



SUMMER – 2016 EXAMINATION

MODEL ANSWER

Subject: BASIC MATHEMATICS

Subject Code: 17104

Important Instructions to examiners:

- 1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
- 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
- 3) The language errors such as grammatical, spelling errors should not be given more importance. (Not applicable for subject English and Communication Skills.)
- 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by the candidate and those in the model answer may vary. The examiner may give credit for any equivalent figure drawn.
- 5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and the model answer.
- 6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidate's understanding.
- 7) For programming language papers, credit may be given to any other program based on equivalent concept.



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1.		Attempt any TEN of the following:		20
	(a)	Find x if $\begin{vmatrix} 1 & x & x^2 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{vmatrix} = 0$		
	Ans.	$\begin{vmatrix} 1 & x & x^2 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{vmatrix} = 0$ $\therefore 1(18 - 12) - x(9 - 4) + x^2(3 - 2) = 0$ $\therefore 6 - 5x + x^2 = 0$ $\therefore (x - 3)(x - 2) = 0$ $\therefore x = 3 \text{ or } x = 2$	1 ½ ½	2
(b)	Find the value if a and b If $\begin{bmatrix} a - 4b & 5 \\ 6 & -a + b \end{bmatrix} = \begin{bmatrix} 11 & 5 \\ 6 & -5 \end{bmatrix}$			
Ans.	$\begin{bmatrix} a - 4b & 5 \\ 6 & -a + b \end{bmatrix} = \begin{bmatrix} 11 & 5 \\ 6 & -5 \end{bmatrix}$ $a - 4b = 11$ $-a + b = -5$ $\therefore a = 3, b = -2$	1 1	2	
(c)	If $A = \begin{bmatrix} 3 & 2 \\ 1 & -1 \\ 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -1 \\ 3 & 2 \\ 4 & -2 \end{bmatrix}$ verify that $A + B = B + A$			
Ans.	$A + B = \begin{bmatrix} 3 & 2 \\ 1 & -1 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} -1 & -1 \\ 3 & 2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 1 \\ 4 & 2 \end{bmatrix}$ $B + A = \begin{bmatrix} -1 & -1 \\ 3 & 2 \\ 4 & -2 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ 1 & -1 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 1 \\ 4 & 2 \end{bmatrix}$	1 1	2	



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1.	d)	<p>If $A = \begin{bmatrix} 7 & 0 & 2 \\ 1 & 2 & 6 \\ 4 & 5 & 3 \end{bmatrix}$, find whether matrix A is singular or non-singular.</p>		
	Ans.	<p>$A = \begin{bmatrix} 7 & 0 & 2 \\ 1 & 2 & 6 \\ 4 & 5 & 3 \end{bmatrix}$</p> <p>$A = \begin{vmatrix} 7 & 0 & 2 \\ 1 & 2 & 6 \\ 4 & 5 & 3 \end{vmatrix}$</p> <p>$= 7(6 - 30) - 0(3 - 24) + 2(5 - 8) = -174 \neq 0$</p> <p>$\therefore$ Matrix A is non singular</p> <hr/>	1	2
e)	<p>Resolve into the partial fraction $1 + \frac{1}{x^2 - 1}$</p>			
Ans.	<p>Let $\frac{1}{x^2 - 1} = \frac{1}{(x - 1)(x + 1)}$</p> <p>$\frac{1}{(x - 1)(x + 1)} = \frac{A}{x - 1} + \frac{B}{x + 1}$</p> <p>$\therefore 1 = A(x + 1) + B(x - 1)$</p> <p>put $x = 1$</p> <p>$\therefore 1 = A(1 + 1)$</p> <p>$\therefore A = \frac{1}{2}$</p> <p>put $x = -1$</p> <p>$\therefore 1 = B(-1 - 1)$</p> <p>$\therefore B = -\frac{1}{2}$</p> <p>$\therefore 1 + \frac{1}{x^2 - 1} = 1 + \frac{1}{(x - 1)(x + 1)} = 1 + \frac{1}{2(x - 1)} + \frac{-1}{2(x + 1)}$</p> <hr/>	1/2	1/2	1/2
				2



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1.	f)	Find the value of $\sin^2 60^\circ + \tan^2 45^\circ - \operatorname{cosec}^2 30^\circ$		
	Ans.	$\sin^2 60^\circ + \tan^2 45^\circ - \operatorname{cosec}^2 30^\circ$ $= \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2 - (2)^2$ $= -\frac{9}{4} \text{ or } -2.25$	1 1	2
	g)	Prove that $\sin 2A = 2 \sin A \cos A$		
	Ans.	$\text{L.H.S} = \sin 2A = \sin(A + A)$ $= \sin A \cos A + \cos A \sin A$ $= 2 \sin A \cos A = \text{R.H.S}$	½ 1 ½	2
h)	If $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$, find $\tan(A + B)$			
Ans.	$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ $= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}} = 1$	1 1	2	
i)	Evaluate without using calculator $\frac{\tan 32^\circ + \tan 88^\circ}{1 - \tan 32^\circ \cdot \tan 88^\circ}$			
Ans.	$\frac{\tan 32^\circ + \tan 88^\circ}{1 - \tan 32^\circ \cdot \tan 88^\circ} = \tan(32^\circ + 88^\circ)$ $= \tan 120^\circ$ $= \tan(90^\circ + 30^\circ)$ $= -\cot 30^\circ$ $= -\sqrt{3}$	1 ½ ½	2	



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1.	j)	Find the principal value of $\tan^{-1}(\sqrt{3})$		
	Ans.	$\tan^{-1}(\sqrt{3}) = \theta$ $\sqrt{3} = \tan \theta$ $\text{since } \tan 60^\circ = \sqrt{3}$ $\therefore \theta = 60^\circ$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	2
	k)	Find the angle between the lines $3x + 2y = 6$ and $2x - 3y = 5$		
Ans.	$m_1 = \frac{-3}{2} \quad m_2 = \frac{2}{3}$ $\tan \theta = \left \frac{m_1 - m_2}{1 + m_1 m_2} \right $ $= \left \frac{\frac{-3}{2} - \frac{2}{3}}{1 + \frac{-3}{2} \times \frac{2}{3}} \right = \infty$ $\theta = \tan^{-1} \infty = 90^\circ = \frac{\pi}{2}$	1 $\frac{1}{2}$ $\frac{1}{2}$	2	
l)	Find the range from the following data: 800, 725, 750, 900, 925, 910, 1000, 790, 870, 920			
Ans	Range = Largest Value - Smallest value $= 1000 - 725$ $= 275$		1 1	2
2.		Attempt any <u>FOUR</u> of the following:		16
a)	Solve the following equations by Cramer's rule:			
Ans	$x + y = 3, y + z = 5, z + x = 4$ $D = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 1(1-0) - 1(0-1) + 0(0-1) = 2$		1	



