



SUMMER- 18 EXAMINATION

Subject Name: Applied Mathematics

Model Answer

Subject Code:

22210

Important Instructions to examiners:

- 1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
- 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
- 3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills).
- 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
- 5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
- 6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
- 7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.		Solve any <u>FIVE</u> of the following:	10
	a)	If $f(x) = 64^x + \log_3 x$, find $f\left(\frac{1}{3}\right)$	02
	Ans	$f\left(\frac{1}{3}\right) = (64)^{\frac{1}{3}} + \log_3\left(\frac{1}{3}\right)$	½
		$\therefore f\left(\frac{1}{3}\right) = 4 - \log_3 3$	½
		$\therefore f\left(\frac{1}{3}\right) = 4 - 1$	½
		$\therefore f\left(\frac{1}{3}\right) = 3$	½
	b)	If $f(x) = \sin x$, show that $f(3x) = 3f(x) - 4f^3(x)$	02
	Ans	$3f(x) - 4f^3(x)$	½
		$= 3\sin x - 4\sin^3 x$	
		$= \sin 3x$	1
		$= f(3x)$	½
		<i>OR</i>	
		$f(3x)$	



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1.	b)	$= \sin 3x$ $= 3 \sin x - 4 \sin^3 x$ $= 3f(x) - 4f^3(x)$	<p>1/2</p> <p>1</p> <p>1/2</p>
	c)	Find $\frac{dy}{dx}$ if $y = e^x \sin^{-1} x$	02
	Ans	$y = e^x \sin^{-1} x$ $\therefore \frac{dy}{dx} = e^x \frac{1}{\sqrt{1-x^2}} + \sin^{-1} x e^x$ $\therefore \frac{dy}{dx} = e^x \left(\frac{1}{\sqrt{1-x^2}} + \sin^{-1} x \right)$	1+1
	d)	Evaluate: $\int x(x-1)^2 dx$	02
	Ans	$\int x(x-1)^2 dx$ $= \int x(x^2 - 2x + 1) dx$ $= \int (x^3 - 2x^2 + x) dx$ $= \frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2} + c$	<p>1/2</p> <p>1/2</p> <p>1</p>
	e)	Evaluate: $\int \sin^2 2x dx$	02
Ans	$\int \sin^2 2x dx$ $= \frac{1}{2} \int 2 \sin^2 2x dx$ $= \frac{1}{2} \int (1 - \cos 4x) dx$ $= \frac{1}{2} \left(x - \frac{\sin 4x}{4} \right) + c$	<p>1/2</p> <p>1/2</p> <p>1</p>	
f)	Find the area bounded by the curve $y = x^2$, x -axis and ordinates $x = 0$ to $x = 3$	02	



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1.	f)	Area $A = \int_a^b y \, dx$	
	Ans	$= \int_0^3 x^2 \, dx$ $= \left[\frac{x^3}{3} \right]_0^3$ $= \left(\frac{3^3}{3} - 0 \right)$ $= 9$	<p>½</p> <p>½</p> <p>½</p> <p>½</p>
	g)	Express $z = \frac{1-i}{1+i}$ in $a+ib$ form, where $i = \sqrt{-1}$ and a, b are real number	02
	Ans	$z = \frac{1-i}{1+i}$ $\therefore z = \frac{1-i}{1+i} \times \frac{1-i}{1-i}$ $\therefore z = \frac{1-2i+i^2}{1^2-i^2}$ $\therefore z = \frac{1-2i-1}{1+1}$ $\therefore z = \frac{-2i}{2}$ $\therefore z = -i = 0-i$	<p>½</p> <p>½</p> <p>½</p> <p>½</p>
2.		Attempt any <u>THREE</u> of the following:	12
	a)	If $13x^2 + 2x^2y + y^3 = 1$, find $\frac{dy}{dx}$ at $(1, -2)$	04
	Ans	$13x^2 + 2x^2y + y^3 = 1$ $\therefore 26x + 2\left(x^2 \frac{dy}{dx} + y2x\right) + 3y^2 \frac{dy}{dx} = 0$ $\therefore 26x + 2x^2 \frac{dy}{dx} + 4xy + 3y^2 \frac{dy}{dx} = 0$	1



