end{bmatrix}$		
		$\therefore\begin{bmatrix}5&-3\\2&3\end{bmatrix}=\begin{bmatrix}a&b\\c&d\end{bmatrix}$	1	
		$\therefore a=5\ ,\ b=-3\ ,\ c=2\ ,\ d=3$	1	2
	c)	If $A=\begin{bmatrix}3&-1\\2&4\end{bmatrix},B=\begin{bmatrix}1&2\\-3&0\end{bmatrix}$ find X such that $2X+3A-4B=I$		
	Ans.	$2X+3A-4B=I$		
		$2X+3\begin{bmatrix}3&-1\\2&4\end{bmatrix}-4\begin{bmatrix}1&2\\-3&0\end{bmatrix}=\begin{bmatrix}1&0\\0&1\end{bmatrix}$		
		$2X+\begin{bmatrix}9&-3\\6&12\end{bmatrix}-\begin{bmatrix}4&8\\-12&0\end{bmatrix}=\begin{bmatrix}1&0\\0&1\end{bmatrix}$	½	
		$2X+\begin{bmatrix}5&-11\\18&12\end{bmatrix}=\begin{bmatrix}1&0\\0&1\end{bmatrix}$	½	
		$2X=\begin{bmatrix}1&0\\0&1\end{bmatrix}-\begin{bmatrix}5&-11\\18&12\end{bmatrix}$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1.		$2X=\begin{bmatrix}-4 & 11\\ -18 & -11\end{bmatrix}$	½	
		$\therefore X=\frac{1}{2}\begin{bmatrix}-4 & 11\\ -18 & -11\end{bmatrix}$	½	**2**
		--		
	d)	If $A=\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}, B=\begin{bmatrix}1 & 0\\ 3 & -1\end{bmatrix}$, find A^T+B^T and A^T-B^T		
	Ans.	$A^T+B^T=\begin{bmatrix}2 & 3\\ -1 & 4\end{bmatrix}+\begin{bmatrix}1 & 3\\ 0 & -1\end{bmatrix}$	½	
		$A^T+B^T=\begin{bmatrix}3 & 6\\ -1 & 3\end{bmatrix}$	½	
		$A^T-B^T=\begin{bmatrix}2 & 3\\ -1 & 4\end{bmatrix}-\begin{bmatrix}1 & 3\\ 0 & -1\end{bmatrix}$	½	
		$A^T-B^T=\begin{bmatrix}1 & 0\\ -1 & 5\end{bmatrix}$	½	**2**
		--		
	(e)	Resolve into the partial fraction $\frac{1}{x^3+3x^2+2x}$		
	Ans.	$\frac{1}{x^3+3x^2+2x}=\frac{1}{x\left(x+1\right)\left(x+2\right)}$ $\frac{1}{x\left(x+1\right)\left(x+2\right)}=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{x+2}$	½	
		$\therefore 1=\left(x+1\right)\left(x+2\right)A+x\left(x+2\right)B+x\left(x+1\right)C$ Put $x=0$ $1=\left(1\right)\left(2\right)A$ $\therefore A=\frac{1}{2}$	½	
		Put $x=-1$ $1=\left(-1\right)\left(1\right)B$ $\therefore B=-1$	½	
		Put $x=-2$ $1=\left(-2\right)\left(-2+1\right)C$ $C=\frac{1}{2}$	½	**2**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1.	.	$\therefore \frac{1}{x(x+1)(x+2)}=\frac{\frac{1}{2}}{x}+\frac{-1}{x+1}+\frac{\frac{1}{2}}{x+2}$		
		--		
	f) Ans	Prove that $\cos A=\cos^2\left(\frac{A}{2}\right)-\sin^2\left(\frac{A}{2}\right)$		
		$\cos A=\cos\left(\frac{A}{2}+\frac{A}{2}\right)$	½	
		$=\cos\frac{A}{2}\cos\frac{A}{2}-\sin\frac{A}{2}\sin\frac{A}{2}$	1	
		$=\cos^2\left(\frac{A}{2}\right)-\sin^2\left(\frac{A}{2}\right)$	½	**2**
		OR		
		$\cos 2A=\cos^2 A-\sin^2 A$		
		Replace A by $\frac{A}{2}$	½	
		$\cos 2\left(\frac{A}{2}\right)=\cos^2\frac{A}{2}-\sin^2\frac{A}{2}$	1	
		$\cos A=\cos^2\left(\frac{A}{2}\right)-\sin^2\left(\frac{A}{2}\right)$	½	**2**
		--		
	g) Ans.	Without using calculator find the value of $\sin 75^0$		
		$\sin 75^0=\sin\left(45^0+30^0\right)$	½	
		$=\sin 45^0\cos 30^0+\cos 45^0\sin 30^0$	½	
		$=\frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\frac{1}{2}$	1	
		$=\frac{\sqrt{3}+1}{2\sqrt{2}}$		**2**
		--		
	h) Ans	Without using calculator find the value of $\cos(3660)$		
		$\cos(3660)=\cos(3600+60)$		
		$=\cos(40\times 90+60)$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1.		$=\cos\left(60\right)$	½	
		$=\frac{1}{2}$	½	**2**
		--		
	i)	Prove that $\sin\left(A+\frac{\pi}{6}\right)-\sin\left(A-\frac{\pi}{6}\right)=\cos A$		
	Ans.	$\sin\left(A+\frac{\pi}{6}\right)-\sin\left(A-\frac{\pi}{6}\right)$		
		$=\sin A\cos\frac{\pi}{6}+\cos A\sin\frac{\pi}{6}-\sin A\cos\frac{\pi}{6}+\cos A\sin\frac{\pi}{6}$	1	
		$=2\cos A\sin\frac{\pi}{6}$	½	
		$=2\cos A\frac{1}{2}$		
		$=\cos A$	½	**2**
		OR		
		$\sin\left(A+\frac{\pi}{6}\right)-\sin\left(A-\frac{\pi}{6}\right)$		
		$=2\cos\left(\dfrac{A+\frac{\pi}{6}+A-\frac{\pi}{6}}{2}\right)\sin\left(\dfrac{A+\frac{\pi}{6}-A+\frac{\pi}{6}}{2}\right)$	1	
		$=2\cos A\sin\frac{\pi}{6}$	½	
		$=2\cos A\frac{1}{2}$		
		$=\cos A$	½	**2**
		--		
	j)	Prove that $\cos\left[\sin^{-1}\left(\frac{3}{5}\right)\right]=\frac{4}{5}$ 		
	Ans	Put $\sin^{-1}\left(\frac{3}{5}\right)=A$	½	
		$\therefore\sin A=\frac{3}{5}$	½	
		$\therefore\cos A=\frac{4}{5}$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1.		$\therefore \cos\left[\sin^{-1}\left(\frac{3}{5}\right)\right]=\frac{4}{5}$	½	**2**
		OR		
		Put $\sin^{-1}\left(\frac{3}{5}\right)=\theta$	½	
		$\therefore \sin\theta=\frac{3}{5}$	½	
		$\therefore \cos^2\theta=1-\sin^2\theta=1-\left(\frac{3}{5}\right)^2$		
		$\therefore \cos^2\theta=1-\frac{9}{25}=\frac{16}{25}$		
		$\therefore \cos\theta=\frac{4}{5}$	½	
		$\therefore \cos\left[\sin^{-1}\left(\frac{3}{5}\right)\right]=\frac{4}{5}$	½	**2**
	k)	State the condition of two lines are parallel and perpendicular to each other.		
	Ans.	For parallel lines , $m_1=m_2$	1	
		For perpendicular lines , $m_1m_2=-1$	1	**2**
	l)	Calculte the range from the following data:		
	Ans	Weight in kg : 70, 75, 69, 80, 85, 83, 65, 89, 73, 84, 90		
		$\text{Range}=\text{Largest Value}-\text{Smallest value}$		
		$\text{Range}=\text{L}-\text{S}=90-65$	1	
		$=25$	1	**2**
2.		**Attempt any <u>FOUR</u> of the following:**		**16**
	a)	Solve the following equations by Cramer's Rule:		
		$\frac{5}{x+2}+\frac{3}{y+1}=2$, $\frac{10}{x+2}-\frac{3}{y+1}=1$		
	Ans	Let $\frac{1}{x+2}=p$ and $\frac{1}{y+1}=q$		
		$\therefore 5p+3q=2$		
		$10p-3q=1$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2.		$D=\begin{vmatrix}5 & 3\\10 & -3\end{vmatrix}=-15-30=-45$	½	
		$D_p=\begin{vmatrix}2 & 3\\1 & -3\end{vmatrix}=-6-3=-9$	½	
		$D_q=\begin{vmatrix}5 & 2\\10 & 1\end{vmatrix}=5-20=-15$	½	
		$\therefore\ p=\dfrac{D_p}{D}=\dfrac{-9}{-45}=\dfrac{1}{5}$	½	
		$q=\dfrac{D_q}{D}=\dfrac{-15}{-45}=\dfrac{1}{3}$	½	
		But $\dfrac{1}{x+2}=p$		
		$\dfrac{1}{x+2}=\dfrac{1}{5}$		
		$\therefore x+2=5$		
		$x=3$	½	
		$\dfrac{1}{y+1}=q$		
		$\dfrac{1}{y+1}=\dfrac{1}{3}$		
		$y+1=3$		
		$y=2$	½	**4**
		--		
	b)	Find x,y,z if $\begin{bmatrix}2+x & -1 & 3\\0 & y & z\\4 & 1 & 3\end{bmatrix}+\begin{bmatrix}1+x & 2 & 3\\0 & 1+y & 4\\2 & 3 & 5\end{bmatrix}=\begin{bmatrix}6 & 1 & 6\\0 & -1 & 6\\6 & 4 & 8\end{bmatrix}$		
	Ans	$\begin{bmatrix}3+2x & 1 & 6\\0 & 1+2y & z+4\\6 & 4 & 8\end{bmatrix}=\begin{bmatrix}6 & 1 & 6\\0 & -1 & 6\\6 & 4 & 8\end{bmatrix}$	1	
		$\because 3+2x=6$	½	
		$2x=3$	½	
		$\therefore x=\dfrac{3}{2}$		
		$1+2y=-1$	½	
		$2y=-2$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2.		$\therefore y=-1$	½	
		$z+4=6$		
		$\therefore z=2$	½	**4**
		--		
	c)	If $A=\begin{bmatrix}2&1&0\\-1&3&2\end{bmatrix}, B=\begin{bmatrix}1&3\\3&0\\0&1\end{bmatrix}, C=\begin{bmatrix}1&2\\3&-1\end{bmatrix}$ find $(AB)C$		
	Ans	$AB=\begin{bmatrix}2&1&0\\-1&3&2\end{bmatrix}\begin{bmatrix}1&3\\3&0\\0&1\end{bmatrix}$		
		$AB=\begin{bmatrix}2+3+0&6+0+0\\-1+9+0&-3+0+2\end{bmatrix}$	1	
		$AB=\begin{bmatrix}5&6\\8&-1\end{bmatrix}$	1	
		$\therefore (AB)C=\begin{bmatrix}5&6\\8&-1\end{bmatrix}\begin{bmatrix}1&2\\3&-1\end{bmatrix}$		
		$=\begin{bmatrix}5+18&10-6\\8-3&16+1\end{bmatrix}$	1	
		$=\begin{bmatrix}23&4\\5&17\end{bmatrix}$	1	**4**
		--		
	d)	Find inverse of matrix, $\begin{bmatrix}3&-3&4\\2&-3&4\\0&-1&1\end{bmatrix}$		
	Ans	Let $A=\begin{bmatrix}3&-3&4\\2&-3&4\\0&-1&1\end{bmatrix}$		
		$\therefore \lvert A\rvert=\begin{vmatrix}3&-3&4\\2&-3&4\\0&-1&1\end{vmatrix}$		
		$\lvert A\rvert=3(-3+4)+3(2-0)+4(-2-0)$		
		$=3+6-8$	½	
		$\lvert A\rvert=1\neq 0$		
		$\therefore A^{-1}$ exists		