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2.		$D_x = \begin{vmatrix} 3 & 1 & 0 \\ 5 & 1 & 1 \\ 4 & 0 & 1 \end{vmatrix} = 3(1-0) - 1(5-4) + 0 = 2$ $D_y = \begin{vmatrix} 1 & 3 & 0 \\ 0 & 5 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 1(5-4) - 3(0-1) + 0 = 4$ $D_z = \begin{vmatrix} 1 & 1 & 3 \\ 0 & 1 & 5 \\ 1 & 0 & 4 \end{vmatrix} = 1(4-0) - 1(0-5) + 3(0-1) = 6$ $x = \frac{D_x}{D} = \frac{2}{2} = 1$ $y = \frac{D_y}{D} = \frac{4}{2} = 2$ $z = \frac{D_z}{D} = \frac{6}{2} = 3$	1 1 1	4
	b)	<p>If $A = \begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$, verify that $(AB)^T = B^T \cdot A^T$</p> <p>A is 2×2, B is 2×3.</p> <p>Ans $AB = \begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 3 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$</p> $AB = \begin{bmatrix} 6-3 & -2 & 4-3 \\ 3+5 & -1 & 2+5 \end{bmatrix}$ $AB = \begin{bmatrix} 3 & -2 & 1 \\ 8 & -1 & 7 \end{bmatrix} \quad \therefore (AB)^T = \begin{bmatrix} 3 & 8 \\ -2 & -1 \\ 1 & 7 \end{bmatrix}$ $B^T = \begin{bmatrix} 3 & 1 \\ -1 & 0 \\ 2 & 1 \end{bmatrix}, A^T = \begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix}$ $B^T A^T = \begin{bmatrix} 3 & 1 \\ -1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix}$ $B^T A^T = \begin{bmatrix} 6-3 & 3+5 \\ -2 & -1 \\ 4-3 & 2+5 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ -2 & -1 \\ 1 & 7 \end{bmatrix}$	1 1 1	4



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2.	c)	<p>Find inverse of matrix, $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$</p> <p>Ans</p> <p>Let $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$</p> <p>$\therefore A = \begin{vmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{vmatrix}$</p> <p>$A = 1(3+0) - 2(-1-0) - 2(2-0)$</p> <p>$A = 1 \neq 0$</p> <p>$\therefore A^{-1}$ exists</p> <p>Matrix of minors = $\begin{bmatrix} \begin{vmatrix} 3 & 0 \\ -2 & 1 \end{vmatrix} & \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} -1 & 3 \\ 0 & -2 \end{vmatrix} \\ \begin{vmatrix} 2 & -2 \\ -2 & 1 \end{vmatrix} & \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 0 & -2 \end{vmatrix} \\ \begin{vmatrix} 2 & -2 \\ 3 & 0 \end{vmatrix} & \begin{vmatrix} 1 & -2 \\ -1 & 0 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ -1 & 3 \end{vmatrix} \end{bmatrix}$</p> <p>$= \begin{bmatrix} 3 & -1 & 2 \\ -2 & 1 & -2 \\ 6 & -2 & 5 \end{bmatrix}$</p> <p>Matrix of cofactors = $\begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}$</p> <p>$\text{Adj.}A = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$</p> <p>$A^{-1} = \frac{1}{ A } \text{Adj.}A$</p> <p>$A^{-1} = \frac{1}{1} \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$</p>	1	
			1	
			1/2	
			1/2	
			1	4



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2.	d)	<p>If $\left\{ \begin{array}{l} \left[\begin{array}{cc} 3 & 1 \end{array} \right] \left[\begin{array}{cc} 0 & 2 \end{array} \right] \\ 3 \left[\begin{array}{ccc} 4 & 0 & -2 \end{array} \right] - 2 \left[\begin{array}{ccc} -2 & 3 & 3 \end{array} \right] \\ \left[\begin{array}{cc} 3 & -3 \end{array} \right] \left[\begin{array}{cc} -5 & 4 \end{array} \right] \end{array} \right\} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ then find x, y, z</p>		
	Ans	$\left\{ \begin{array}{l} \left[\begin{array}{cc} 9 & 3 \end{array} \right] \left[\begin{array}{cc} 0 & 4 \end{array} \right] \\ \left[\begin{array}{ccc} 12 & 0 & -4 \end{array} \right] - \left[\begin{array}{ccc} -4 & 6 & 6 \end{array} \right] \\ \left[\begin{array}{cc} 9 & -9 \end{array} \right] \left[\begin{array}{cc} -10 & 8 \end{array} \right] \end{array} \right\} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$	1	
		$\left[\begin{array}{cc} 9 & -1 \end{array} \right] \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$	1	
		$\left[\begin{array}{cc} -9 & -2 \end{array} \right] \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$	1	
		$\begin{bmatrix} -11 \\ -28 \\ -53 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$	1	4
		$x = -11, y = -28, z = -53$		
	e)	<p>Resolve into partial fraction $\frac{x-5}{x^3+x^2-6x}$</p>		
	Ans	$\frac{x-5}{x^3+x^2-6x} = \frac{x-5}{x(x^2+x-6)} = \frac{x-5}{x(x-2)(x+3)}$	1/2	
		$\frac{x-5}{x(x-2)(x+3)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+3}$		
		$x-5 = (x-2)(x+3)A + x(x+3)B + x(x-2)C$	1/2	
		<p>Put $x = 0$</p>		
		$-5 = (-2)(3)A$		
		$\therefore A = \frac{5}{6}$		
		<p>Put $x = 2$</p>		
		$-3 = (2)(5)B$	1/2	