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2.	a)	$\therefore 2x^2 \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = -26x - 4xy$ $\therefore (2x^2 + 3y^2) \frac{dy}{dx} = -26x - 4xy$ $\therefore \frac{dy}{dx} = \frac{-26x - 4xy}{2x^2 + 3y^2}$ at (1, -2) $\frac{dy}{dx} = \frac{-18}{14} = \frac{-9}{7}$	1 1 1
	b)	If $x = a(\theta + \sin \theta)$, $y = a(1 - \cos \theta)$, find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{2}$	04
	Ans	$x = a(\theta + \sin \theta) \qquad y = a(1 - \cos \theta)$ $\frac{dx}{d\theta} = a(1 + \cos \theta) \qquad \frac{dy}{d\theta} = a(0 + \sin \theta) = a \sin \theta$ $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a \sin \theta}{a(1 + \cos \theta)}$ $\frac{dy}{dx} = \frac{\sin \theta}{1 + \cos \theta}$ at $\theta = \frac{\pi}{2}$ $\therefore \frac{dy}{dx} = \frac{\sin \frac{\pi}{2}}{1 + \cos \frac{\pi}{2}} = 1$	1+1 1
c)	The rate of working of an engine is given by the expression $10V + \frac{4000}{V}$, where V is the speed of the engine. Find the speed at which the rate of working is the least.	04	
Ans	The rate of working is, $W = 10V + \frac{4000}{V}$ $\therefore \frac{dW}{dV} = 10 - \frac{4000}{V^2}$	½	



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2.	c)	$\therefore \frac{d^2W}{dV^2} = \frac{8000}{V^3}$ <p>Consider $\frac{dW}{dV} = 0$</p> $\therefore 10 - \frac{4000}{V^2} = 0$ $\therefore 10 = \frac{4000}{V^2}$ $\therefore V^2 = 400$ $\therefore V = 20, -20$ <p>at $V = 20$</p> $\therefore \frac{d^2W}{dV^2} = \frac{8000}{(20)^3} = 1 > 0$ <p>\therefore The speed is $V = 20$ at which the rate of working is least</p>	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>1</p>
	d)	<p>A telegraph wire hangs in the form of a curve $y = 2 \sin x - \sin 2x$. Find the radius of curvature of the wire at the point $x = \frac{\pi}{2}$</p> <p>Ans $y = 2 \sin x - \sin 2x$</p> $\therefore \frac{dy}{dx} = 2 \cos x - 2 \cos 2x$ $\therefore \frac{d^2y}{dx^2} = -2 \sin x + 4 \sin 2x$ <p>at $x = \frac{\pi}{2}$</p> $\therefore \frac{dy}{dx} = 2 \cos\left(\frac{\pi}{2}\right) - 2 \cos 2\left(\frac{\pi}{2}\right) = 2$ $\therefore \frac{d^2y}{dx^2} = -2 \sin\left(\frac{\pi}{2}\right) + 4 \sin 2\left(\frac{\pi}{2}\right) = -2$ $\therefore \text{Radius of curvature} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{\left[1 + (2)^2\right]^{\frac{3}{2}}}{-2}$	<p>04</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p>