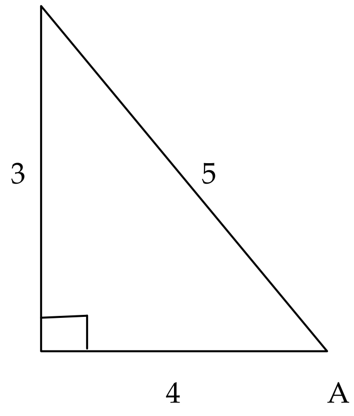
Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2.		Matrix of minors $=\begin{bmatrix} \begin{vmatrix} -3 & 4 \\ -1 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 4 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} 2 & -3 \\ 0 & -1 \end{vmatrix} \\ \begin{vmatrix} -3 & 4 \\ -1 & 1 \end{vmatrix} & \begin{vmatrix} 3 & 4 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} 3 & -3 \\ 0 & -1 \end{vmatrix} \\ \begin{vmatrix} -3 & 4 \\ -3 & 4 \end{vmatrix} & \begin{vmatrix} 3 & 4 \\ 2 & 4 \end{vmatrix} & \begin{vmatrix} 3 & -3 \\ 2 & -3 \end{vmatrix} \end{bmatrix}$		
		$=\begin{bmatrix} 1 & 2 & -2 \\ 1 & 3 & -3 \\ 0 & 4 & -3 \end{bmatrix}$	1	
		Matrix of cofactors $=\begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{bmatrix}$	1	
		$\text{Adj}.A=\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$	1	
		$A^{-1}=\frac{1}{\lvert A\rvert}\text{Adj}.A$		
		$A^{-1}=\frac{1}{1}\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$	½	
		--		**4**
	e)	Resolve into partial fraction $\frac{x^3+1}{x^2+2x}$		
	Ans	$\begin{array}{r} x-2 \\ x^2+2x\overline{\smash{)}x^3+1} \\ x^3+2x^2 \\ - \quad - \\ \hline -2x^2+1 \\ -2x^2-4x \\ + \qquad + \\ \hline 4x+1 \end{array}$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2.		$\therefore \frac{x^3+1}{x^2+2x}=\left(x-2\right)+\frac{4x+1}{x^2+2x}$	1	
		$\therefore \frac{4x+1}{x\left(x+2\right)}=\frac{A}{x}+\frac{B}{x+2}$	½	
		$\therefore 4x+1=\left(x+2\right)A+xB$		
		Put $x=0$		
		$1=2A$		
		$A=\frac{1}{2}$	1	
		Put $x=-2$		
		$-7=-2B$		
		$B=\frac{7}{2}$	1	
		$\therefore \frac{4x+1}{x\left(x+2\right)}=\frac{\frac{1}{2}}{x}+\frac{\frac{7}{2}}{x+2}$		
		$\frac{x^3+1}{x^2+2x}=\left(x-2\right)+\frac{\frac{1}{2}}{x}+\frac{\frac{7}{2}}{x+2}$	½	**4**
		--		
	f)	Resolve into partial fractions $\frac{2x+3}{x^2\left(x-1\right)}$		
	Ans	$\frac{2x+3}{x^2\left(x-1\right)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-1}$	½	
		$2x+3=x\left(x-1\right)A+\left(x-1\right)B+x^2C$		
		Put $x=0$		
		$3=\left(-1\right)B$		
		$\therefore B=-3$	1	
		Put $x=1$		
		$\therefore 5=C$	1	
		Put $x=-1$		
		$\therefore 1=\left(-1\right)\left(-1-1\right)A+\left(-1-1\right)B+\left(-1\right)^2C$		
		$\therefore 1=2A-2B+C$		
		$\therefore 1=2A+6+5$		
		$\therefore -10=2A$		
		$\therefore A=-5$	1	