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2.		$\therefore B = \frac{-3}{10}$ <p>Put $x = -3$</p> $-8 = (-3)(-5)C$ $\therefore C = -\frac{8}{15}$ $\frac{x-5}{x(x-2)(x+3)} = \frac{5}{x} + \frac{-3}{x-2} + \frac{-8}{x+3}$	1 1 1/2	4
	f) Ans	<p>Resolve into partial fraction $\frac{x^3}{x^2-1}$</p> $x^2-1 \overline{) x^3}$ $\begin{array}{r} x^3 - x \\ - \quad + \\ \hline x \end{array}$ $\therefore \frac{x^3}{x^2-1} = x + \frac{x}{x^2-1}$ $\therefore \frac{x}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1}$ $\therefore x = (x+1)A + (x-1)B$ <p>Put $x = 1$</p> $1 = 2A$ $A = \frac{1}{2}$ <p>Put $x = -1$</p> $-1 = -2B$ $B = \frac{1}{2}$ $\frac{x^3}{x^2-1} = x + \frac{x}{x^2-1} = x + \frac{1}{x-1} + \frac{1}{x+1}$	1 1/2 1 1/2	



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3.	a)	<p>Attempt any <u>FOUR</u> of the following:</p> <p>Solve the equation by inverse matrix method: $3x + y + 2z = 3$, $2x - 3y - z = -3$, $x + 2y + z = 4$</p> <p>Ans</p> $\text{Let } A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$ $ A = 3(-3 + 2) - 1(2 + 1) + 2(4 + 3)$ $ A = -3 - 3 + 14$ $\therefore A = 8 \neq 0$ $\therefore A^{-1} \text{ exists}$ $A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$ $\text{Matrix of minors} = \begin{bmatrix} \begin{vmatrix} -3 & -1 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix} \\ \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} \\ \begin{vmatrix} 1 & 2 \\ -3 & -1 \end{vmatrix} & \begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} 3 & 1 \\ 2 & -3 \end{vmatrix} \end{bmatrix}$ $= \begin{bmatrix} -1 & 3 & 7 \\ -3 & 1 & 5 \\ 5 & -7 & -11 \end{bmatrix}$ $\text{Matrix of cofactors} = \begin{bmatrix} -1 & -3 & 7 \\ 3 & 1 & -5 \\ 5 & 7 & -11 \end{bmatrix}$ $\text{Adj.}A = \begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}$ $A^{-1} = \frac{1}{ A } \text{Adj.}A$	<p>1/2</p> <p>1</p> <p>1/2</p>	16



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3.		$A^{-1} = \frac{1}{8} \begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}$ $\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8} \begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix} \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$ $\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8} \begin{bmatrix} -3 - 9 + 20 \\ -9 - 3 + 28 \\ 21 + 15 - 44 \end{bmatrix}$ $\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 8 \\ 16 \\ -8 \end{bmatrix}$ $\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ <p>$\therefore x = 1, y = 2, z = -1$</p>	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p>	4
	b)	<p>Resolve into partial fractions: $\frac{3x - 2}{(x + 2)(x^2 + 4)}$</p> <p>Ans $\therefore \frac{3x - 2}{(x + 2)(x^2 + 4)} = \frac{A}{x + 2} + \frac{Bx + C}{x^2 + 4}$</p> <p>$\therefore 3x - 2 = (x^2 + 4)A + (x + 2)(Bx + C)$</p> <p>Put $x = -2$</p> <p>$\therefore -8 = 8A$</p> <p>$\therefore A = -1$</p> <p>Put $x = 0$</p> <p>$-2 = (4)A + (2)C$</p> <p>$-2 = -4 + 2C$</p> <p>$\therefore C = 1$</p> <p>Put $x = 1$</p> <p>$\therefore 1 = (5)A + 3(B + C)$</p> <p>$\therefore 1 = -5 + 3B + 3$</p>	<p>$\frac{1}{2}$</p> <p>1</p> <p>1</p>	



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3.		$\therefore 1 - 3 + 5 = 3B$ $\therefore 3 = 3B$ $\therefore B = 1$ $\therefore \frac{3x - 2}{(x + 2)(x^2 + 4)} = \frac{-1}{x + 2} + \frac{x + 1}{x^2 + 4}$ <hr/>	1 1/2	4
	c)	Resolve into partial fractions $\frac{2x + 1}{x^2(x + 1)}$ $\frac{2x + 1}{x^2(x + 1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x + 1}$ $2x + 1 = x(x + 1)A + (x + 1)B + x^2C$ Put $x = 0$ $1 = (1)B$ $\therefore B = 1$ Put $x = -1$ $\therefore -1 = C$ Put $x = 1$ $\therefore 3 = (1)(1 + 1)A + (1 + 1)B + (1)^2C$ $\therefore 3 = 2A + 2B + C$ $\therefore 3 = 2A + 2 - 1$ $\therefore 2 = 2A$ $\therefore A = 1$ $\therefore \frac{2x + 1}{x^2(x + 1)} = \frac{1}{x} + \frac{1}{x^2} + \frac{-1}{x + 1}$ <hr/>	1/2 1 1 1 1/2	
	d)	Given $\tan(A + B) = \frac{3}{4}$, $\tan(A - B) = \frac{77}{36}$, find $\tan 2A$ Let $A + B = x$, $A - B = y$ $x + y = A + B + A - B = 2A$ $\therefore \tan(x + y) = \tan 2A$	1	4