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2.	d)	\therefore Radius of curvature $= -5.59$ i.e. 5.59	1
3.		Solve any <u>THREE</u> of the following:	12
	a)	Find the equation of the tangent to the curve $y = 9x^2 - 12x + 7$ which is parallel to the x -axis	04
	Ans	$y = 9x^2 - 12x + 7$ $\therefore \frac{dy}{dx} = 18x - 12$ tangent is parallel to the x -axis $\therefore \frac{dy}{dx} = 0$ $\therefore 18x - 12 = 0$ $\therefore x = \frac{2}{3}$ $\therefore y = 3$ $\therefore (x_1, y_1) = \left(\frac{2}{3}, 3\right)$ \therefore slope of tangent, $m = 0$ Equation of tangent at $\left(\frac{2}{3}, 3\right)$ is $y - 3 = 0\left(x - \frac{2}{3}\right)$ $\therefore y - 3 = 0$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$
	b)	Find $\frac{dy}{dx}$ if $y = \log\left(\frac{\sin x}{1 + \cos x}\right)$	04
	Ans	$y = \log\left(\frac{\sin x}{1 + \cos x}\right)$ $\therefore \frac{dy}{dx} = \frac{1}{\frac{\sin x}{1 + \cos x}} \cdot \frac{d}{dx}\left(\frac{\sin x}{1 + \cos x}\right)$ $\therefore \frac{dy}{dx} = \frac{1 + \cos x}{\sin x} \left(\frac{(1 + \cos x)\cos x - \sin x(0 - \sin x)}{(1 + \cos x)^2}\right)$	1 1



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3.	b)	$\therefore \frac{dy}{dx} = \frac{1}{\sin x} \left(\frac{\cos x + \cos^2 x + \sin^2 x}{1 + \cos x} \right)$	1
		$\therefore \frac{dy}{dx} = \frac{1}{\sin x} (\cos x + 1)$	1
		$\therefore \frac{dy}{dx} = \frac{1}{\sin x} = \operatorname{cosec} x$	
		<i>OR</i>	
		$y = \log \left(\frac{\sin x}{1 + \cos x} \right)$	
		$\therefore y = \log \left(\frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \right)$	1
		$\therefore y = \log \left(\tan \frac{x}{2} \right)$	1
		$\therefore \frac{dy}{dx} = \frac{1}{\tan \frac{x}{2}} \left(\sec^2 \frac{x}{2} \right) \left(\frac{1}{2} \right)$	2
		$\therefore \frac{dy}{dx} = \frac{\cos \frac{x}{2}}{2 \sin \frac{x}{2} \cos^2 \frac{x}{2}} = \frac{1}{2 \sin \frac{x}{2} \cos \frac{x}{2}}$	
		$\therefore \frac{dy}{dx} = \frac{1}{\sin x} = \operatorname{cosec} x$	
	<i>OR</i>		
	$y = \log \left(\frac{\sin x}{1 + \cos x} \right)$		
	$\therefore y = \log (\sin x) - \log (1 + \cos x)$	1	
	$\therefore \frac{dy}{dx} = \frac{1}{\sin x} \cos x - \frac{1}{1 + \cos x} (-\sin x)$	1	
	$\therefore \frac{dy}{dx} = \frac{\cos x (1 + \cos x)}{\sin x} + \frac{\sin x}{1 + \cos x}$	½	
	$\therefore \frac{dy}{dx} = \frac{\cos x + \cos^2 x + \sin^2 x}{\sin x (1 + \cos x)}$	1	



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3.	b)	$\therefore \frac{dy}{dx} = \frac{\cos x + 1}{\sin x(1 + \cos x)} = \frac{1}{\sin x} = \operatorname{cosec} x$	½
	c)	If $x^y = e^{x-y}$, then prove that $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$	04
	Ans	$x^y = e^{x-y}$ $\log x^y = \log e^{x-y}$ $y \log x = (x - y) \log e$ $\therefore y \log x = x - y$ $\therefore y \log x + y = x$ $\therefore y(\log x + 1) = x$ $\therefore y = \frac{x}{\log x + 1}$ $\therefore \frac{dy}{dx} = \frac{(\log x + 1) \frac{d}{dx}(x) - x \frac{d}{dx}(\log x + 1)}{(\log x + 1)^2}$ $\therefore \frac{dy}{dx} = \frac{(\log x + 1) \cdot 1 - x \left(\frac{1}{x} + 0 \right)}{(\log x + 1)^2}$ $\therefore \frac{dy}{dx} = \frac{\log x + 1 - 1}{(\log x + 1)^2}$ $\therefore \frac{dy}{dx} = \frac{\log x}{(\log x + 1)^2}$	½ ½ ½ ½ ½ 1
d)	Evaluate: $\int \frac{\cos x}{1 + \sin^2 x} dx$	04	
	Ans	Put $\sin x = t$ $\therefore \cos x dx = dt$ $= \int \frac{dt}{1 + t^2}$ $= \tan^{-1} t + c$ $= \tan^{-1}(\sin x) + c$	1 1 1 1