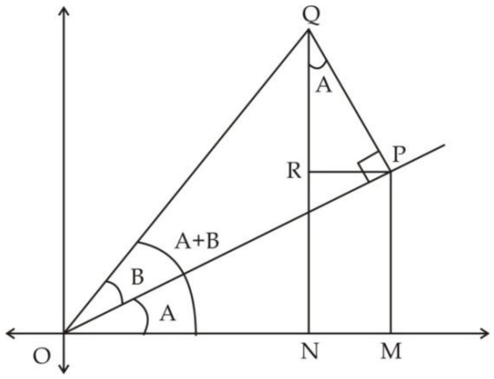
Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2.		$\therefore \frac{2x+3}{x^2\left(x-1\right)}=\frac{-5}{x}+\frac{-3}{x^2}+\frac{5}{x-1}$	½	**4**
		--		
3.		**Attempt any FOUR of the following:**		**16**
	a)	Solve the following equations by using matrix intersection method: $x+3y+2z=6$, $3x-2y+5z=5$, $2x-3y+6z=7$		
	Ans	$Let\ A=\begin{bmatrix}1&3&2\\3&-2&5\\2&-3&6\end{bmatrix}$ $\lvert A\rvert=1\left(-12+15\right)-3\left(18-10\right)+2\left(-9+4\right)$		
		$\lvert A\rvert=3-24-10$ $\therefore \lvert A\rvert=-31\neq 0$ $\therefore A^{-1}$ exists	½	
		$A=\begin{bmatrix}1&3&2\\3&-2&5\\2&-3&6\end{bmatrix}$ Matrix of minors $=\begin{bmatrix}\begin{vmatrix}-2&5\\-3&6\end{vmatrix}&\begin{vmatrix}3&5\\2&6\end{vmatrix}&\begin{vmatrix}3&-2\\2&-3\end{vmatrix}\\\begin{vmatrix}3&2\\-3&6\end{vmatrix}&\begin{vmatrix}1&2\\2&6\end{vmatrix}&\begin{vmatrix}1&3\\2&-3\end{vmatrix}\\\begin{vmatrix}3&2\\-2&5\end{vmatrix}&\begin{vmatrix}1&2\\3&5\end{vmatrix}&\begin{vmatrix}1&3\\3&-2\end{vmatrix}\end{bmatrix}$		
		$=\begin{bmatrix}3&8&-5\\24&2&-9\\19&-1&-11\end{bmatrix}$	½	
		Matrix of cofactors $=\begin{bmatrix}3&-8&-5\\-24&2&9\\19&1&-11\end{bmatrix}$	½	
		$\text{Adj}.A=\begin{bmatrix}3&-24&19\\-8&2&1\\-5&9&-11\end{bmatrix}$	½	
		$A^{-1}=\frac{1}{\lvert A\rvert}\text{Adj}.A$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3.		$A^{-1}=\frac{1}{-31}\begin{bmatrix}3 & -24 & 19\\ -8 & 2 & 1\\ -5 & 9 & -11\end{bmatrix}$	½	
		$\therefore \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{-31}\begin{bmatrix}3 & -24 & 19\\ -8 & 2 & 1\\ -5 & 9 & -11\end{bmatrix}\begin{bmatrix}6\\ 5\\ 7\end{bmatrix}$		
		$\therefore \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{-31}\begin{bmatrix}18-120+133\\ -48+10+7\\ -30+45-77\end{bmatrix}$	½	
		$\therefore \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{-31}\begin{bmatrix}31\\ -31\\ -62\end{bmatrix}$	½	
		$\therefore \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}-1\\ 1\\ 2\end{bmatrix}$		
		$\therefore\ x=-1,\ y=1,\ z=2$	½	**4**
		--		
	b)	Resolve into partial fractions $\frac{x}{x^3+1}$		
	Ans	$\frac{x}{x^3+1}=\frac{x}{(x+1)(x^2-x+1)}$		
		$\therefore \frac{x}{(x+1)(x^2-x+1)}=\frac{A}{x+1}+\frac{Bx+C}{x^2-x+1}$	½	
		$\therefore x=(x^2-x+1)A+(x+1)(Bx+C)$		
		Put $x=-1$		
		$\therefore -1=3A$		
		$\therefore A=\frac{-1}{3}$	1	
		Put $x=0$		
		$0=(1)A+(1)C$		
		$0=\frac{-1}{3}+C$		
		$\therefore C=\frac{1}{3}$	1	
		Put $x=1$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3.		$\therefore 1=(1)A+2(B+C)$		
		$\therefore 1=\frac{-1}{3}+2B+\frac{2}{3}$		
		$\therefore 1-\frac{1}{3}=2B$		
		$\therefore \frac{2}{3}=2B$		
		$\therefore B=\frac{1}{3}$	1	
		$\therefore \frac{x}{(x+1)(x^2-x+1)}=\frac{\frac{-1}{3}}{x+1}+\frac{\frac{1}{3}x+\frac{1}{3}}{x^2-x+1}$	½	**4**
		---		
	c)	Resolve into partial fractions, $\frac{e^x+1}{(e^x+2)(e^x+3)}$		
	Ans	Put $e^x=m$	½	
		$\therefore \frac{m+1}{(m+2)(m+3)}=\frac{A}{m+2}+\frac{B}{m+3}$	½	
		$\therefore m+1=(m+3)A+(m+2)B$		
		Put $m=-2$		
		$\therefore -1=A$	1	
		Put $m=-3$		
		$\therefore -2=(-1)B$		
		$\therefore B=2$	1	
		$\therefore \frac{m+1}{(m+2)(m+3)}=\frac{-1}{m+2}+\frac{2}{m+3}$	½	
		$\therefore \frac{e^x+1}{(e^x+2)(e^x+3)}=\frac{-1}{e^x+2}+\frac{2}{e^x+3}$	½	**4**
		---		
	d)	Prove that $\sin(\pi+\theta)=-\sin\theta$		
	Ans	$\sin(\pi+\theta)=\sin\pi\cos\theta+\cos\pi\sin\theta$	1	
		$=0\cos\theta+(-1)\sin\theta$	2	**4**
		$=-\sin\theta$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3.	e)	Find value of $\frac{\sec^2 135^0}{\cos\left(-240^0\right)-2\sin\left(930^0\right)}$		
	Ans	$\frac{\sec^2 135^0}{\cos\left(-240^0\right)-2\sin\left(930^0\right)}=\frac{\sec^2 135^0}{\cos\left(240^0\right)-2\sin\left(930^0\right)}$		
		$=\frac{\sec^2\left(90^0+45^0\right)}{\cos\left(2\times90^0+60^0\right)-2\sin\left(10\times90^0+30^0\right)}$	1	
		$=\frac{\cos ec^2\left(45^0\right)}{-\cos\left(60^0\right)+2\sin\left(30^0\right)}$	1	
		$=\frac{2}{-\frac{1}{2}+2\frac{1}{2}}=\frac{2}{-\frac{1}{2}+1}$	1	
		$=4$	1	**4**
		---		
	f)	Prove that $\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{27}{11}\right)$		
	Ans	Let $\cos^{-1}\left(\frac{4}{5}\right)=A$		
		$\therefore \cos A=\frac{4}{5}$		
		$\therefore \tan A=\frac{3}{4}$		
		$\therefore A=\tan^{-1}\left(\frac{3}{4}\right)$ 	1	
		$\therefore \cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{3}{5}\right)$		
		$\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{3}{5}\right)$		
		$=\tan^{-1}\left(\frac{\frac{3}{4}+\frac{3}{5}}{1-\frac{3}{4}\frac{3}{5}}\right)$	1	