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks	
3.		$\therefore \tan 2A = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$ $= \frac{\frac{3}{4} + \frac{77}{36}}{1 - \frac{3}{4} \times \frac{77}{36}}$ $= -\frac{416}{87} = -4.7816$	1 1 1	4	
	e)	<p>If $A + B = \frac{\pi}{4}$, show that $(1 + \tan A)(1 + \tan B) = 2$</p> <p>Ans Let $A + B = \frac{\pi}{4}$</p> $\therefore \tan(A + B) = \tan \frac{\pi}{4}$ $\therefore \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = 1$ $\tan A + \tan B = 1 - \tan A \cdot \tan B$ $\tan A + \tan B + \tan A \cdot \tan B = 1$ $\tan A(1 + \tan B) + \tan B + 1 = 1 + 1$ $(1 + \tan B)(1 + \tan A) = 2$	1/2 1 1/2 1/2 1 1/2		4
	f)	<p>Prove that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$</p> <p>Ans Let $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right)$</p> $= \tan^{-1}\left\{\frac{\frac{1}{2} + \frac{1}{5}}{1 - \frac{1}{2} \times \frac{1}{5}}\right\} + \tan^{-1}\left(\frac{1}{8}\right)$ $= \tan^{-1}\left(\frac{7}{9}\right) + \tan^{-1}\left(\frac{1}{8}\right)$ $= \tan^{-1}\left\{\frac{\frac{7}{9} + \frac{1}{8}}{1 - \frac{7}{9} \times \frac{1}{8}}\right\} = \tan^{-1}(1) = \frac{\pi}{4}$	1 1 2		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks															
4.		<p>Attempt any <u>FOUR</u> of the following:</p> <p>a) Prove that $\cos(A + B) = \cos A \cos B - \sin A \sin B$</p> <p>Ans.</p> <table border="1" style="width: 100%; margin-top: 10px;"> <thead> <tr> <th style="width: 25%;">Right Angled Triangle</th> <th style="width: 25%;">Acute Angle</th> <th style="width: 50%;">Trigonometric Ratios</th> </tr> </thead> <tbody> <tr> <td>ΔOMP</td> <td>$\angle MOP = A$</td> <td>$\sin A = \frac{PM}{OP}, \cos A = \frac{OM}{OP}$</td> </tr> <tr> <td>$\Delta OPQ$</td> <td>$\angle POQ = B$</td> <td>$\sin B = \frac{PQ}{OQ}, \cos B = \frac{OP}{OQ}$</td> </tr> <tr> <td>$\Delta PRQ$</td> <td>$\angle PQR = A$</td> <td>$\sin A = \frac{PR}{PQ}, \cos A = \frac{QR}{PQ}$</td> </tr> <tr> <td>$\Delta ONQ$</td> <td>$\angle NOQ = A+B$</td> <td>$\sin(A+B) = \frac{QN}{OQ}, \cos(A+B) = \frac{ON}{OQ}$</td> </tr> </tbody> </table> <p style="margin-top: 10px;"> $\therefore \cos(A + B) = \frac{ON}{OQ}$ $= \frac{OM - MN}{OQ}$ $= \frac{OM - PR}{OQ}$ $= \frac{OM}{OQ} - \frac{PR}{OQ}$ $= \frac{OM}{OP} \times \frac{OP}{OQ} - \frac{PR}{PQ} \times \frac{PQ}{OQ}$ $= \cos A \cos B - \sin A \sin B.$ </p>	Right Angled Triangle	Acute Angle	Trigonometric Ratios	ΔOMP	$\angle MOP = A$	$\sin A = \frac{PM}{OP}, \cos A = \frac{OM}{OP}$	ΔOPQ	$\angle POQ = B$	$\sin B = \frac{PQ}{OQ}, \cos B = \frac{OP}{OQ}$	ΔPRQ	$\angle PQR = A$	$\sin A = \frac{PR}{PQ}, \cos A = \frac{QR}{PQ}$	ΔONQ	$\angle NOQ = A+B$	$\sin(A+B) = \frac{QN}{OQ}, \cos(A+B) = \frac{ON}{OQ}$	1	16
Right Angled Triangle	Acute Angle	Trigonometric Ratios																	
ΔOMP	$\angle MOP = A$	$\sin A = \frac{PM}{OP}, \cos A = \frac{OM}{OP}$																	
ΔOPQ	$\angle POQ = B$	$\sin B = \frac{PQ}{OQ}, \cos B = \frac{OP}{OQ}$																	
ΔPRQ	$\angle PQR = A$	$\sin A = \frac{PR}{PQ}, \cos A = \frac{QR}{PQ}$																	
ΔONQ	$\angle NOQ = A+B$	$\sin(A+B) = \frac{QN}{OQ}, \cos(A+B) = \frac{ON}{OQ}$																	
			1																
			$\frac{1}{2}$																
			$\frac{1}{2}$																
			$\frac{1}{2}$																
			$\frac{1}{2}$	4															

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4.		<p>OR</p> <p>Consider a standard unit circle</p> <p>Let P,Q,R,S be points such that</p> <p>$\angle XOP = A$, $\angle XOQ = B$, $\angle XOR = A - B$</p> <p>From fig.</p> <p>$\angle POQ = A - B$</p> <p>$\therefore \angle POQ = \angle XOR$</p> <p style="text-align: center;"> $P(\cos A, \sin A)$, $Q(\cos B, \sin B)$ $R(\cos(A - B), \sin(A - B))$, $S(1, 0)$ </p> <p>\therefore Chord PQ = Chord RS</p> $\sqrt{(\cos A - \cos B)^2 + (\sin A - \sin B)^2} = \sqrt{[\cos(A - B) - 1]^2 + [\sin(A - B) - 0]^2}$ $(\cos A - \cos B)^2 + (\sin A - \sin B)^2 = [\cos(A - B) - 1]^2 + [\sin(A - B) - 0]^2$ <p>$\therefore \cos^2 A + \cos^2 B - 2 \cos A \cos B + \sin^2 A + \sin^2 B - 2 \sin A \sin B =$</p> $\cos^2(A - B) + 1 - 2 \cos(A - B) + \sin^2(A - B)$ <p>$\therefore 1 + 1 - 2(\cos A \cos B + \sin A \sin B) = 1 + 1 - 2 \cos(A - B)$</p> <p>$\therefore \cos A \cos B + \sin A \sin B = \cos(A - B)$</p> <p>Replace B by $-B$ in above equation</p> <p>$\therefore \cos A \cos(-B) + \sin A \sin(-B) = \cos(A - (-B))$</p> <p>$\therefore \cos A \cos B - \sin A \sin B = \cos(A + B)$</p> <hr style="border-top: 1px dashed black;"/> <p>b) Without using calculator prove that</p> <p>$\sin 420^\circ \times \cos 390^\circ + \cos(-300^\circ) \times \sin(-330^\circ) = 1.$</p> <p>Ans.</p> $\sin 420^\circ = \sin[4 \times 90^\circ + 60^\circ] = \sin 60^\circ = \frac{\sqrt{3}}{2}$ $\cos 390^\circ = \cos[4 \times 90^\circ + 30^\circ] = \cos 30^\circ = \frac{\sqrt{3}}{2}$ $\cos(-300^\circ) = \cos 300^\circ \quad \dots \text{ Since } \cos(-\theta) = \cos \theta$ $= \cos[3 \times 90^\circ + 30^\circ]$ $= \sin 30^\circ$ $= \frac{1}{2}$ <hr style="border-top: 1px dashed black;"/>	<p>1</p> <p>1</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>	4