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4.		<p>Solve any <u>THREE</u> of the following:</p> <p>a) Evaluate: $\int \frac{\log x}{x(2 + \log x)} \frac{dx}{(3 + \log x)}$</p> <p>Ans $\int \frac{\log x}{x(2 + \log x)(3 + \log x)} dx$</p> <p>Put $\log x = t$</p> <p>$\therefore \frac{1}{x} dx = dt$</p> <p>$\int \frac{t}{(2+t)(3+t)} dt$</p> <p>consider $\frac{t}{(2+t)(3+t)} = \frac{A}{2+t} + \frac{B}{3+t}$</p> <p>$\therefore t = A(3+t) + B(2+t)$</p> <p>Put $t = -2$</p> <p>$A = -2$</p> <p>Put $t = -3$</p> <p>$B = 3$</p> <p>$\therefore \frac{t}{(2+t)(3+t)} = \frac{-2}{2+t} + \frac{3}{3+t}$</p> <p>$\therefore \int \frac{t}{(2+t)(3+t)} dt = \int \left(\frac{-2}{2+t} + \frac{3}{3+t} \right) dt$</p> <p>$= -2 \log(2+t) + 3 \log(3+t) + c$</p> <p>$= -2 \log(2 + \log x) + 3 \log(3 + \log x) + c$</p>	<p>12</p> <p>04</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p>
4.		<p>b) Evaluate: $\int \frac{dx}{3 - 2 \sin x}$</p> <p>Ans $\int \frac{dx}{3 - 2 \sin x}$</p> <p>Put $\tan \frac{x}{2} = t$, $dx = \frac{2dt}{1+t^2}$, $\sin x = \frac{2t}{1+t^2}$</p> <p>$\int \frac{\frac{2dt}{1+t^2}}{3 - 2 \left(\frac{2t}{1+t^2} \right)}$</p>	<p>04</p> <p>1</p>



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4.	b)	$= 2 \int \frac{dt}{3(1+t^2) - 2(2t)}$ $= 2 \int \frac{dt}{3+3t^2-4t}$ $= 2 \int \frac{dt}{3t^2-4t+3}$ $= \frac{2}{3} \int \frac{dt}{t^2 - \frac{4}{3}t + 1}$ $T.T. = \left(\frac{1}{2} \times \frac{4}{3}\right)^2 = \frac{4}{9}$ $= \frac{2}{3} \int \frac{dt}{t^2 - \frac{4}{3}t + \frac{4}{9} - \frac{4}{9} + 1}$ $= \frac{2}{3} \int \frac{dt}{\left(t - \frac{2}{3}\right)^2 + \frac{5}{9}}$ $= \frac{2}{3} \int \frac{dt}{\left(t - \frac{2}{3}\right)^2 + \left(\frac{\sqrt{5}}{3}\right)^2}$ $= \frac{2}{3} \frac{1}{\frac{\sqrt{5}}{3}} \tan^{-1} \left(\frac{t - \frac{2}{3}}{\frac{\sqrt{5}}{3}} \right) + c$ $= \frac{2}{\sqrt{5}} \tan^{-1} \left(\frac{\tan\left(\frac{x}{2}\right) - \frac{2}{3}}{\frac{\sqrt{5}}{3}} \right) + c$	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>
	c)	<p>Evaluate: $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$</p>	04
Ans		$\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$	