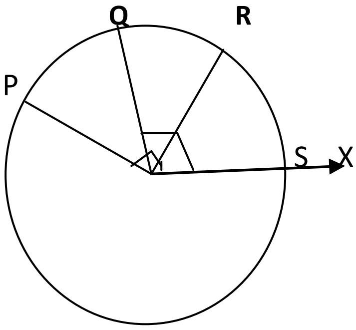
Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3.		$=\tan^{-1}\left(\dfrac{\frac{15+12}{20}}{\frac{20-9}{20}}\right)$	1	
		$=\tan^{-1}\left(\dfrac{27}{11}\right)$	1	**4**
		OR		
		$\text{Let } \cos^{-1}\left(\dfrac{4}{5}\right)=\theta$		
		$\therefore \cos\theta=\dfrac{4}{5} \qquad \therefore \sec\theta=\dfrac{5}{4}$		
		$\tan^2\theta=\sec^2\theta-1$		
		$=\left(\dfrac{5}{4}\right)^2-1$		
		$\tan^2\theta=\dfrac{9}{16}$		
		$\tan\theta=\dfrac{3}{4}$		
		$\therefore \theta=\tan^{-1}\left(\dfrac{3}{4}\right)$	1	
		$\therefore \tan^{-1}\left(\dfrac{3}{4}\right)+\tan^{-1}\left(\dfrac{3}{5}\right)$		
		$=\tan^{-1}\left(\dfrac{\frac{3}{4}+\frac{3}{5}}{1-\frac{3}{4}\frac{3}{5}}\right)$	1	
		$=\tan^{-1}\left(\dfrac{\frac{15+12}{20}}{1-\frac{9}{20}}\right)$		
		$=\tan^{-1}\left(\dfrac{\frac{15+12}{20}}{\frac{20-9}{20}}\right)$	1	
		$=\tan^{-1}\left(\dfrac{27}{11}\right)$	1	**4**
		--		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4.		**Attempt any FOUR of the following:**		**16**
	a)	Prove that $\cos(A+B)=\cos A\cos B-\sin A\sin B$		
	Ans		1	

Right Angled Triangle	**Acute Angle**	**Trigonometric Ratios**
Δ OMP	∠MOP = A	$\sin A=\frac{PM}{OP}$, $\cos A=\frac{OM}{OP}$
Δ OPQ	∠POQ = B	$\sin B=\frac{PQ}{OQ}$, $\cos B=\frac{OP}{OQ}$
Δ PRQ	∠PQR = A	$\sin A=\frac{PR}{PQ}$, $\cos A=\frac{QR}{PQ}$
Δ ONQ	∠NOQ = A+B	$\sin(A+B)=\frac{QN}{OQ}$, $\cos(A+B)=\frac{ON}{OQ}$

Marks: 1

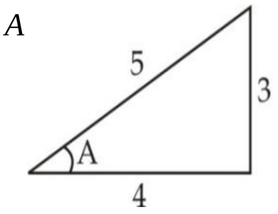
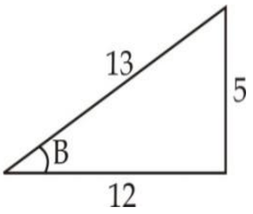
Model answers	Marks	Total Marks
$\therefore\cos(A+B)=\frac{ON}{OQ}$	½	
$=\frac{OM-MN}{OQ}$		
$=\frac{OM-PR}{OQ}$	½	
$=\frac{OM}{OQ}-\frac{PR}{OQ}$		
$=\frac{OM}{OP}\times\frac{OP}{OQ}-\frac{PR}{PQ}\times\frac{PQ}{OQ}$	½	
$=\cos A\cos B-\sin A\sin B.$	½	**4**

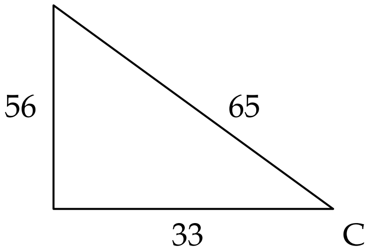
Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4.		**OR**		
				