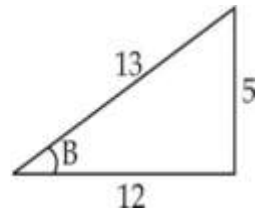
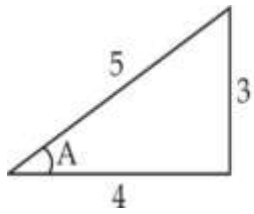


Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4.		$\sin(-330^\circ) = -\sin 330^\circ \quad \dots \text{ Since } \sin(-\theta) = -\sin \theta$ $= -\sin[4 \times 90^\circ - 30^\circ]$ $= -(-\sin 30^\circ)$ $= \frac{1}{2}$ $\text{L.H.S.} = \sin 420^\circ \times \cos 390^\circ + \cos(-300^\circ) \times \sin(-330^\circ)$ $= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ $= 1$ $= \text{R.H.S.}$	<p>1/2</p> <p>1/2</p>	4
	c)	<p>Prove that $\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$</p>		
	Ans.	$\tan 3A = \tan(A + 2A)$ $= \frac{\tan A + \tan 2A}{1 - \tan A \tan 2A}$ $= \frac{\tan A + \frac{2 \tan A}{1 - \tan^2 A}}{1 - \tan A \left(\frac{2 \tan A}{1 - \tan^2 A}\right)}$ $= \frac{\tan A(1 - \tan^2 A) + 2 \tan A}{1 - \tan^2 A}$ $= \frac{1 - \tan^2 A - \tan A(2 \tan A)}{1 - \tan^2 A}$ $= \frac{\tan A - \tan^3 A + 2 \tan A}{1 - \tan^2 A - 2 \tan^2 A}$ $= \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$	<p>1</p> <p>1</p> <p>1/2</p> <p>1</p> <p>1/2</p>	4
	d)	<p>Prove that : $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$</p>		

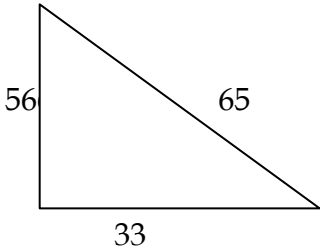


Que. No.	Sub. Que.	Model Answers	Marks	Total Marks	
4.	Ans.	$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}}\right)$ $= \tan^{-1}(1)$ $= \frac{\pi}{4}$	2 1 1	4	
	e) Ans.	<p>Prove that $\frac{\sin 4A + \sin 5A + \sin 6A}{\cos 4A + \cos 5A + \cos 6A} = \tan 5A$</p> $\frac{\sin 4A + \sin 5A + \sin 6A}{\cos 4A + \cos 5A + \cos 6A} = \frac{\sin 4A + \sin 6A + \sin 5A}{\cos 4A + \cos 6A + \cos 5A}$ $= \frac{2 \sin 5A \cos(-A) + \sin 5A}{2 \cos 5A \cos(-A) + \cos 5A}$ $= \frac{\sin 5A [2 \cos(-A) + 1]}{\cos 5A [2 \cos(-A) + 1]}$ $= \tan 5A$	2 1 1		4
	f) Ans.	<p>Prove that $\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$</p> <p>Let $\cos^{-1}\left(\frac{4}{5}\right) = A$</p> $\therefore \cos A = \frac{4}{5}$ $\therefore \sin^2 A = 1 - \cos^2 A$ $= 1 - \frac{16}{25}$ $= \frac{9}{25}$ $\therefore \sin A = \frac{3}{5}$ <p>$\cos^{-1}\left(\frac{12}{13}\right) = B$</p>	1		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4.		$\therefore \cos B = \frac{12}{13}$ $\therefore \sin^2 B = 1 - \cos^2 B$ $= 1 - \frac{144}{169}$ $= \frac{25}{169}$ $\therefore \sin B = \frac{5}{13}$ $\therefore \cos(A + B) = \cos A \cos B - \sin A \sin B$ $= \frac{4}{5} \cdot \frac{12}{13} - \frac{3}{5} \cdot \frac{5}{13}$ $= \frac{48}{65} - \frac{15}{65}$ $\therefore \cos(A + B) = \frac{33}{65}$ $\therefore A + B = \cos^{-1}\left(\frac{33}{65}\right)$ $\therefore \cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$ <p style="text-align: center;"><i>OR</i></p> <p>Let $\cos^{-1}\left(\frac{4}{5}\right) = A$</p> $\therefore \cos A = \frac{4}{5}$ $\therefore \tan A = \frac{3}{4}$ $A = \tan^{-1}\left(\frac{3}{4}\right)$ $\therefore \cos^{-1}\left(\frac{4}{5}\right) = \tan^{-1}\left(\frac{3}{4}\right)$ $\cos^{-1}\left(\frac{12}{13}\right) = B$	<p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>1</p>	<p>4</p>





Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4.		$\therefore \cos B = \frac{12}{13}$ $\therefore \tan B = \frac{5}{12}$ $B = \tan^{-1}\left(\frac{5}{12}\right)$ $\therefore \cos^{-1}\left(\frac{12}{13}\right) = \tan^{-1}\left(\frac{5}{12}\right)$ $L.H.S. = \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{5}{12}\right)$ $= \tan^{-1}\left(\frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \times \frac{5}{12}}\right)$ $= \tan^{-1}\left(\frac{\frac{36 + 20}{48}}{1 - \frac{15}{48}}\right)$ $= \tan^{-1}\left(\frac{\frac{56}{48}}{\frac{48 - 15}{48}}\right)$ $= \tan^{-1}\left(\frac{56}{33}\right)$ <p>Let $\tan^{-1}\left(\frac{56}{33}\right) = C$</p> $\therefore \tan C = \frac{56}{33}$ $\therefore \cos C = \frac{33}{65}$ $\therefore C = \cos^{-1}\left(\frac{33}{65}\right)$ $\therefore R.H.S. = \cos^{-1}\left(\frac{33}{65}\right)$ 	<p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p>	<p>4</p>