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4.	c)	Put $\sin^{-1} x = t \quad \therefore x = \sin t$ $\therefore \frac{1}{\sqrt{1-x^2}} dx = dt$ $\therefore \int \sin t \cdot t \cdot dt$ $= \int t \sin t \cdot dt$ $= t(-\cos t) - \int (-\cos t) \cdot 1 dt$ $= -t \cos t + \int \cos t \cdot dt$ $= -t \cos t + \sin t + c$ $= -\sin^{-1} x \cos(\sin^{-1} x) + x + c$ OR $-\sin^{-1} x \sqrt{1-x^2} + x + c$	1 1 1 1
	d)	Evaluate: $\int \frac{x+1}{x^2(x-2)} dx$	04
	Ans	$\int \frac{x+1}{x^2(x-2)} dx$ Consider $\frac{x+1}{x^2(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$ $\therefore x+1 = Ax(x-2) + B(x-2) + Cx^2$ Put $x = 0$ $\therefore B = -\frac{1}{2}$ Put $x = 2$ $\therefore C = \frac{3}{4}$ Put $x = 1$ $2 = -A - B + C$ $\therefore 2 = -A + \frac{1}{2} + \frac{3}{4}$ $\therefore A = \frac{-3}{4}$ $\frac{x+1}{x^2(x-2)} = \frac{-3}{4x} + \frac{-1}{x^2} + \frac{3}{x-2}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$



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4.	d)	$\therefore \int \frac{x+1}{x^2(x-2)} dx = \int \left(\frac{-3}{x} + \frac{-1}{x^2} + \frac{3}{x-2} \right) dx$ $\therefore \int \frac{x+1}{x^2(x-2)} dx = \frac{-3}{4} \log x + \frac{1}{2x} + \frac{3}{4} \log(x-2) + c$	<p>1/2</p> <p>1</p>
	e)	<p>Evaluate: $\int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5} + \sqrt[3]{9-x}} dx$</p> <p>Ans $I = \int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5} + \sqrt[3]{9-x}} dx$ ----- (1)</p> $I = \int_1^3 \frac{\sqrt[3]{(1+3-x)+5}}{\sqrt[3]{(1+3-x)+5} + \sqrt[3]{9-(1+3-x)}} dx$ $\therefore I = \int_1^3 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x} + \sqrt[3]{x+5}} dx$ ----- (2) <p>add (1) and (2)</p> $I + I = \int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5} + \sqrt[3]{9-x}} dx + \int_1^3 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x} + \sqrt[3]{x+5}} dx$ $2I = \int_1^3 \frac{\sqrt[3]{x+5} + \sqrt[3]{9-x}}{\sqrt[3]{9-x} + \sqrt[3]{x+5}} dx$ $2I = \int_1^3 1 dx$ $2I = [x]_1^3$ $2I = 3 - 1$ $I = 1$	<p>04</p> <p>1</p> <p>1</p> <p>1/2</p> <p>1</p>
5.		<p>Solve any TWO of the following:</p> <p>a) Find the area enclosed between the parabola $y = x^2$ and the line $y = 4$.</p>	<p>12</p>
	Ans	$y = x^2$ $4 = x^2$ $\therefore x = \pm 2$	<p>04</p> <p>1/2</p>



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5.	a)	$\therefore A = \int_{-2}^2 (x^2 - 4)$ $A = \left(\frac{x^3}{3} - 4x \right)_{-2}^2$ $A = \left(\frac{(2)^3}{3} - 4(2) \right) - \left(\frac{(-2)^3}{3} - 4(-2) \right)$ $\therefore A = \frac{16}{3} - 16$ $\therefore A = \frac{32}{3} \text{ or } 10.667$	<p>1/2</p> <p>1</p> <p>1</p> <p>1</p>
	b)	Attempt the following:	06
	(i)	Find the order and degree of the differential equation	02
	Ans	$\frac{d^2 y}{dx^2} = \left(y + \frac{dy}{dx} \right)^{3/2}$ $\frac{d^2 y}{dx^2} = \left(y + \frac{dy}{dx} \right)^{3/2}$ <p>squaring</p> $\left(\frac{d^2 y}{dx^2} \right)^2 = \left(y + \frac{dy}{dx} \right)^3$ <p>Order of D.E. = 2</p> <p>Degree of D.E. = 2</p>	<p>1</p> <p>1</p>
ii)	Solve: $x \frac{dy}{dx} - y = x^2$	04	
Ans	$x \frac{dy}{dx} - y = x^2$ <p>Divide by x</p> $\frac{dy}{dx} - \frac{y}{x} = x$ <p>\therefore Comparing with $\frac{dy}{dx} + Py = Q$</p>	1/2	