		Consider a standard unit circle		
		Let P,Q,R,S be points such that	1	
		$\angle XOP = A$, $\angle XOQ = B$, $\angle XOR = A - B$		
		From fig.		
		$\angle POQ = A - B$		
		$\therefore \angle POQ = \angle XOR$ $\quad$ $\text{P}\left(\cos A, \sin A\right)$, $\text{Q}\left(\cos B, \sin B\right)$		
		$R\left(\cos\left(A-B\right), \sin\left(A-B\right)\right)$, $\text{S}\left(1,0\right)$		
		$\therefore$ Chord PQ = Chord RS		
		$\sqrt{\left(\cos A - \cos B\right)^2 + \left(\sin A - \sin B\right)^2} = \sqrt{\left[\cos\left(A-B\right)-1\right]^2 + \left[\sin\left(A-B\right)-0\right]^2}$	1	
		$\left(\cos A - \cos B\right)^2 + \left(\sin A - \sin B\right)^2 = \left[\cos\left(A-B\right)-1\right]^2 + \left[\sin\left(A-B\right)-0\right]^2$		
		$\therefore \cos^2 A + \cos^2 B - 2\cos A\cos B + \sin^2 A + \sin^2 B - 2\sin A\sin B =$		
		$\cos^2\left(A-B\right) + 1 - 2\cos\left(A-B\right) + \sin^2\left(A-B\right)$	1	
		$\therefore 1 + 1 - 2\left(\cos A\cos B + \sin A\sin B\right) = 1 + 1 - 2\cos\left(A-B\right)$		
		$\therefore \cos A\cos B + \sin A\sin B = \cos\left(A-B\right)$	½	
		Replace B by $-B$ in above equation		
		$\therefore \cos A\cos\left(-B\right) + \sin A\sin\left(-B\right) = \cos\left(A-\left(-B\right)\right)$		
		$\therefore \cos A\cos B - \sin A\sin B = \cos\left(A+B\right)$	½	**4**
		---		
	b)	Prove that tanA $\tan\left(60^0 - A\right)\tan\left(60^0 + A\right) = \tan 3A$		
	Ans	tanA $\tan\left(60^0 - A\right)\tan\left(60^0 + A\right)$		
		$= \tan A\left(\frac{\tan 60^0 - \tan A}{1 + \tan 60^0 \tan A}\right)\left(\frac{\tan 60^0 + \tan A}{1 - \tan 60^0 \tan A}\right)$	1	
		$= \tan A\left(\frac{\sqrt{3} - \tan A}{1 + \sqrt{3}\tan A}\right)\left(\frac{\sqrt{3} + \tan A}{1 - \sqrt{3}\tan A}\right)$	1	
		$= \tan A\left(\frac{3 - \tan^2 A}{1 - 3\tan^2 A}\right)$	1	
		$= \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$	½	
		$= \tan 3A$	½	**4**
		OR		
		tanA $\tan\left(60^0 - A\right)\tan\left(60^0 + A\right)$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4.		$= \tan A \dfrac{\sin\left(60^0 - A\right)}{\cos\left(60^0 - A\right)} \dfrac{\sin\left(60^0 + A\right)}{\cos\left(60^0 + A\right)}$		
		$= \tan A \dfrac{\left[\sin 60^0 \cos A - \cos 60^0 \sin A\right]}{\left[\cos 60^0 \cos A + \sin 60^0 \sin A\right]} \dfrac{\left[\sin 60^0 \cos A + \cos 60^0 \sin A\right]}{\left[\cos 60^0 \cos A - \sin 60^0 \sin A\right]}$	1	
		$= \tan A \dfrac{\left[\frac{\sqrt{3}}{2}\cos A - \frac{1}{2}\sin A\right]\left[\frac{\sqrt{3}}{2}\cos A + \frac{1}{2}\sin A\right]}{\left[\frac{1}{2}\cos A + \frac{\sqrt{3}}{2}\sin A\right]\left[\frac{1}{2}\cos A - \frac{\sqrt{3}}{2}\sin A\right]}$	1	
		$= \tan A \dfrac{\left[\frac{3}{4}\cos^2 A - \frac{1}{4}\sin^2 A\right]}{\left[\frac{1}{4}\cos^2 A - \frac{3}{4}\sin^2 A\right]}$		
		$= \tan A \dfrac{\left[3\cos^2 A - \sin^2 A\right]}{\left[\cos^2 A - 3\sin^2 A\right]}$	½	
		$= \tan A \left[\dfrac{3 - \tan^2 A}{1 - 3\tan^2 A}\right]$	½	
		$= \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$	½	
		$= \tan 3A$	½	**4**
		--		
	c)	By using principal value , prove that $\sin^{-1}\left(\dfrac{-1}{\sqrt{2}}\right) + 2\cos^{-1}\left(\dfrac{-1}{\sqrt{2}}\right) + 3\sin^{-1}\left(-1\right) = \dfrac{-\pi}{4}$		
	Ans	$\sin^{-1}\left(\dfrac{-1}{\sqrt{2}}\right) + 2\cos^{-1}\left(\dfrac{-1}{\sqrt{2}}\right) + 3\sin^{-1}\left(-1\right)$		
		$= -\sin^{-1}\dfrac{1}{\sqrt{2}} + 2\left(\pi - \cos^{-1}\dfrac{1}{\sqrt{2}}\right) - 3\sin^{-1} 1$	2	
		$= -\dfrac{\pi}{4} + 2\left(\pi - \dfrac{\pi}{4}\right) - 3\dfrac{\pi}{2}$	1	
		$= -\dfrac{\pi}{4}$	1	**4**
		OR		
		$\therefore \sin^{-1}\left(\dfrac{-1}{\sqrt{2}}\right) + \cos^{-1}\left(\dfrac{-1}{\sqrt{2}}\right) + \cos^{-1}\left(\dfrac{-1}{\sqrt{2}}\right) + 3\sin^{-1}\left(-1\right)$		
		$= \dfrac{\pi}{2} + \pi - \cos^{-1}\left(\dfrac{1}{\sqrt{2}}\right) - 3\sin^{-1}\left(1\right)$	2	

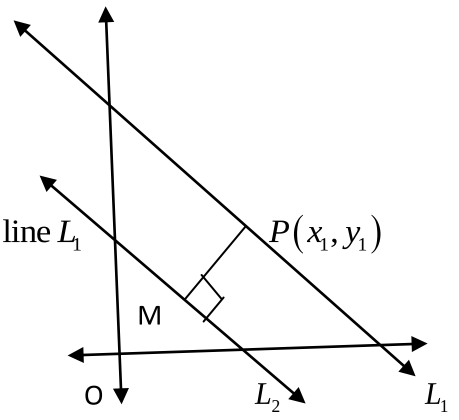
Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4.		$=\frac{\pi}{2}+\pi-\frac{\pi}{4}-3\frac{\pi}{2}$	1	
		$=\frac{3\pi}{2}-\frac{\pi}{4}-\frac{3\pi}{2}$		
		$=\frac{-\pi}{4}$	1	**4**
		--		
	d)	Prove that , (without using calculator) $\sin 20^0 \sin 40^0 \sin 60^0 \sin 80^0 = \frac{3}{16}$		
	Ans	$\sin 20^0 \sin 40^0 \sin 60^0 \sin 80^0$		
		$=\sin 20^0 \sin 40^0 \frac{\sqrt{3}}{2}\sin 80^0$	½	
		$=\frac{\sqrt{3}}{4}\left[2\sin 20^0 \sin 40^0\right]\sin 80^0$		
		$=\frac{\sqrt{3}}{4}\left[\cos\left(-20^0\right)-\cos 60^0\right]\sin 80^0$	1	
		$=\frac{\sqrt{3}}{4}\left[\cos 20^0-\frac{1}{2}\right]\sin 80^0$		
		$=\frac{\sqrt{3}}{4}\left[\cos 20^0 \sin 80^0-\frac{1}{2}\sin 80^0\right]$		
		$=\frac{\sqrt{3}}{4}\left[\frac{1}{2}2\cos 20^0 \sin 80^0-\frac{1}{2}\sin 80^0\right]$	½	
		$=\frac{\sqrt{3}}{8}\left[\sin 100^0-\sin\left(-60\right)-\sin 80^0\right]$	1	
		$=\frac{\sqrt{3}}{8}\left[\sin\left(2\times 90^0-80\right)+\frac{\sqrt{3}}{2}-\sin 80^0\right]$		
		$=\frac{\sqrt{3}}{8}\left[\sin 80^0+\frac{\sqrt{3}}{2}-\sin 80^0\right]$	½	
		$=\frac{3}{16}$	½	**4**
		--		
	e)	Prove that $\cos C-\cos D=-2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$		
	Ans	We know that		
		$\cos\left(A+B\right)-\cos\left(A-B\right)=-2\sin A\sin B$	1	
		Let $A+B=C$		
		$A-B=D$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4.		$\therefore 2A=C+D$		
		$\therefore A=\dfrac{C+D}{2}$	1	
		$\therefore B=\dfrac{C-D}{2}$	1	
		$\cos C-\cos D=-2\sin\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{C-D}{2}\right)$	1	**4**
		--		
	f)	Prove that $\cos^{-1}\left(\dfrac{4}{5}\right)+\cos^{-1}\left(\dfrac{12}{13}\right)=\cos^{-1}\left(\dfrac{33}{65}\right)$		
	Ans	Let $\cos^{-1}\left(\dfrac{4}{5}\right)=A$		
		$\therefore \cos A=\dfrac{4}{5}$		
		$\therefore \sin^2 A=1-\cos^2 A$		
		$=1-\dfrac{16}{25}$		
		$=\dfrac{9}{25}$		
		$\therefore \sin A=\dfrac{3}{5}$	1	
		$\cos^{-1}\left(\dfrac{12}{13}\right)=B$		
		$\therefore \cos B=\dfrac{12}{13}$		
		$\therefore \sin^2 B=1-\cos^2 B$		
		$=1-\dfrac{144}{169}$		
		$=\dfrac{25}{169}$		
		$\therefore \sin B=\dfrac{5}{13}$	1	
		$\therefore \cos\left(A+B\right)=\cos A\cos B-\sin A\sin B$		
		$=\dfrac{4}{5}\dfrac{12}{13}-\dfrac{3}{5}\dfrac{5}{13}$		
		$=\dfrac{48}{65}-\dfrac{15}{65}$		