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5.	a)	<p>Attempt any <u>FOUR</u> of the following:</p> <p>Without using calculator ,Prove that : $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = \frac{3}{16}$</p>		
	Ans.	$\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ$ $= \sin 20^\circ \sin 40^\circ \frac{\sqrt{3}}{2} \sin 80^\circ$ $= \frac{\sqrt{3}}{4} [2 \sin 20^\circ \sin 40^\circ] \sin 80^\circ$ $= \frac{\sqrt{3}}{4} [\cos(-20^\circ) - \cos 60^\circ] \sin 80^\circ$ $= \frac{\sqrt{3}}{4} \left[\cos 20^\circ - \frac{1}{2} \right] \sin 80^\circ$ $= \frac{\sqrt{3}}{4} \left[\cos 20^\circ \sin 80^\circ - \frac{1}{2} \sin 80^\circ \right]$ $= \frac{\sqrt{3}}{4} \left[\frac{1}{2} 2 \cos 20^\circ \sin 80^\circ - \frac{1}{2} \sin 80^\circ \right]$ $= \frac{\sqrt{3}}{8} [\sin 100^\circ - \sin(-60) - \sin 80^\circ]$ $= \frac{\sqrt{3}}{8} \left[\sin(2 \times 90^\circ - 80) + \frac{\sqrt{3}}{2} - \sin 80^\circ \right]$ $= \frac{\sqrt{3}}{8} \left[\sin 80^\circ + \frac{\sqrt{3}}{2} - \sin 80^\circ \right]$ $= \frac{3}{16}$	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>	4
	b)	<p>Prove that $\sin C + \sin D = 2 \sin \left(\frac{C+D}{2} \right) \cos \left(\frac{C-D}{2} \right)$</p>		
	Ans.	<p>We know that,</p> $2 \sin A \cos B = \sin(A+B) + \sin(A-B)$ <p>Put $A+B = C$ $A-B = D$</p> $\therefore A = \frac{C+D}{2} \quad \text{and} \quad B = \frac{C-D}{2}$ $\therefore \sin C + \sin D = 2 \sin \left(\frac{C+D}{2} \right) \cos \left(\frac{C-D}{2} \right)$	<p>1</p> <p>2</p> <p>1</p>	4

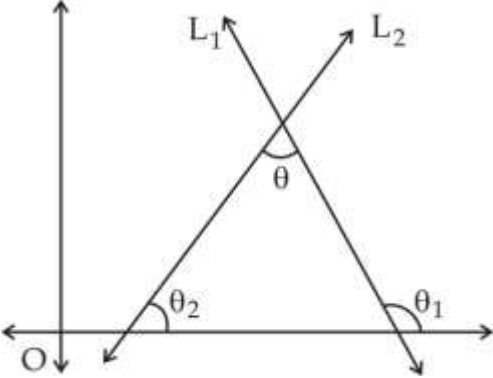


Que. No.	Sub. Que.	Model Answers	Marks	Total Marks	
5.	c) Ans.	<p>Prove that $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, $x > 0, y > 0$ and $xy < 1$</p> <p>Put $\tan^{-1}x = A$ and $\tan^{-1}y = B$</p> <p>$\therefore x = \tan A$ and $y = \tan B$</p> <p>$\therefore \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$</p> <p style="text-align: center;">$= \frac{x+y}{1-xy}$</p> <p>$\therefore A+B = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$</p> <p>$\therefore \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$</p> <p>-----</p>	1 1 1 1/2	4	
	d) Ans.	<p>Find the distance between the lines $3x + 2y = 5$ and $6x + 4y = 6$.</p> <p>Let $L_1 : 3x + 2y = 5$</p> <p>and $L_2 : 6x + 4y = 6 \Rightarrow 3x + 2y = 3$</p> <p>$\therefore L_1 : 3x + 2y - 5 = 0$</p> <p>$\therefore L_2 : 3x + 2y - 3 = 0$</p> <p>$\therefore a = 3, b = 2, c_1 = -5, c_2 = -3$</p> <p>distance between the lines is given by,</p> $d = \left \frac{c_2 - c_1}{\sqrt{a^2 + b^2}} \right $ $= \left \frac{-3 - (-5)}{\sqrt{3^2 + 2^2}} \right $ $= \left \frac{2}{\sqrt{13}} \right = \frac{2}{\sqrt{13}} \text{ units}$ <p>-----</p>	1/2 1 1/2 1 1		4
	e)	<p>Prove that the length of perpendicular on the line $Ax + By + C = 0$ from the point $P(x_1, y_1)$ is $P = \left \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}} \right$</p>			

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5.	Ans.		1/2	
		<p>Let $Q \left(\frac{-C}{A}, 0 \right)$ and $R \left(0, \frac{-C}{B} \right)$</p> $A(\Delta PQR) = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ \frac{-C}{A} & 0 & 1 \\ 0 & \frac{-C}{B} & 1 \end{vmatrix} = \frac{1}{2} \left[x_1 \left(0 + \frac{C}{B} \right) - y_1 \left(\frac{-C}{A} - 0 \right) + 1 \left(\frac{C^2}{AB} \right) \right]$ $= \frac{1}{2} \left[\frac{x_1 C}{B} + \frac{y_1 C}{A} + \frac{C^2}{AB} \right] = \frac{1}{2} \left[\frac{C}{AB} (Ax_1 + By_1 + C) \right]$	1	
		$d(QR) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{\left(\frac{-C}{A} - 0 \right)^2 + \left(0 + \frac{C}{B} \right)^2}$ $= \sqrt{\frac{C^2}{A^2} + \frac{C^2}{B^2}} = \sqrt{\frac{B^2 C^2 + A^2 C^2}{A^2 B^2}}$ $= \frac{C}{AB} \sqrt{A^2 + B^2}$	1/2	
		$A(\Delta PQR) = \frac{1}{2} \times d(QR) \times PM$ $= \frac{1}{2} \times \frac{C}{AB} \sqrt{A^2 + B^2} \times PM$	1/2	
		$\therefore \frac{1}{2} \frac{C}{AB} (Ax_1 + By_1 + C) = \frac{1}{2} \frac{C}{AB} \sqrt{A^2 + B^2} \times PM$ $\therefore PM = \left \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}} \right \quad \because \text{distance is always positive}$	1/2	4