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5.	b)(ii)	$P = \frac{-1}{x}, Q = x$ <p>Integrating factor $IF = e^{\int \frac{-1}{x} dx} = e^{-\log x} = \frac{1}{x}$</p> $y \cdot IF = \int Q \cdot IF dx + c$ $y \frac{1}{x} = \int x \cdot \frac{1}{x} dx$ $\frac{y}{x} = \int 1 dx$ $\frac{y}{x} = x + c$	<p>1/2</p> <p>1</p> <p>1</p> <p>1</p>
	c)	<p>The current 'i' is given by $L \frac{di}{dt} = 30 \sin(10\pi t)$, where L is inductance and t is time. Find 'i' in terms of t, given that $L = 2$ and $i = 0$ at $t = 0$</p>	04
	Ans	$L di = 30 \sin(10\pi t) dt$ $\int L di = \int 30 \sin(10\pi t) dt$ $Li = 30 \left(\frac{-\cos(10\pi t)}{10\pi} \right) + c$ $Li = \frac{-3 \cos(10\pi t)}{\pi} + c$ <p>at $t = 0, i = 0$</p> $L(0) = \frac{-3 \cos(0)}{\pi} + c$ $0 = \frac{-3}{\pi} + c$ $\therefore c = \frac{3}{\pi}$ $\therefore Li = \frac{-3 \cos(10\pi t)}{\pi} + \frac{3}{\pi}$ <p>at $L = 2$</p> $\therefore 2i = \frac{-3 \cos(10\pi t)}{\pi} + \frac{3}{\pi}$	<p>1/2</p> <p>2</p> <p>1/2</p> <p>1</p> <p>1/2</p> <p>1/2</p>



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5.	c)	$\therefore i = \frac{3}{2\pi}(-\cos(10\pi t) + 1)$	1
6.		Solve any TWO of the following:	12
	a)	Attempt the following:	06
	(i)	If $z_1 = -3 + 4i$, $z_2 = 5 - 3i$ express $\frac{z_1}{z_2}$ in $x + iy$ form.	03
	Ans	$\frac{z_1}{z_2} = \frac{-3 + 4i}{5 - 3i}$ $\therefore \frac{z_1}{z_2} = \frac{-3 + 4i}{5 - 3i} \times \frac{5 + 3i}{5 + 3i}$ $\therefore \frac{z_1}{z_2} = \frac{-15 - 9i + 20i + 12i^2}{25 - 9i^2}$ $\therefore \frac{z_1}{z_2} = \frac{-15 - 9i + 20i + 12(-1)}{25 - 9(-1)}$ $\therefore \frac{z_1}{z_2} = \frac{-27 + 11i}{34}$ $\therefore \frac{z_1}{z_2} = \frac{-27}{34} + \frac{11}{34}i$	<p>1/2</p> <p>1</p> <p>1/2</p> <p>1/2</p>
	(ii)	Find $L\{e^{-3t} \sin 2t\}$	03
	Ans	$L\{e^{-3t} \sin 2t\}$ $L\{\sin 2t\} = \frac{2}{s^2 + 2^2} = \frac{2}{s^2 + 4}$ $\therefore L\{e^{-3t} \sin 2t\} = \frac{2}{(s+3)^2 + 4}$ $\therefore L\{e^{-3t} \sin 2t\} = \frac{2}{s^2 + 6s + 9 + 4}$ $\therefore L\{e^{-3t} \sin 2t\} = \frac{2}{s^2 + 6s + 13}$	<p>1/2</p> <p>1</p> <p>1</p> <p>1/2</p>