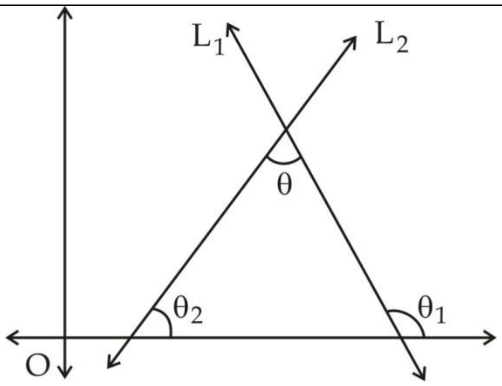
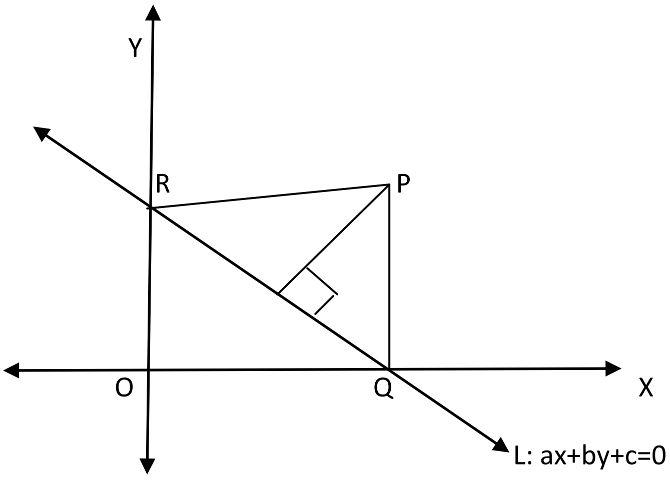
Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4.		$\therefore \cos\left(A+B\right)=\frac{33}{65}$	1	
		$\therefore A+B=\cos^{-1}\left(\frac{33}{65}\right)$		
		$\therefore \cos^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\cos^{-1}\left(\frac{33}{65}\right)$	1	**4**
		OR		
		OR		
		Let $\cos^{-1}\left(\frac{4}{5}\right)=A$		
		  		
		$\therefore \cos A=\frac{4}{5}$		
		$\therefore \tan A=\frac{3}{4}$		
		$B=\tan^{-1}\left(\frac{3}{4}\right)$		
		$\therefore \cos^{-1}\left(\frac{4}{5}\right)=\tan^{-1}\left(\frac{3}{4}\right)$	1	
		$\cos^{-1}\left(\frac{12}{13}\right)=B$		
		$\therefore \cos B=\frac{12}{13}$		
		$\therefore \tan B=\frac{5}{12}$		
		$B=\tan^{-1}\left(\frac{5}{12}\right)$		
		$\therefore \cos^{-1}\left(\frac{12}{13}\right)=\tan^{-1}\left(\frac{5}{12}\right)$	1	
		$L.H.S.=\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{5}{12}\right)$		
		$=\tan^{-1}\left(\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\frac{5}{12}}\right)$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4.		$L.H.S.=\tan^{-1}\left(\dfrac{\frac{36+20}{48}}{1-\frac{15}{48}}\right)$ $=\tan^{-1}\left(\dfrac{\frac{56}{48}}{\frac{48-15}{48}}\right)$ $=\tan^{-1}\left(\dfrac{56}{33}\right)$	½	
		Let $\tan^{-1}\left(\dfrac{56}{33}\right)=C$ $\therefore \tan C=\dfrac{56}{33}$ $\therefore \cos C=\dfrac{33}{65}$ $\therefore C=\cos^{-1}\left(\dfrac{33}{65}\right)$ $\therefore R.H.S.=\cos^{-1}\left(\dfrac{33}{65}\right)$  	1	4
		--		
5.		**Attempt any <u>FOUR</u> of the following:**		**16**
	a)	Prove that $\dfrac{\sec 4\theta-1}{\sec 2\theta-1}=\dfrac{\tan 4\theta}{\tan\theta}$		
	Ans	$\dfrac{\sec 4\theta-1}{\sec 2\theta-1}$ $=\dfrac{\frac{1}{\cos 4\theta}-1}{\frac{1}{\cos 2\theta}-1}$	½	
		$=\dfrac{\cos 2\theta\left(1-\cos 4\theta\right)}{\cos 4\theta\left(1-\cos 2\theta\right)}$ $=\dfrac{2\cos 2\theta\sin^2 2\theta}{2\cos 4\theta\sin^2\theta}$	1	
		$=\dfrac{\sin 4\theta\sin 2\theta}{2\cos 4\theta\sin^2\theta}$	1	
		$=\dfrac{2\tan 4\theta\sin\theta\cos\theta}{2\sin^2\theta}$ $=\tan 4\theta\cot\theta$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5.		$=\frac{\tan 4\theta}{\tan\theta}$	1	**4**
		--		
	b)	Prove that $\frac{\sin\theta-\sin 5\theta+\sin 9\theta-\sin 3\theta}{\cos\theta-\cos 5\theta-\cos 9\theta+\cos 13\theta}=\cot 4\theta$		
	Ans	Consider $\frac{\sin\theta-\sin 5\theta+\sin 9\theta-\sin 13\theta}{\cos\theta-\cos 5\theta-\cos 9\theta+\cos 13\theta}$		
		$=\frac{(\sin\theta-\sin 13\theta)-(\sin 5\theta-\sin 9\theta)}{\cos\theta+\cos 13\theta-(\cos 5\theta+\cos 9\theta)}$		
		$=\frac{2\cos\left(\frac{\theta+13\theta}{2}\right)\sin\left(\frac{\theta-13\theta}{2}\right)-2\cos\left(\frac{5\theta+9\theta}{2}\right)\sin\left(\frac{5\theta-9\theta}{2}\right)}{2\cos\left(\frac{\theta+13\theta}{2}\right)\cos\left(\frac{\theta-13\theta}{2}\right)-2\cos\left(\frac{5\theta+9\theta}{2}\right)\cos\left(\frac{5\theta-9\theta}{2}\right)}$	1	
		$=\frac{2\cos 7\theta\sin(-6\theta)-2\cos 7\theta\sin(-2\theta)}{2\cos 7\theta\cos(-6\theta)-2\cos 7\theta\cos(-2\theta)}$		
		$=\frac{2\cos 7\theta\left[\sin(-6\theta)-\sin(-2\theta)\right]}{2\cos 7\theta\left[\cos(-6\theta)-\cos(-2\theta)\right]}$	½	
		$=\frac{\sin 2\theta-\sin 6\theta}{\cos 6\theta-\cos 2\theta}$	½	
		$=\frac{2\cos\left(\frac{2\theta+6\theta}{2}\right)\sin\left(\frac{2\theta-6\theta}{2}\right)}{-2\sin\left(\frac{6\theta+2\theta}{2}\right)\sin\left(\frac{6\theta-2\theta}{2}\right)}$	1	
		$=\frac{\cos 4\theta\sin(-2\theta)}{-\sin 4\theta\sin 2\theta}=\frac{-\cos 4\theta\sin 2\theta}{-\sin 4\theta\sin 2\theta}$	½	**4**
		$=\cot 4\theta$	½	
		--		
	c)	Prove that $\sin^{-1}x=\cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$		
	Ans	Consider $\cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$		
		Put $x=\sin\theta$		
		$=\cot^{-1}\left(\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}\right)$	1	
		$=\cot^{-1}\left(\frac{\cos\theta}{\sin\theta}\right)$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5.		$=\cot^{-1}\left(\cot\theta\right)$	1	
		$=\theta$		**4**
		$=\sin^{-1}x$	1	
		OR		
		Consider $\cot^{-1}\left(\dfrac{\sqrt{1-x^2}}{x}\right)$		
		Put $x=\cos\theta$		
		$=\cot^{-1}\left(\dfrac{\sqrt{1-\cos^2\theta}}{\cos\theta}\right)$	1	
		$=\cot^{-1}\left(\dfrac{\sin\theta}{\cos\theta}\right)$	1	
		$=\cot^{-1}\left(\tan\theta\right)$		
		$=\cot^{-1}\left(\cot\left(\dfrac{\pi}{2}-\theta\right)\right)$	½	
		$=\dfrac{\pi}{2}-\theta$	½	
		$=\dfrac{\pi}{2}-\cos^{-1}x$	½	
		$=\sin^{-1}x$	½	**4**
		--		
	d)	Prove that distance between two parallel lines $ax+by+c_1=0$ and $ax+by+c_2=0$ is $\left\lvert\dfrac{c_1-c_2}{\sqrt{A^2+B^2}}\right\rvert$		
	Ans			
		$L_1: ax+by+c_1=0$		
		$L_2: ax+by+c_2=0$		
		Let P$\left(x_1,y_1\right)$ be any point on the line L_1		
		$\therefore ax_1+by_1+c_1=0$		
		$\therefore ax_1+by_1=-c_1$	1	
		PM is perpendicular on the line L_2		
		$\therefore PM=\left\lvert\dfrac{ax_1+by_1+c_2}{\sqrt{a^2+b^2}}\right\rvert$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5.		$\therefore PM = \left\|\frac{-c_1+c_2}{\sqrt{a^2+b^2}}\right\|$	1	
		$\therefore PM = \left\|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right\|$	1	**4**
	e)	Find equation of lines passing through $(12,-4)$ and whose sum of the intercept is equal to 10		
	Ans	Let $x-$intercept is a		
		$y-$intercept is b		
		$a+b=10$	½	
		Equation of line is,		
		$\therefore \frac{x}{a}+\frac{y}{b}=1$		
		line passing through $(12,-4)$		
		$\therefore \frac{12}{a}+\frac{(-4)}{10-a}=1$	1	
		$\therefore a^2-26a+120=0$	½	
		$\therefore a=20, a=6$	½	
		$\therefore b=-10, b=4$		
		When $a=20, b=-10$	½	
		Equation of line is,		
		$\therefore \frac{x}{20}-\frac{y}{10}=1$	½	
		$i.e. x-2y=20$		
		When $a=6, b=4$		
		Equation of line is,		
		$\therefore \frac{x}{6}+\frac{y}{4}=1$	½	
		$i.e. 2x+3y=12$		**4**
	f)	If m_1 and m_2 are the slope of two lines then prove that angle between two lines is $\theta=\tan^{-1}\left\|\frac{m_1-m_2}{1+m_1m_2}\right\|$		
	Ans	Let θ_1 = Inclination of L_1		
		θ_2 =Inclination of L_2		
		$\therefore$ Slope of L_1 is $m_1=\tan\theta_1$		
		Slope of L_2 is $m_2=\tan\theta_2$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5.			1	
		$\therefore$ from figure, $\theta=\theta_1-\theta_2$ $\therefore \tan\theta=\tan\left(\theta_1-\theta_2\right)$	1	
		$=\dfrac{\tan\theta_1-\tan\theta_2}{1+\tan\theta_1\tan\theta_2}$ $\therefore \tan\theta=\dfrac{m_1-m_2}{1+m_1\cdot m_2}$	1	
		Since θ is acute $\therefore \tan\theta=\left\|\dfrac{m_1-m_2}{1+m_1\cdot m_2}\right\|$	½	
		$\therefore\theta=\tan^{-1}\left\|\dfrac{m_1-m_2}{1+m_1\cdot m_2}\right\|$	½	4
		--		
6.		**Attempt any FOUR of the following:**		16
	a)	Prove that the length of perpendicular from the point $\mathrm{P}\left(x_1,y_1\right)$ to the line $\mathrm{A}x+By+C=0$ is $\left\|\dfrac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right\|$		
	Ans			