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5.	f)	Find equation of the line passing through the point of intersection of lines $x + y = 0$ and $2x - y = 9$ and point $(4, 5)$.		
	Ans.	$x + y = 0, 2x - y = 9$ $\therefore x + y = 0$ $\quad \underline{2x - y = 9}$ $\therefore 3x = 9$ $\therefore x = 3$ $\quad y = -3$ \therefore point of intersection $= (3, -3) = (x_1, y_1)$ and given point $= (4, 5) = (x_2, y_2)$ its equation in two points form is $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$ $\therefore \frac{y - (-3)}{5 - (-3)} = \frac{x - 3}{4 - 3}$ $\therefore \frac{y + 3}{8} = x - 3$ $\therefore y + 3 = 8(x - 3)$ $\therefore y + 3 = 8x - 24$ $\therefore -8x + y + 27 = 0$ or $8x - y - 27 = 0$	$\frac{1}{2}$	
		OR $x + y = 0, 2x - y = 9$ $\therefore x + y = 0$ $\quad \underline{2x - y = 9}$ $\therefore 3x = 9$ $\therefore x = 3$ $\quad y = -3$ \therefore point of intersection $= (3, -3) = (x_1, y_1)$ and given point $= (4, 5) = (x_2, y_2)$ \therefore slope of line is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-3)}{4 - 3} = 8$ \therefore the equation is, $y - y_1 = m(x - x_1)$ $\therefore y - (-3) = 8(x - 3)$ $\therefore y + 3 = 8x - 24$ $\therefore -8x + y + 27 = 0$ or $8x - y - 27 = 0$	1	
			$\frac{1}{2}$	
			1	
			$\frac{1}{2}$	
			1	
			1	
			1	4

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6	a)	<p>Attempt any <u>FOUR</u> of the following:</p> <p>If m_1 and m_2 are the slope of two lines then prove that angle between two lines is $\theta = \tan^{-1} \left \frac{m_1 - m_2}{1 + m_1 m_2} \right$</p>		16
	Ans.	<p>Let θ_1 = Inclination of L_1 θ_2 =Inclination of L_2 \therefore Slope of L_1 is $m_1 = \tan \theta_1$ Slope of L_2 is $m_2 = \tan \theta_2$</p>  <p>\therefore from figure, $\theta = \theta_1 - \theta_2$ $\therefore \tan \theta = \tan (\theta_1 - \theta_2)$ $= \frac{\tan \theta_1 - \tan \theta_2}{1 + \tan \theta_1 \tan \theta_2}$ $\therefore \tan \theta = \frac{m_1 - m_2}{1 + m_1 \cdot m_2}$</p> <p>Since θ is acute $\therefore \tan \theta = \left \frac{m_1 - m_2}{1 + m_1 \cdot m_2} \right$ $\therefore \theta = \tan^{-1} \left \frac{m_1 - m_2}{1 + m_1 \cdot m_2} \right$</p> <p style="text-align: center;">-----</p>	<p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>	4



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks														
6.	b)	<p>Find the equation of line passing through point of intersection of lines $2x + 3y = 13$, $5x - y = 7$ and perpendicular to $3x - y + 7 = 0$</p> <p>Ans. $2x + 3y = 13$ $5x - y = 7$ $\therefore 2x + 3y = 13$ $\underline{15x - 3y = 21}$ $\therefore 17x = 34$ $\therefore x = 2, y = 3 \quad \therefore \text{Point of intersection} = (2, 3)$</p> <p>Slope of the line $3x - y + 7 = 0$ is, $m_0 = -\frac{a}{b} = -\frac{3}{-1} = 3$</p> <p>$\therefore$ Slope of the required line is, $m = -\frac{1}{m_0} = -\frac{1}{3}$</p> <p>$\therefore y - y_1 = m(x - x_1)$ $y - 3 = -\frac{1}{3}(x - 2)$ $\therefore x + 3y - 11 = 0$</p> <p>-----</p>	1 1 $\frac{1}{2}$ 1 $\frac{1}{2}$	4														
	c)	<p>From the following data ,Calculate range and coefficient of range:</p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse; text-align: center;"> <tr> <td>Marks</td> <td>10-19</td> <td>20-29</td> <td>30-39</td> <td>40-49</td> <td>50-59</td> <td>60-69</td> </tr> <tr> <td>No.of student</td> <td>06</td> <td>10</td> <td>16</td> <td>14</td> <td>08</td> <td>04</td> </tr> </table>	Marks	10-19	20-29	30-39	40-49	50-59	60-69	No.of student	06	10	16	14	08	04		
Marks	10-19	20-29	30-39	40-49	50-59	60-69												
No.of student	06	10	16	14	08	04												
	Ans.	<p>Range=Upper boundary of the last class – lower boundary of first class</p> <p style="margin-left: 40px;">$= 69.5 - 9.5$ $= 60$</p> <p style="text-align: center;">coefficient of range = $\frac{\text{Range}}{\text{sum of the highest and lowest value}}$</p> <p style="margin-left: 40px;">$= \frac{60}{69.5 + 9.5}$ $= \frac{60}{79}$ or 0.759</p>	1 1 1 1	4														