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Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	b)	Find $L^{-1} \left\{ \frac{3s+1}{(s-1)(s^2+1)} \right\}$	06
	Ans	<p>Let</p> $\frac{3s+1}{(s-1)(s^2+1)} = \frac{A}{s-1} + \frac{Bs+C}{s^2+1}$ $3s+1 = (s^2+1)A + (s-1)(Bs+C)$ <p>Put $s = 1$</p> $\therefore A = 2$ <p>Put $s = 0$</p> $1 = A + (-1)C$ $\therefore 1 = 2 - C$ $\therefore C = 1$ <p>Put $s = -1$</p> $-2 = 2A + (-2)(-B+C)$ $\therefore -2 = 2(2) + 2B - 2(1)$ $\therefore -2 = 2 + 2B$ $\therefore B = -2$ $\therefore \frac{3s+1}{(s-1)(s^2+1)} = \frac{2}{s-1} + \frac{-2s+1}{s^2+1}$ $\therefore L^{-1} \left\{ \frac{3s+1}{(s-1)(s^2+1)} \right\} = 2L^{-1} \left\{ \frac{1}{s-1} \right\} - 2L^{-1} \left\{ \frac{s}{s^2+1} \right\} + L^{-1} \left\{ \frac{1}{s^2+1} \right\}$ $= 2e^t - 2\cos t + \sin t$	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1+1+1</p>
	c)	Solve the differential equation using Laplace transform.	06
	Ans	$L \frac{di}{dt} + Ri = V, i(0) = 0$ $L \frac{di}{dt} + Ri = V$ <p>Apply laplace transform on both sides,</p>	



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6.	c)	$\therefore L\left\{L\frac{di}{dt} + Ri\right\} = L\{V\}$ $\therefore L\left\{L\left(\frac{di}{dt}\right)\right\} + R\{L(i)\} = VL\{1\}$ $\therefore L\{sL(i) - i(0)\} + R\{L(i)\} = V\left(\frac{1}{s}\right)$ $\therefore L\{sL(i) - 0\} + R\{L(i)\} = V\left(\frac{1}{s}\right)$ $\therefore (Ls + R)L(i) = \frac{V}{s}$ $\therefore L(i) = \frac{V}{s(Ls + R)}$ $\therefore L(i) = \frac{V}{L} \frac{1}{s\left(s + \frac{R}{L}\right)}$ <p>Partial fraction is</p> $\frac{1}{s\left(s + \frac{R}{L}\right)} = \frac{A}{s} + \frac{B}{s + \frac{R}{L}}$ $1 = \left(s + \frac{R}{L}\right)A + sB$ <p>Put $s = 0$</p> $\therefore A = \frac{L}{R}$ <p>Put $s = -\frac{R}{L}$</p> $\therefore B = -\frac{L}{R}$ $\therefore \frac{1}{s\left(s + \frac{R}{L}\right)} = \frac{L}{R} \frac{1}{s} - \frac{L}{R} \frac{1}{s + \frac{R}{L}}$ $\therefore L(i) = \frac{V}{L} \frac{L}{R} \left(\frac{1}{s} - \frac{1}{s + \frac{R}{L}}\right)$	<p>1/2</p> <p>1</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>



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6.	c)	$\therefore i = \frac{V}{R} L^{-1} \left\{ \frac{1}{s} - \frac{1}{s + \frac{R}{L}} \right\}$ $\therefore i = \frac{V}{R} \left(1 - e^{-\frac{R}{L}t} \right)$ <p style="text-align: center;"><u>Important Note</u></p> <p><i>In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.</i></p>	<p style="text-align: center;">½</p> <p style="text-align: center;">½</p>