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6.		Let Q$\left(\frac{-C}{A},0\right)$ and R$\left(0,\frac{-C}{B}\right)$		
		$A(\Delta PQR)=\frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ \frac{-C}{A} & 0 & 1\\ 0 & \frac{-C}{B} & 1\end{vmatrix}=\frac{1}{2}\left[x_1\left(0+\frac{C}{B}\right)-y_1\left(\frac{-C}{A}-0\right)+1\left(\frac{C^2}{AB}\right)\right]$		
		$=\frac{1}{2}\left[\frac{x_1C}{B}+\frac{y_1C}{A}+\frac{C^2}{AB}\right]$		
		$=\frac{1}{2}\left[\frac{C}{AB}(Ax_1+By_1+C)\right]$	1	
		$d(QR)=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}=\sqrt{\left(\frac{-C}{A}-0\right)^2+\left(0+\frac{C}{B}\right)^2}$		
		$=\sqrt{\frac{C^2}{A^2}+\frac{C^2}{B^2}}$		
		$=\sqrt{\frac{B^2C^2+A^2C^2}{A^2B^2}}$		
		$=\frac{C}{AB}\sqrt{A^2+B^2}$	1	
		$A(\Delta PQR)=\frac{1}{2}\times d(QR)\times PM$		
		$=\frac{1}{2}\times\frac{C}{AB}\sqrt{A^2+B^2}\times PM$	1	
		$\therefore\frac{1}{2}\frac{C}{AB}(Ax_1+By_1+C)=\frac{1}{2}\frac{C}{AB}\sqrt{A^2+B^2}\times PM$	½	
		$\therefore PM=\left\lvert\frac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right\rvert$ ∵ distance is always positive	½	**4**
		---		
	b)	Find the length of the perpendicular from the point $(2,3)$ on the line $4x-6y-3=0$		
	Ans	$p=\left\lvert\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right\rvert$		
		$=\left\lvert\frac{4(2)+(-6)(3)-3}{\sqrt{(4)^2+(-6)^2}}\right\rvert$	2	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6.		$p = \left\lvert \frac{8-18-3}{\sqrt{16+36}} \right\rvert$ $= \left\lvert \frac{-13}{\sqrt{52}} \right\rvert$ $= \frac{13}{\sqrt{52}}$ $p = \frac{\sqrt{13}}{2}$	2	**4**
		------------------------------------ --		
	c)	Calculate the mean deviation fror mean for the following data:		

Marks	3	4	5	6	7	8
No. of Student	1	3	7	5	2	2

Ans

x_i	f_i	$f_i x_i$	$d_i = x_i - \bar{x}$	$\lvert d_i \rvert$	$f_i \lvert d_i \rvert$
3	1	3	-2.5	2.5	2.5
4	3	12	-1.5	1.5	4.5
5	7	35	-0.5	0.5	3.5
6	5	30	0.5	0.5	2.5
7	2	14	1.5	1.5	3
8	2	16	2.5	2.5	5
	20	110			21