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks																																																						
6.	d)	<p>Find the mean deviation from median of the following distribution:</p> <table border="1"> <tr> <td>Weight (in gms)</td> <td>10-15</td> <td>15-20</td> <td>20-25</td> <td>25-30</td> <td>30-35</td> <td>35-40</td> <td>40-45</td> </tr> <tr> <td>No. of items</td> <td>07</td> <td>12</td> <td>16</td> <td>25</td> <td>19</td> <td>15</td> <td>06</td> </tr> </table>	Weight (in gms)	10-15	15-20	20-25	25-30	30-35	35-40	40-45	No. of items	07	12	16	25	19	15	06																																								
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No. of items	07	12	16	25	19	15	06																																																			
	Ans.	<table border="1"> <thead> <tr> <th>Class</th> <th>x_i</th> <th>f_i</th> <th>c.f</th> <th>$D_i = x_i - M$</th> <th>$f_i D_i$</th> </tr> </thead> <tbody> <tr> <td>10-15</td> <td>12.5</td> <td>07</td> <td>07</td> <td>15.5</td> <td>108.5</td> </tr> <tr> <td>15-20</td> <td>17.5</td> <td>12</td> <td>19</td> <td>10.5</td> <td>126</td> </tr> <tr> <td>20-25</td> <td>22.5</td> <td>16</td> <td>35</td> <td>5.5</td> <td>88</td> </tr> <tr> <td>25-30</td> <td>27.5</td> <td>25</td> <td>60</td> <td>0.5</td> <td>12.5</td> </tr> <tr> <td>30-35</td> <td>32.5</td> <td>19</td> <td>79</td> <td>4.5</td> <td>85.5</td> </tr> <tr> <td>35-40</td> <td>37.5</td> <td>15</td> <td>94</td> <td>9.5</td> <td>142.5</td> </tr> <tr> <td>40-45</td> <td>42.5</td> <td>06</td> <td>100</td> <td>14.5</td> <td>87</td> </tr> <tr> <td>Total</td> <td></td> <td>100</td> <td></td> <td></td> <td>650</td> </tr> </tbody> </table> $Median = L + \frac{\frac{N}{2} - c.f}{f} \times h$ $= 25 + \frac{50 - 35}{25} \times 5$ $= 28$ $M.D = \frac{\sum f_i D_i}{N}$ $= \frac{650}{100} = 6.5$	Class	x_i	f_i	c.f	$D_i = x_i - M $	$f_i D_i$	10-15	12.5	07	07	15.5	108.5	15-20	17.5	12	19	10.5	126	20-25	22.5	16	35	5.5	88	25-30	27.5	25	60	0.5	12.5	30-35	32.5	19	79	4.5	85.5	35-40	37.5	15	94	9.5	142.5	40-45	42.5	06	100	14.5	87	Total		100			650	2	
Class	x_i	f_i	c.f	$D_i = x_i - M $	$f_i D_i$																																																					
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25-30	27.5	25	60	0.5	12.5																																																					
30-35	32.5	19	79	4.5	85.5																																																					
35-40	37.5	15	94	9.5	142.5																																																					
40-45	42.5	06	100	14.5	87																																																					
Total		100			650																																																					
			1																																																							
			1	4																																																						
	e)	<p>In two factories A and B, engaged in the same area of the industry, the average weekly wages (in Rs.) and the S.D are as below:</p> <table border="1"> <thead> <tr> <th>Factory</th> <th>Average wages</th> <th>S.D</th> </tr> </thead> <tbody> <tr> <td>A</td> <td>34.5</td> <td>5.0</td> </tr> <tr> <td>B</td> <td>28.5</td> <td>4.5</td> </tr> </tbody> </table>	Factory	Average wages	S.D	A	34.5	5.0	B	28.5	4.5																																															
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Que. No.	Sub. Que.	Model Answers	Marks	Total Marks																																										
6.	Ans.	<p>Which Factory A or B has greater variability in individual wages ?</p> <p>For Factory A</p> $C.V_A = \frac{S.D}{x} \times 100$ $= \frac{5.0}{34.5} \times 100 = 14.49$ <p>For Factory B</p> $C.V_B = \frac{S.D}{x} \times 100$ $= \frac{4.5}{28.5} \times 100 = 15.79$ $C.V_A < C.V_B$ <p>∴ Factory B has greater variability</p>	<p>½</p> <p>1</p> <p>½</p> <p>1</p> <p>1</p>	4																																										
	f)	<p>Calculate the mean and standard deviation of the following frequency distribution:</p> <table border="1" style="width: 100%; text-align: center;"> <thead> <tr> <th>Class Interval</th> <th>0-10</th> <th>10-20</th> <th>20-30</th> <th>30-40</th> <th>40-50</th> </tr> </thead> <tbody> <tr> <td>Frequency</td> <td>14</td> <td>23</td> <td>27</td> <td>21</td> <td>15</td> </tr> </tbody> </table>	Class Interval		0-10	10-20	20-30	30-40	40-50	Frequency	14	23	27	21	15																															
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		$= \frac{2500}{100} = 25$ $S.D = \sqrt{\frac{\sum f_i x_i^2}{N} - (\bar{x})^2}$ $= \sqrt{\frac{78500}{100} - (25)^2} = 12.65$	1	4																																																	
		<p style="text-align: center;">OR</p> <table border="1"> <thead> <tr> <th>C.I</th> <th>x_i</th> <th>f_i</th> <th>d_i</th> <th>d_i^2</th> <th>$f_i d_i$</th> <th>$f_i d_i^2$</th> </tr> </thead> <tbody> <tr> <td>0-10</td> <td>5</td> <td>14</td> <td>-2</td> <td>4</td> <td>-28</td> <td>56</td> </tr> <tr> <td>10-20</td> <td>15</td> <td>23</td> <td>-1</td> <td>1</td> <td>-23</td> <td>23</td> </tr> <tr> <td>20-30</td> <td>25</td> <td>27</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>30-40</td> <td>35</td> <td>21</td> <td>1</td> <td>1</td> <td>21</td> <td>21</td> </tr> <tr> <td>40-50</td> <td>45</td> <td>15</td> <td>2</td> <td>4</td> <td>30</td> <td>60</td> </tr> <tr> <td>Total</td> <td></td> <td>100</td> <td></td> <td></td> <td>00</td> <td>160</td> </tr> </tbody> </table>	C.I		x_i	f_i	d_i	d_i^2	$f_i d_i$	$f_i d_i^2$	0-10	5	14	-2	4	-28	56	10-20	15	23	-1	1	-23	23	20-30	25	27	0	0	0	0	30-40	35	21	1	1	21	21	40-50	45	15	2	4	30	60	Total		100			00	160	1
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		$\bar{x} = A + \frac{\sum f_i d_i}{N} \times h = 25 + \frac{0}{100} \times 10 = 25$ $S.D = \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2} \times h$ $= \sqrt{\frac{160}{100} - \left(\frac{0}{100}\right)^2} \times 10 = 12.65$	2																																																		
		<p style="text-align: center;"><u>Important Note</u></p> <p><i>In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.</i></p>	1																																																		
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