2

Mean $\bar{x} = \frac{\sum f_i x_i}{N} = \frac{110}{20}$

$\bar{x} = 5.5$ — 1

$\text{M.D.} = \frac{\sum f_i \lvert d_i \rvert}{\sum f_i}$

$= \frac{21}{20}$

$= 1.05$ — 1 — **4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6.	d)	Find the standard deviation of the following:		

Class	0-20	20-40	40-60	60-80	80-100
Frequency	20	130	220	70	60

Ans

Class	x_i	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
0-20	10	20	200	100	2000
20-40	30	130	3900	900	117000
40-60	50	220	11000	2500	550000
60-80	70	70	4900	4900	343000
80-100	90	60	5400	8100	486000
		500	25400		1498000

2

$$\text{Mean } \bar{x} = \frac{\sum f_i x_i}{N} = \frac{25400}{500}$$

$$\bar{x} = 50.8$$

1

$$\text{S.D.}\sigma = \sqrt{\frac{\sum f_i x_i^2}{N} - \left(\bar{x}\right)^2}$$

$$= \sqrt{\frac{1498000}{500} - (50.8)^2}$$

$$= \sqrt{2996 - 2580.64}$$

$$= \sqrt{415.36}$$

$$\sigma = 20.38$$

1 **4**

OR

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6.		(table below)	2	

Class	x_i	f_i	$d_i = \frac{x_i - a}{h}$	$f_i d_i$	d_i^2	$f_i d_i^2$
0-20	10	20	-2	-40	4	80
20-40	30	130	-1	-130	1	130
40-60	50	220	0	0	0	0
60-80	70	70	1	70	1	70
80-100	90	60	2	120	4	240
		500		20		520

$$S.D.\sigma = \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2} \times h$$

$$= \sqrt{\frac{520}{500} - \left(\frac{20}{500}\right)^2} \times 20$$ 1

$$= 20.38$$ 1

4

----

e) Find variance from the following series:

Age under	10	20	30	40	50	60	70	80
No.of person	15	30	53	75	100	110	115	125

<table>
<tr><th>Que. No.</th><th>Sub. Que.</th><th>Model Answers</th><th>Marks</th><th>Total Marks</th></tr>
<tr><td>6.</td><td>Ans</td><td>
<table>
<tr><th>Class</th><th>x_i</th><th>f_i</th><th>$f_i x_i$</th><th>x_i^2</th><th>$f_i x_i^2$</th></tr>
<tr><td>0-10</td><td>5</td><td>15</td><td>75</td><td>25</td><td>375</td></tr>
<tr><td>10-20</td><td>15</td><td>15</td><td>225</td><td>225</td><td>3375</td></tr>
<tr><td>20-30</td><td>25</td><td>23</td><td>575</td><td>625</td><td>14375</td></tr>
<tr><td>30-40</td><td>35</td><td>22</td><td>770</td><td>1225</td><td>26950</td></tr>
<tr><td>40-50</td><td>45</td><td>25</td><td>1125</td><td>2025</td><td>50625</td></tr>
<tr><td>50-60</td><td>55</td><td>10</td><td>550</td><td>3025</td><td>30250</td></tr>
<tr><td>60-70</td><td>65</td><td>5</td><td>325</td><td>4225</td><td>21125</td></tr>
<tr><td>70-80</td><td>75</td><td>10</td><td>750</td><td>5625</td><td>56250</td></tr>
<tr><td></td><td></td><td>125</td><td>4395</td><td></td><td>203325</td></tr>
</table>

$$\text{Mean } \bar{x} = \frac{\sum f_i x_i}{N} = \frac{4395}{125}$$
$$\bar{x} = 35.16$$

$$\text{S.D.}\sigma = \sqrt{\frac{\sum f_i x_i^2}{N} - \left(\bar{x}\right)^2}$$
$$= \sqrt{\frac{203325}{125} - \left(35.16\right)^2}$$
$$\sigma = 19.75$$

<u>OR</u>
</td><td>2
1
1</td><td>4</td></tr>
</table>

<table>
<tr><th>Que. No.</th><th>Sub. Que.</th><th>Model Answers</th><th>Marks</th><th>Total Marks</th></tr>
<tr><td>6.</td><td></td><td>
<table>
<tr><th>Class</th><th>x_i</th><th>f_i</th><th>$d_i=\frac{x_i-a}{h}$</th><th>f_id_i</th><th>d_i^2</th><th>$f_id_i^2$</th></tr>
<tr><td>0-10</td><td>5</td><td>15</td><td>-3</td><td>-45</td><td>9</td><td>135</td></tr>
<tr><td>10-20</td><td>15</td><td>15</td><td>-2</td><td>-30</td><td>4</td><td>60</td></tr>
<tr><td>20-30</td><td>25</td><td>23</td><td>-1</td><td>-23</td><td>1</td><td>23</td></tr>
<tr><td>30-40</td><td>35</td><td>22</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>40-50</td><td>45</td><td>25</td><td>1</td><td>25</td><td>1</td><td>25</td></tr>
<tr><td>50-60</td><td>55</td><td>10</td><td>2</td><td>20</td><td>4</td><td>40</td></tr>
<tr><td>60-70</td><td>65</td><td>5</td><td>3</td><td>15</td><td>9</td><td>45</td></tr>
<tr><td>70-80</td><td>75</td><td>10</td><td>4</td><td>40</td><td>16</td><td>160</td></tr>
<tr><td></td><td></td><td>125</td><td></td><td>2</td><td></td><td>488</td></tr>
</table>
</td><td>2</td><td></td></tr>
<tr><td></td><td></td><td>$$S.D.\sigma=\sqrt{\frac{\sum f_id_i^2}{N}-\left(\frac{\sum f_id_i}{N}\right)^2}\times h$$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$$=\sqrt{\frac{488}{125}-\left(\frac{2}{125}\right)^2}\times 10$$</td><td>1</td><td></td></tr>
<tr><td></td><td></td><td>σ =19.75</td><td>1</td><td>4</td></tr>
<tr><td></td><td></td><td>---</td><td></td><td></td></tr>
<tr><td></td><td>f)</td><td>The mean and variance of 5 items are 64 and 68 respectively. If two more of values 62 and 66 are added to the data, find the new variance of 7 items.</td><td></td><td></td></tr>
<tr><td></td><td>Ans</td><td>Given $\bar{x}=64$
Variance $=\sigma^2=68$
$$\bar{x}=\frac{\sum x_i}{n}$$
$$64=\frac{\sum x_i}{5}$$</td><td></td><td></td></tr>
</table>

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6.		$\therefore \sum x_i = 320$	1	
		New $\sum x_i = \sum x_i + 62 + 66$		
		$= 320 + 62 + 66$		
		$= 448$	1	
		New Mean $= \dfrac{\text{New} \sum x_i}{n}$		
		$= \dfrac{448}{7}$		
		$= 64$	½	
		$\sigma^2 = \dfrac{\sum {x_i}^2}{n} - \bar{x}^2$		
		$68 = \dfrac{\sum {x_i}^2}{5} - (64)^2$		
		$\sum {x_i}^2 = 20820$	½	
		New $\sum {x_i}^2 = \sum {x_i}^2 + (62)^2 + (66)^2$		
		$= 20820 + 3844 + 4356$		
		$= 29020$	½	
		$\therefore$ New variance $= \dfrac{\text{New} \sum {x_i}^2}{n} - \bar{x}^2$		
		$= \dfrac{29020}{7} - (64)^2$		**4**
		$= 49.71$	½	

Important Note

In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.
